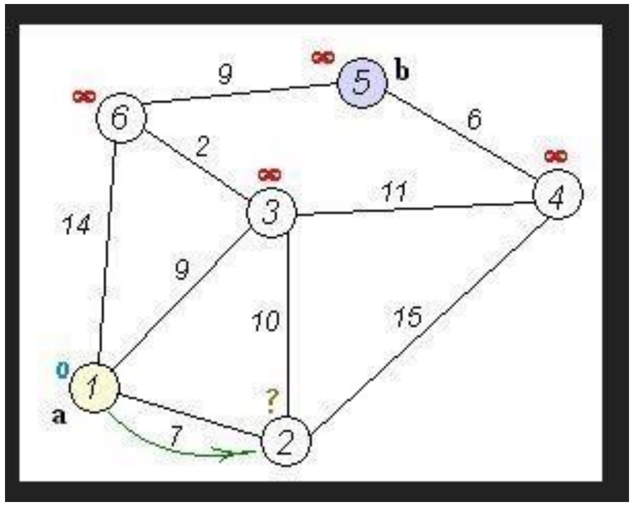
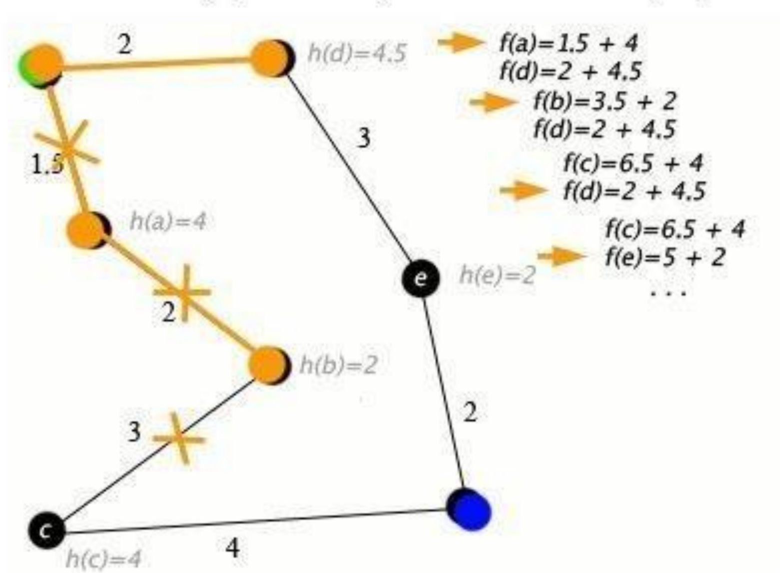


Dijkstra's Algorithm [shortest path]

Distance	Linked List
0	1 → 2 → 3 → 6 7 9 14
0	2 → 1 → 3 → 4 7 10 15
0	3 → 6 → 1 → 2 → 4 2 9 10 11
0	6 → 3 → 5 → 1 2 9 14

Example

An example of an A* algorithm in action where nodes are cities connected with roads and h(x) is the straight-line distance to target point:



A* –Algorithm

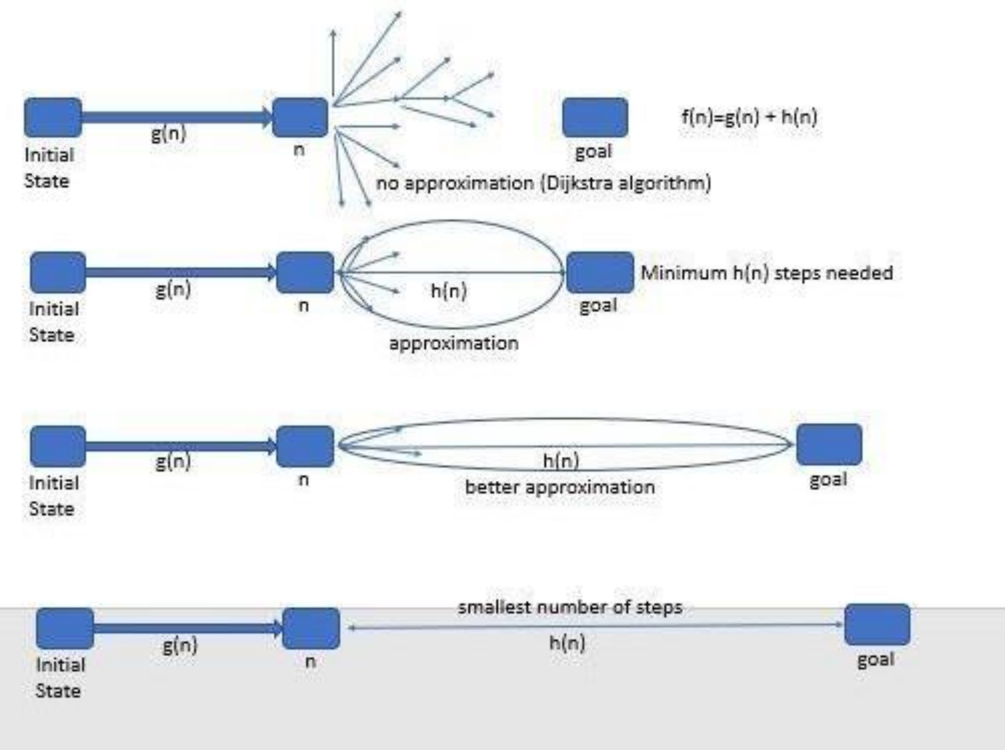
f(n) = g(n) + h(n) ƒexpand node with minimum f(n)ƒ

g(n) = cost to get to n from the root, **h(n)** = cost to get to goal from n (optimistic function)

f(n) never overestimates cost of a solution through n ƒit is **admissible**ƒ

h(n) S c(n, a, n') + h(n') is **monotonic** ƒf(n) is not decreasing along any pathƒ

Remark: A* is simplified to Dijkstra's algorithm if h(n)=O f(n) – **admissible & monotonic -> consistent**



Example (8-puzzle problem)

```
8  1  3      1  2  3     1  2  3  4  5  6  7  8     1  2  3  4  5  6  7  8
4     2      4  5  6     ----------------------     ----------------------
7  6  5      7  8        1  1  0  0  1  1  0  1     1  2  0  0  2  2  0  3

 initial       goal         Hamming = 5 + 0           Manhattan = 10 + 0
```

Option I (Hamming Distance)

```
c(x) = f(x) + h(x) where
   f(x) is the length of the path from root to x
        (the number of moves so far) and
   h(x) is the number of non-blank tiles not in
        their goal position (the number of mis-
        -placed tiles). There are at least h(x)
        moves to transform state x to a goal state
```

Option II (Manhattan Distance – better approximation)

The sum of the Manhattan distances (sum of the vertical and horizontal distance) from the blocks to their goal positions.

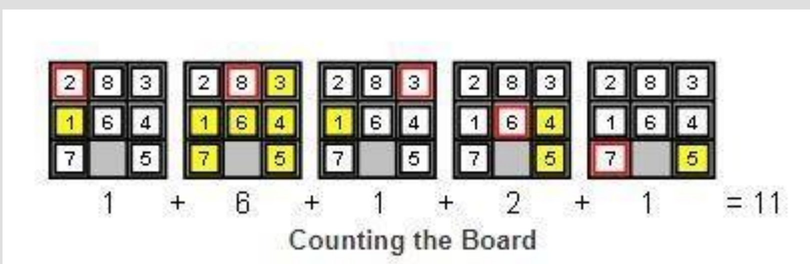
Option I & Option II – both admissible (do not overestimate the cost)

Some 8-puzzles are not solvable

```
 1  2  3         1  2  3  4
 4  5  6         5  6  7  8
 8  7            9 10 11 12
                13 15 14
unsolvable
                unsolvable
```

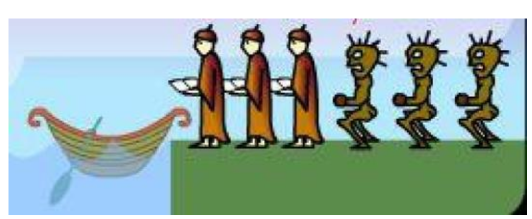
Interesting property

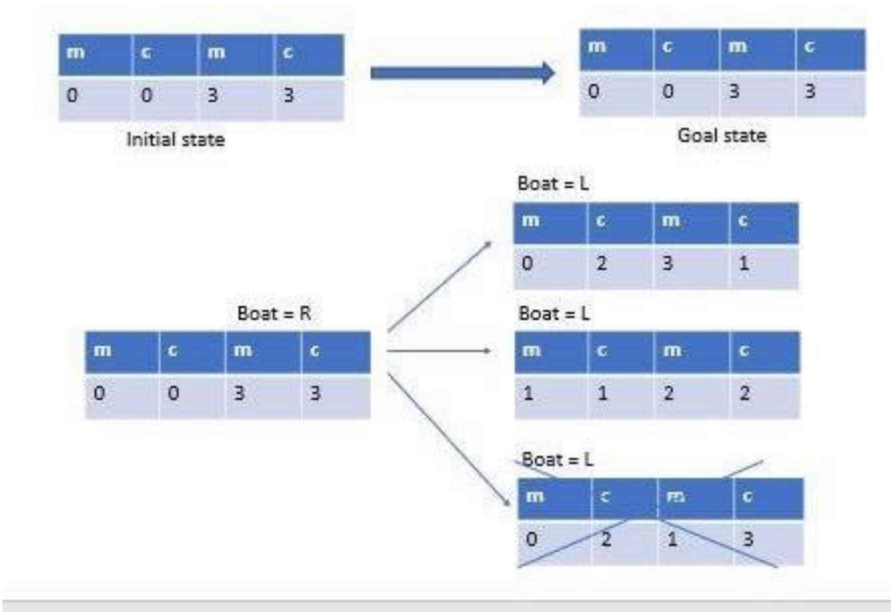
For every digit count the number of smaller digits which follow that digit.



Fact: If this number is odd, then the puzzle is solvable with goal state given below.

1	2	3
8		4
7	6	5

Game 2: [Cannibals & Missionaries] 



f(n)=g(n) + h(n) ; 2 traveling from right to left and 1 traveling from left to right

h(n) = 2 * (m(R) + c(R)) - 1, where

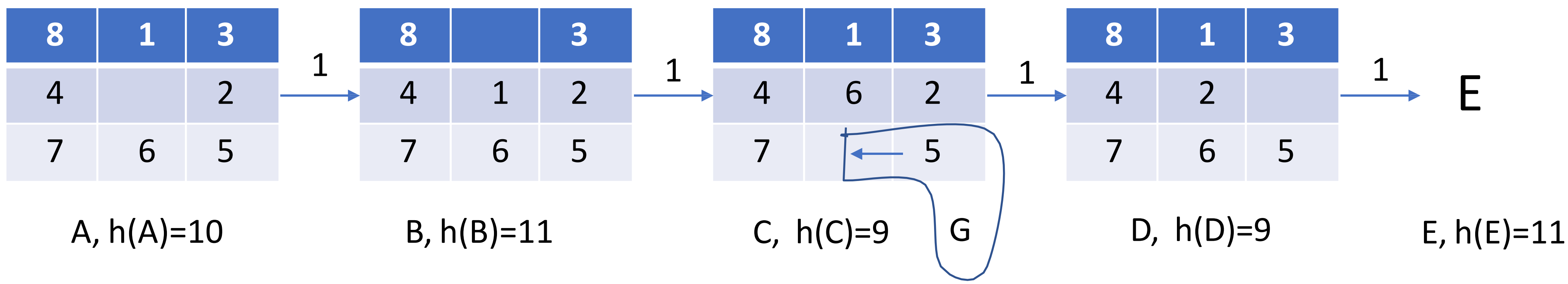
m(R) – number of missionaries on the right site of the river

c(R) – number of cannibals on the right site of the river

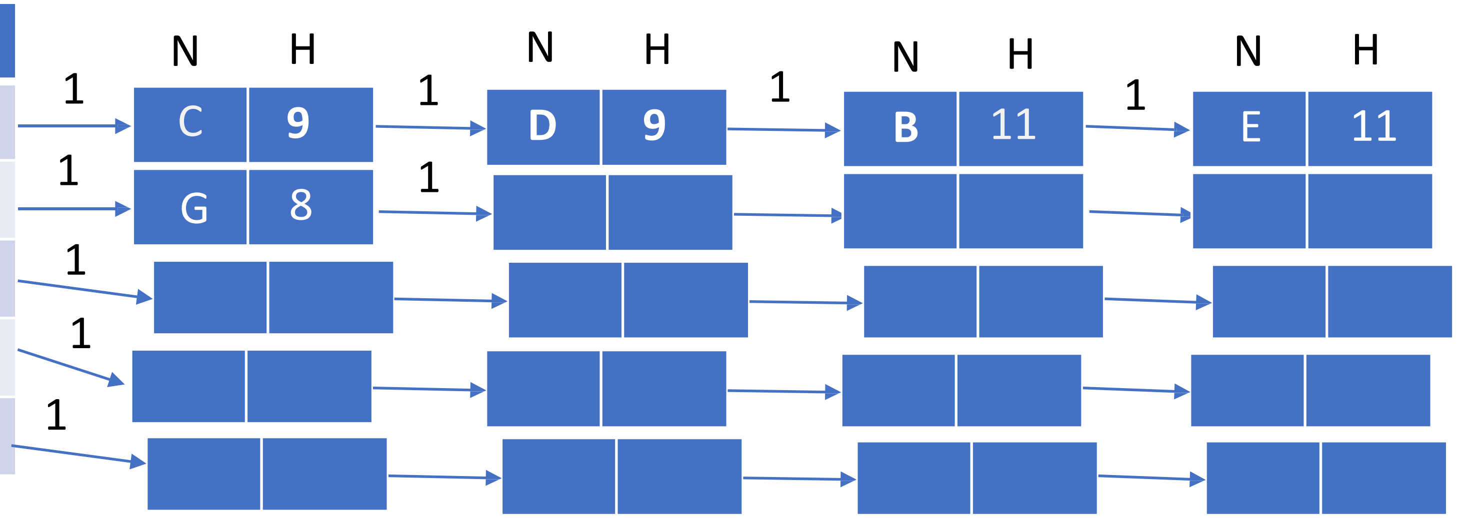
f(n) – admissible (can we find better approximation?)

New Problem: Farmer went to a market and purchased a fox, a goose, and a bag of beans. On his way home, he came to the bank of a river and rented a boat. But crossing the river by boat, the farmer could carry only himself and a single one of his purchases: the fox, the goose, or the bag of beans. If left unattended together, the fox would eat the goose, or the goose would eat the beans.

Eight Puzzle



N	g	h
A	0	10
C	1	9
D	1	9
B	1	11
G	2	8



1	2	3
4	5	6
7	8	

Goal State

H – Manhattan Distance, $f(n)=g(n)+h(n)$, A* - based on f(n)