Induction of Decision Trees

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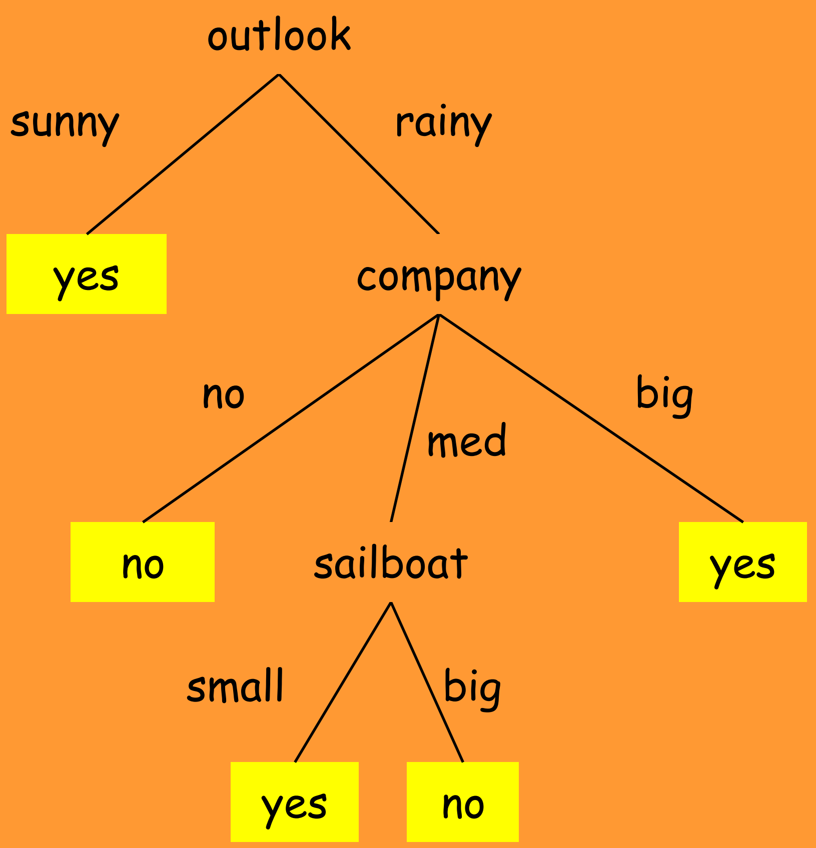
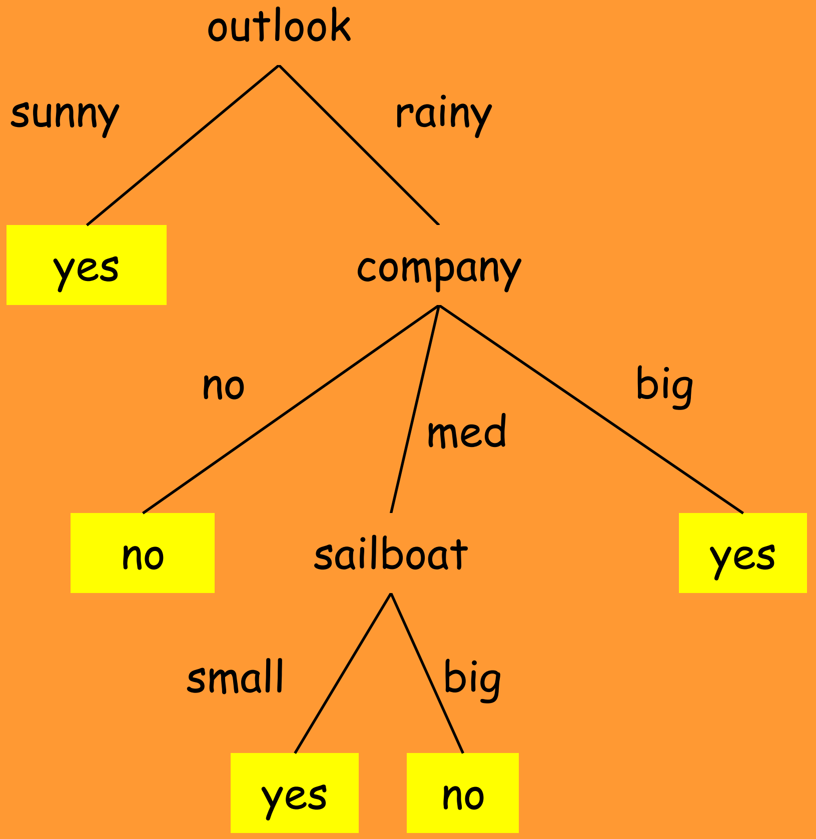
An Example Data Set and Decision Tree

#	*Attribute*			*Class*
	Outlook	Company	Sailboat	Sail?
1	sunny	big	small	yes
2	sunny	med	small	yes
3	sunny	med	big	yes
4	sunny	no	small	yes
5	sunny	big	big	yes
6	rainy	no	small	no
7	rainy	med	small	yes
8	rainy	big	big	yes
9	rainy	no	big	no
10	rainy	med	big	no



Classification

#	Attribute			Class
	Outlook	Company	Sailboat	Sail?
1	sunny	no	big	?
2	rainy	big	small	?



Induction of Decision Trees

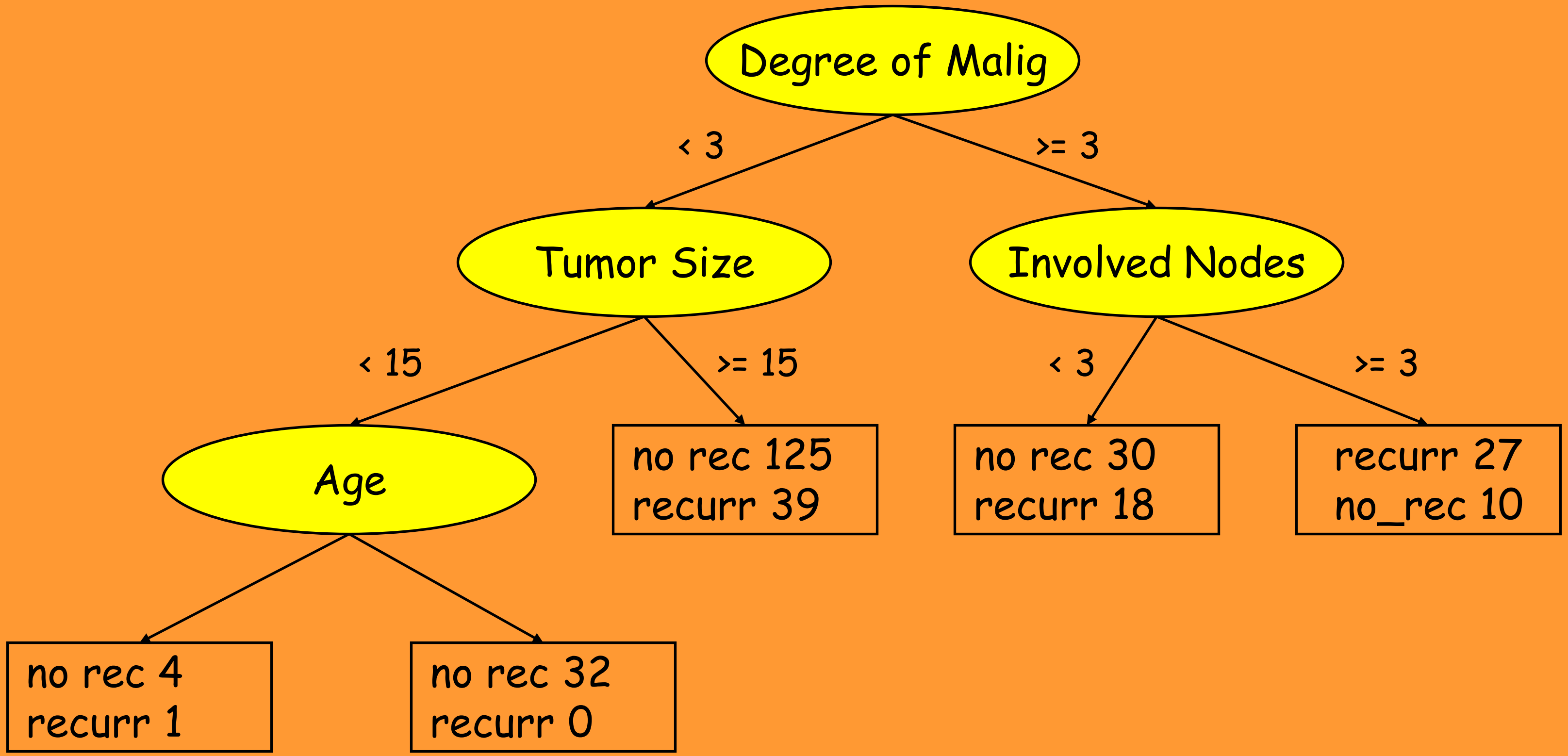
- Data Set (Learning Set)
  - Each example = Attributes + Class
- Induced description = Decision tree

- TDIDT
  - Top Down Induction of Decision Trees
- Recursive Partitioning

Some TDIDT Systems

- ID3 (Quinlan 79)
- CART (Brieman et al. 84)
- C4.5 (Quinlan 93)
- See5 (Quinlan 97)
- ...
- Orange (Demšar, Zupan 98-03)

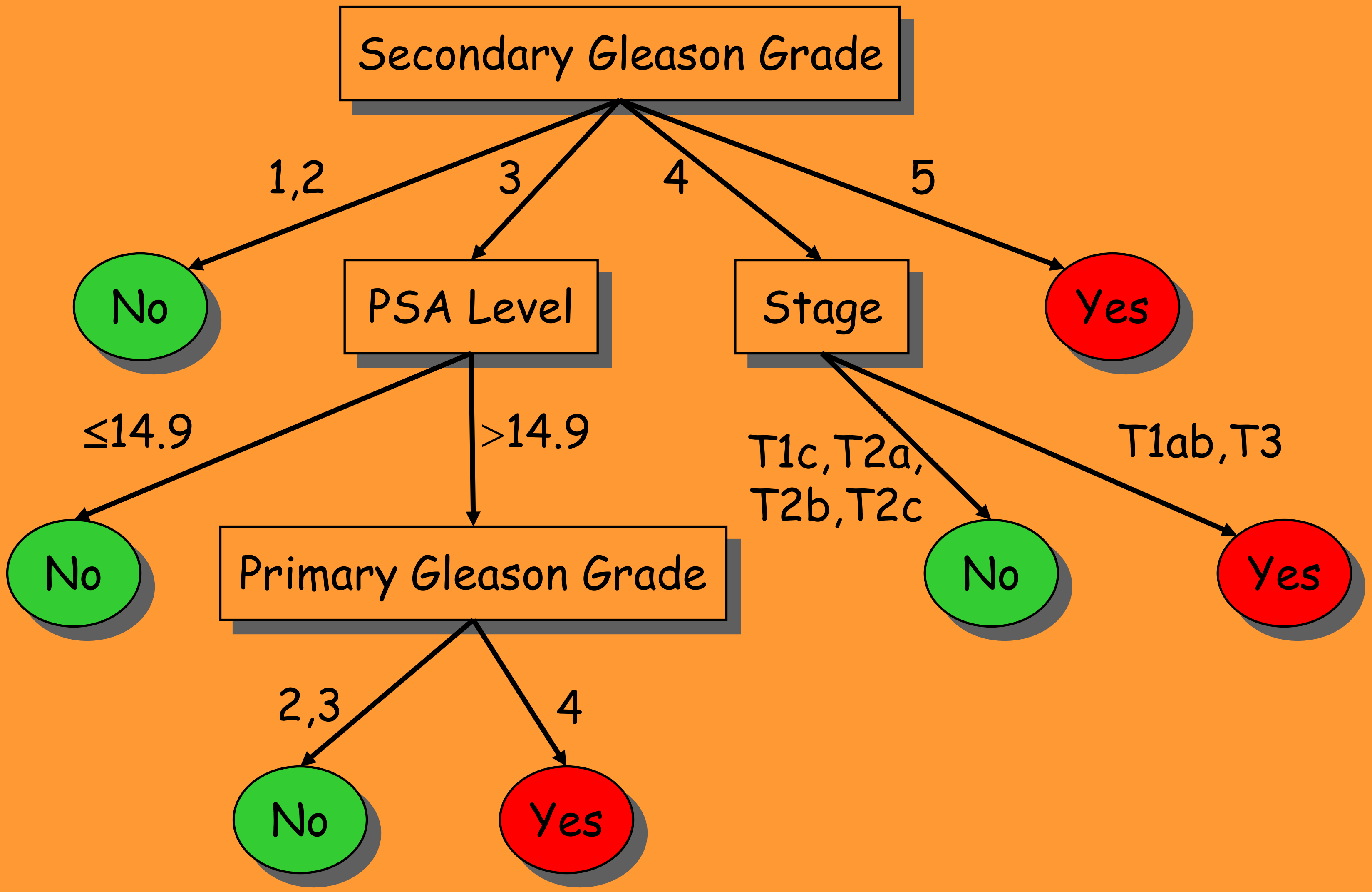
Breast Cancer Recurrence



Tree induced by Assistant Professional

Interesting: Accuracy of this tree compared to medical specialists

Prostate cancer recurrence

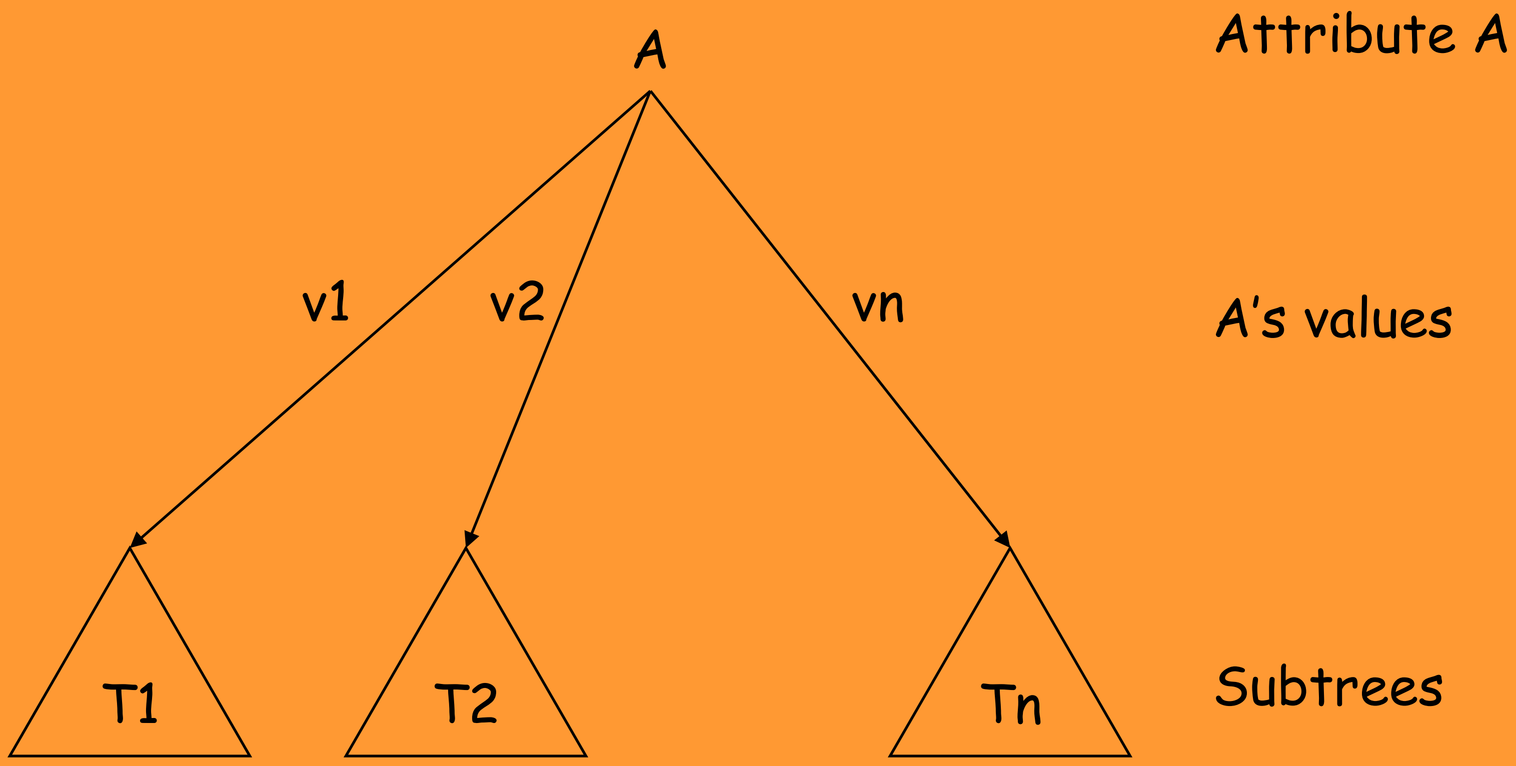


TDIDT Algorithm

- Also known as ID3 (Quinlan)
- To construct decision tree T from learning set S:
  - **If** all examples in S belong to some class C **Then** make leaf labeled C
  - **Otherwise**
    - select the "most informative" attribute A
    - partition S according to A's values
    - recursively construct subtrees T1, T2, ..., for the subsets of S

TDIDT Algorithm

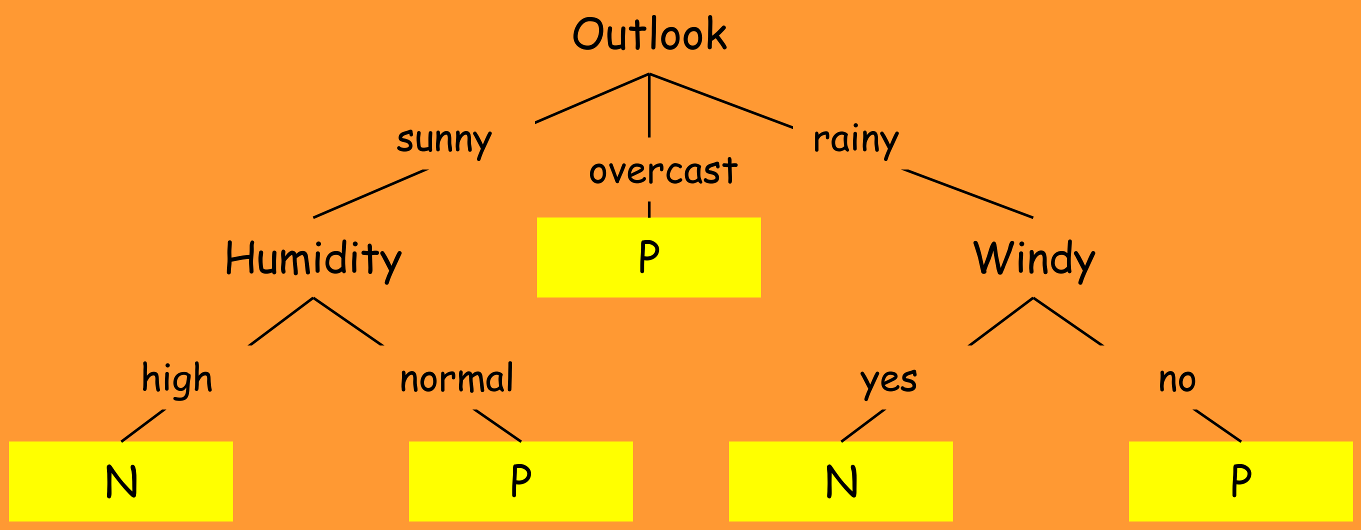
- Resulting tree T is:



Another Example

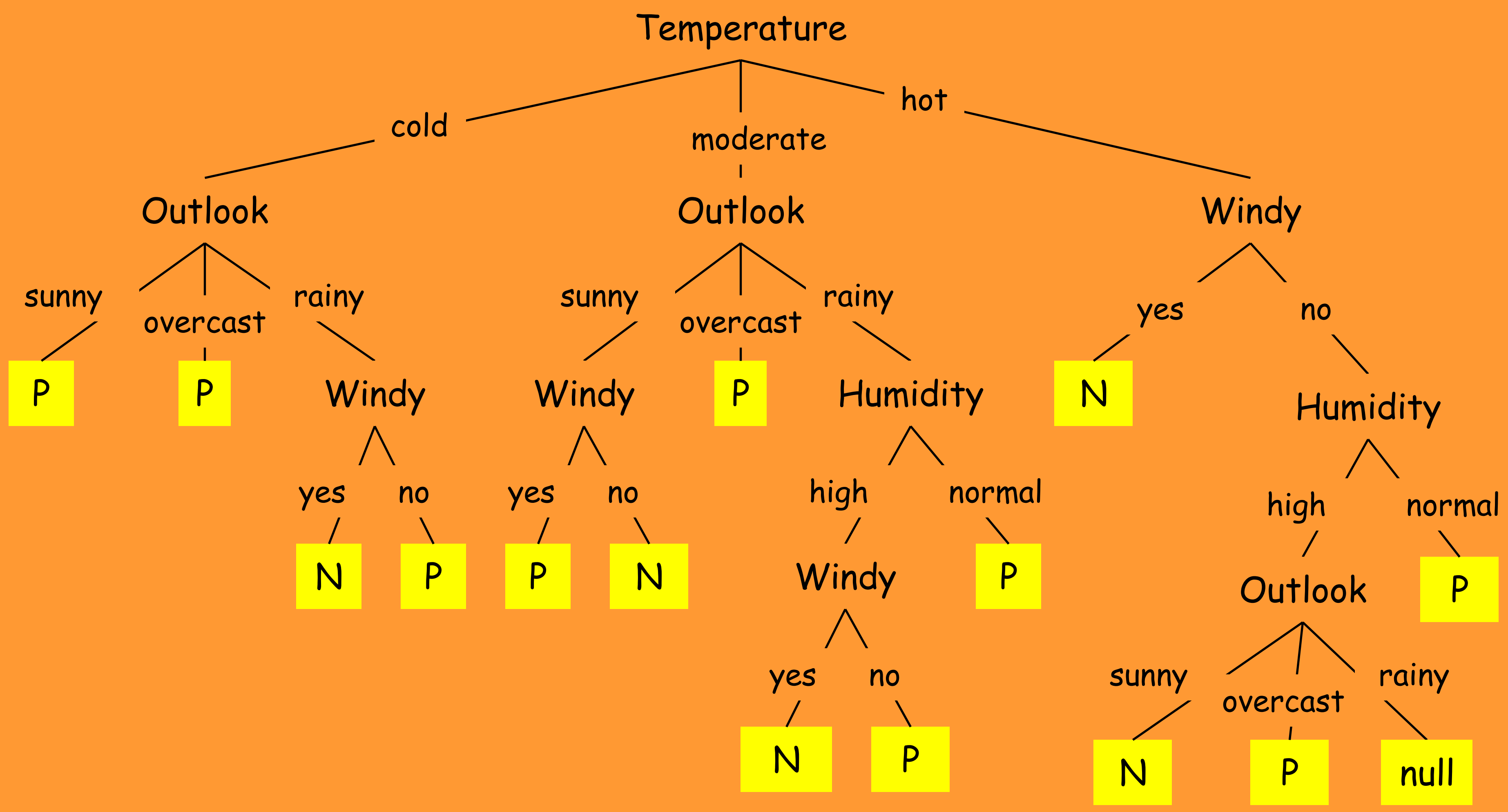
#	Attribute				Class
	Outlook	Temperature	Humidity	Windy	Play
1	sunny	hot	high	no	N
2	sunny	hot	high	yes	N
3	overcast	hot	high	no	P
4	rainy	moderate	high	no	P
5	rainy	cold	normal	no	P
6	rainy	cold	normal	yes	N
7	overcast	cold	normal	yes	P
8	sunny	moderate	high	no	N
9	sunny	cold	normal	no	P
10	rainy	moderate	normal	no	P
11	sunny	moderate	normal	yes	P
12	overcast	moderate	high	yes	P
13	overcast	hot	normal	no	P
14	rainy	moderate	high	yes	N

Simple Tree



Outlook
sunny
overcast
rainy
Humidity
P
Windy
high
normal
yes
no
N
P
N
P

Complicated Tree



Attribute Selection Criteria

- Main principle
  - Select attribute which partitions the learning set into subsets as "pure" as possible
- Various measures of purity
  - **Information-theoretic (entropy)**
  - **Gini index**
  - X^2 (entropy)
  - ReliefF
  - ...

Information-Theoretic Approach

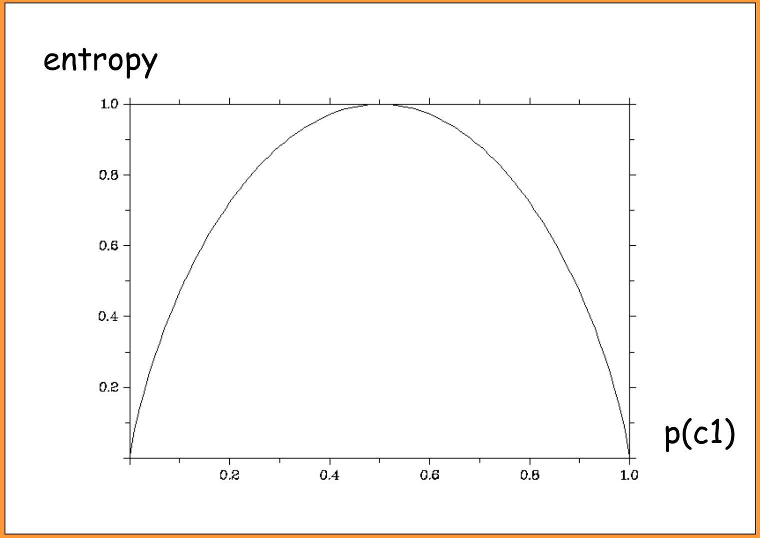
- To classify an object, a certain information is needed
  - I, information
- After we have learned the value of attribute *A*, we only need some remaining amount of information to classify the object
  - Ires, residual information
- Gain
  - Gain(*A*) = I – Ires(*A*)
- The most 'informative' attribute is the one that minimizes Ires, *i.e.*, maximizes Gain

Entropy

- The average amount of information *I* needed to classify an object is given by the entropy measure

$$I = -\sum_{c} p(c) \log_2 p(c)$$

- For a two-class problem:



Residual Information

- After applying attribute *A*, *S* is partitioned into subsets according to values *v* of *A*
- *Ires* is equal to weighted sum of the amounts of information for the subsets

$$I_{res} = -\sum_{v} p(v) \sum_{c} p(c|v) \log_2 p(c|v)$$

Triangles and Squares

#	Attribute			Shape
	Color	Outline	Dot	
1	green	dashed	no	triange
2	green	dashed	yes	triange
3	yellow	dashed	no	square
4	red	dashed	no	square
5	red	solid	no	square
6	red	solid	yes	triange
7	green	solid	no	square
8	green	dashed	no	triange
9	yellow	solid	yes	square
10	red	solid	no	square
11	green	solid	yes	square
12	yellow	dashed	yes	square
13	yellow	solid	no	square
14	red	dashed	yes	triange

Triangles and Squares

#	Attribute			Shape
	Color	Outline	Dot	
1	green	dashed	no	triange
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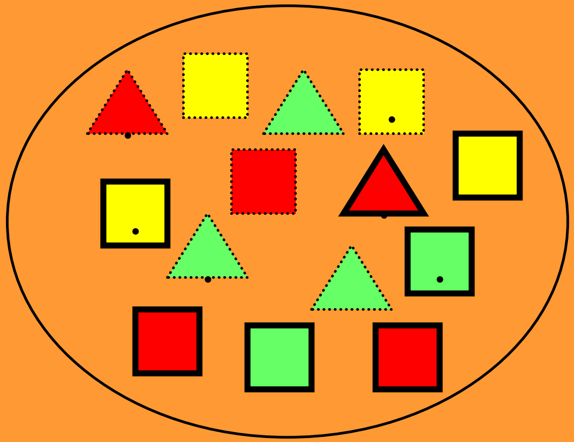
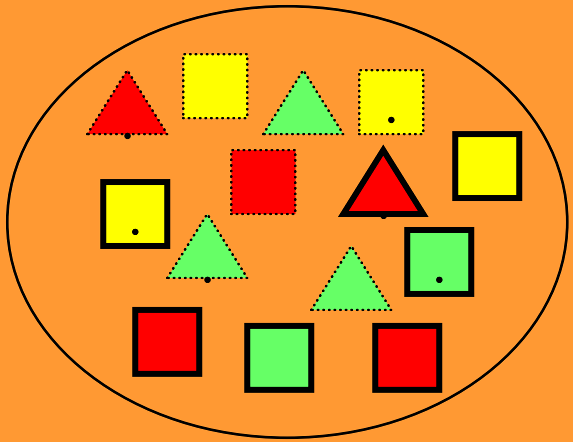
5	red	solid	no	square
6	red	solid	yes	triange
7	green	solid	no	square
8	green	dashed	no	triange
9	yellow	solid	yes	square
10	red	solid	no	square
11	green	solid	yes	square
12	yellow	dashed	yes	square
13	yellow	solid	no	square
14	red	dashed	yes	triange

Data Set:
A set of classified objects



Entropy



- 5 triangles
- 9 squares
- class probabilities

$$p(\square) = \frac{9}{14}$$

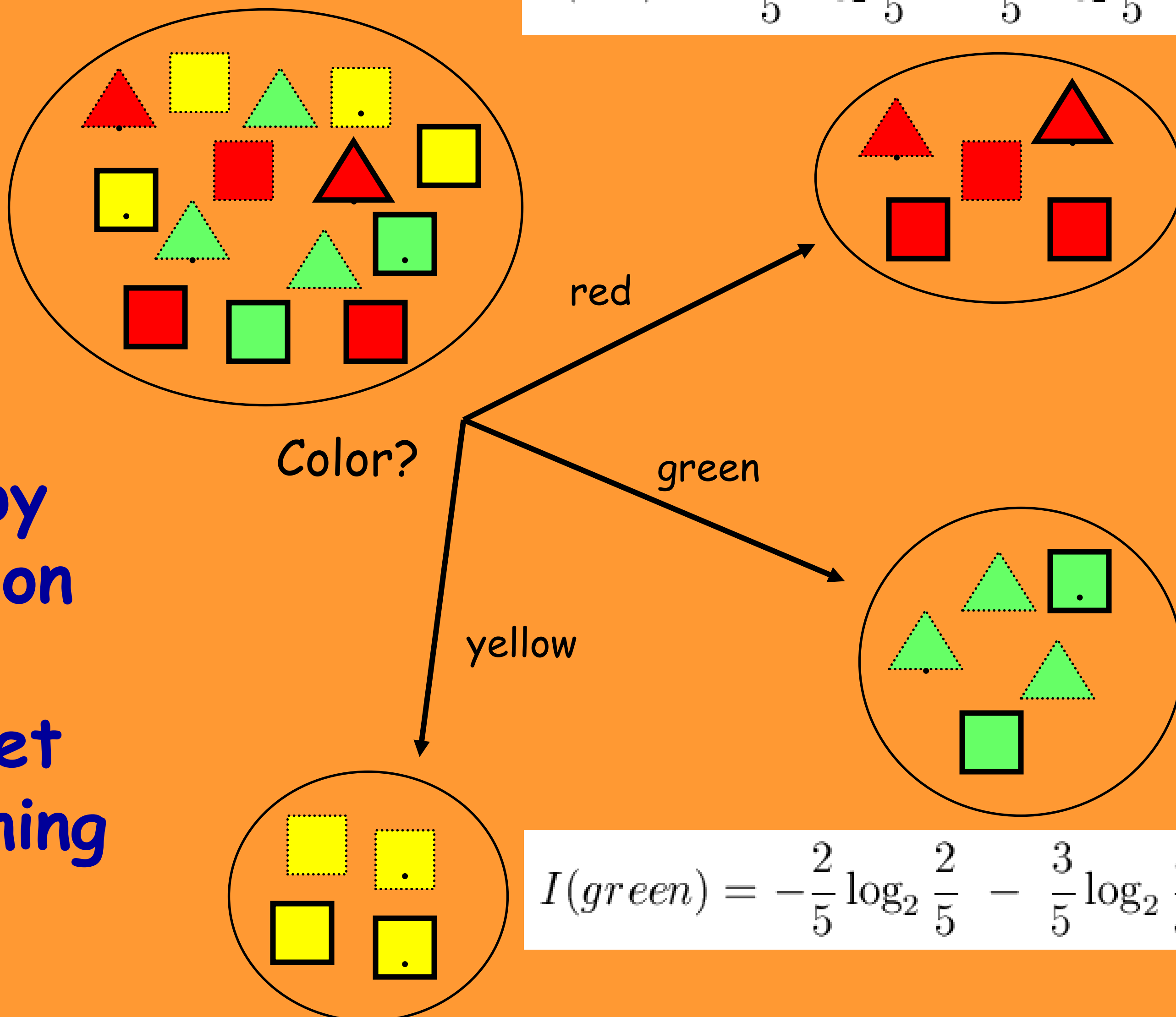
$$p(\triangle) = \frac{5}{14}$$

- entropy

$$I = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.940 \text{ bits}$$

Entropy reduction by data set partitioning

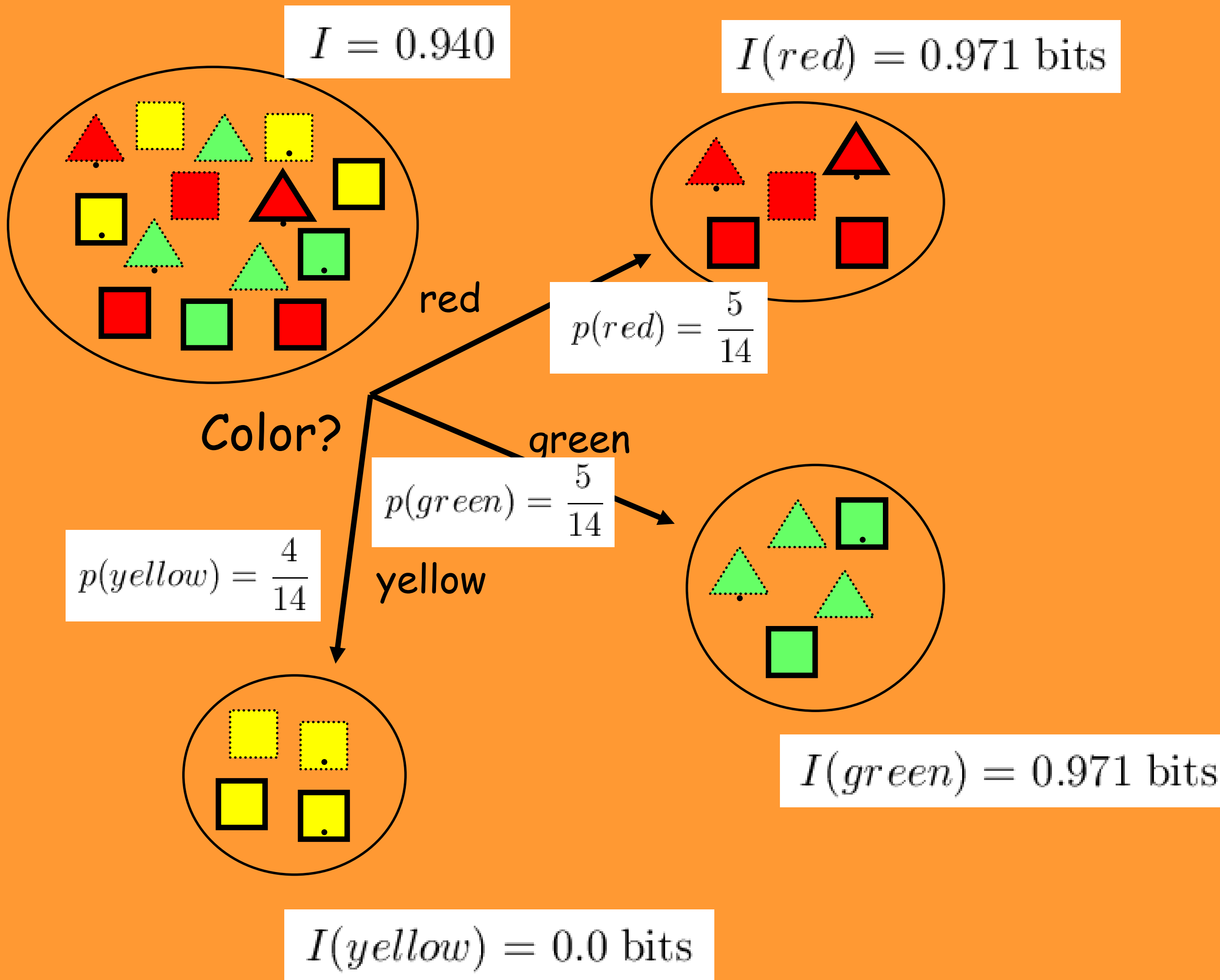
$$I(red) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5} = 0.971 \text{ bits}$$



$$I(green) = -\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5} = 0.971 \text{ bits}$$

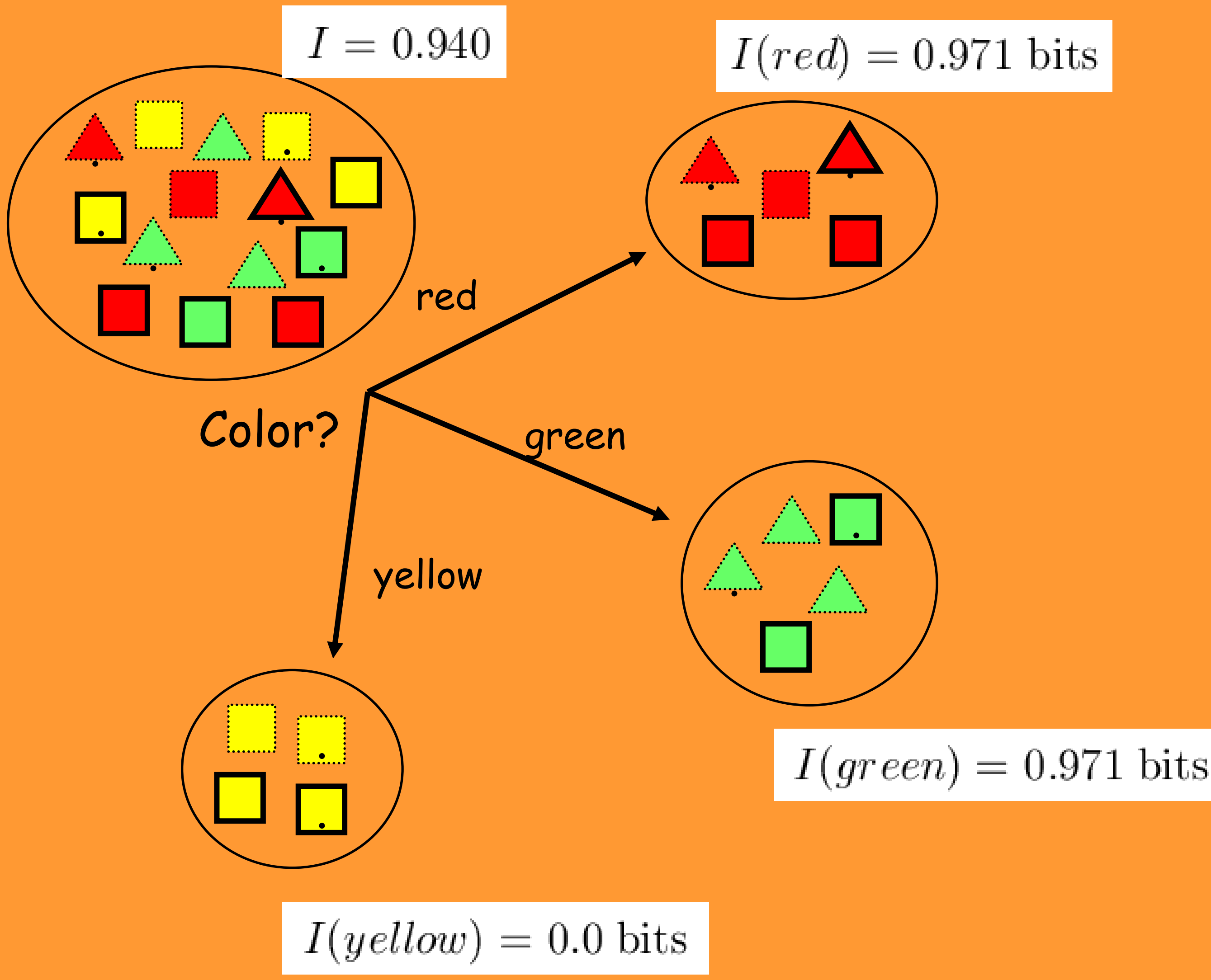
$$I(yellow) = -\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4} = 0.0 \text{ bits}$$

Entropija vrednosti atributa



$$I_{res}(\text{Color}) = \sum p(v)I(v) = \frac{5}{14}0.971 + \frac{5}{14}0.971 + \frac{4}{14}0.0 = 0.694 \ bits$$

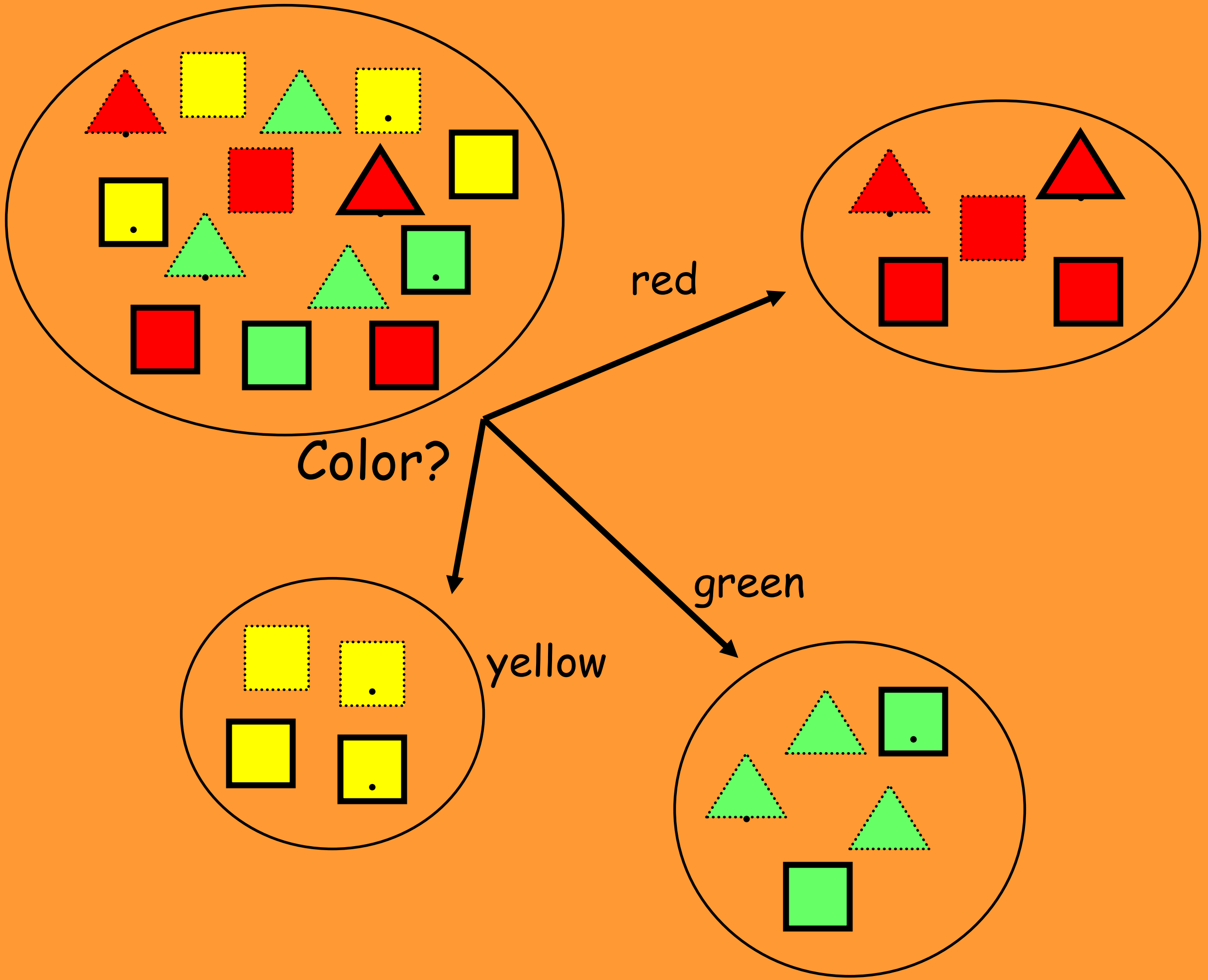
Information Gain



$$Gain(\text{Color}) = I - I_{res}(\text{Color}) = 0.940 - 0.694 = 0.246 \ bits$$

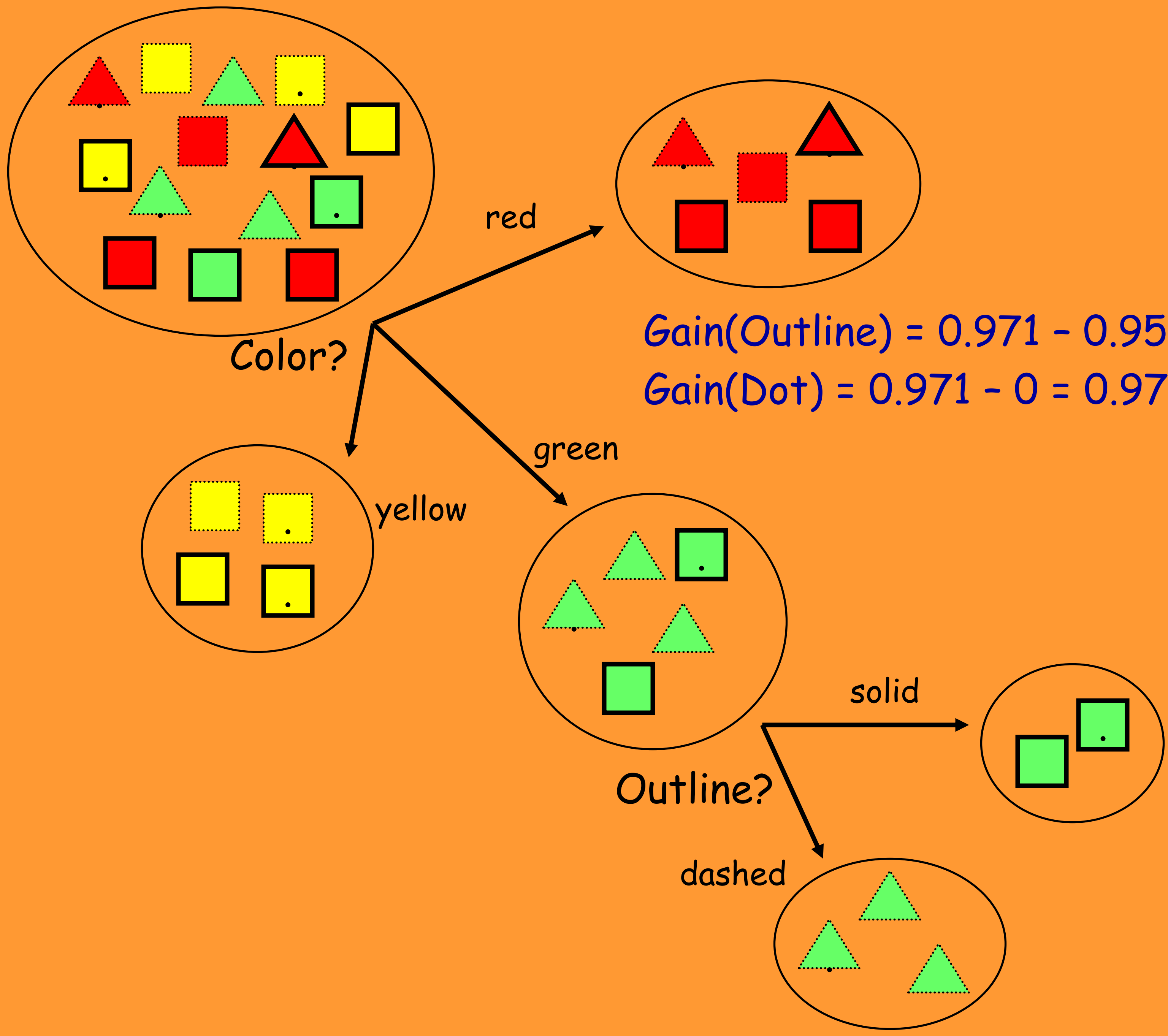
Information Gain of The Attribute

- Attributes
  - Gain(Color) = 0.246
  - Gain(Outline) = 0.151
  - Gain(Dot) = 0.048
- Heuristics: attribute with the highest gain is chosen
- This heuristics is local (local minimization of impurity)



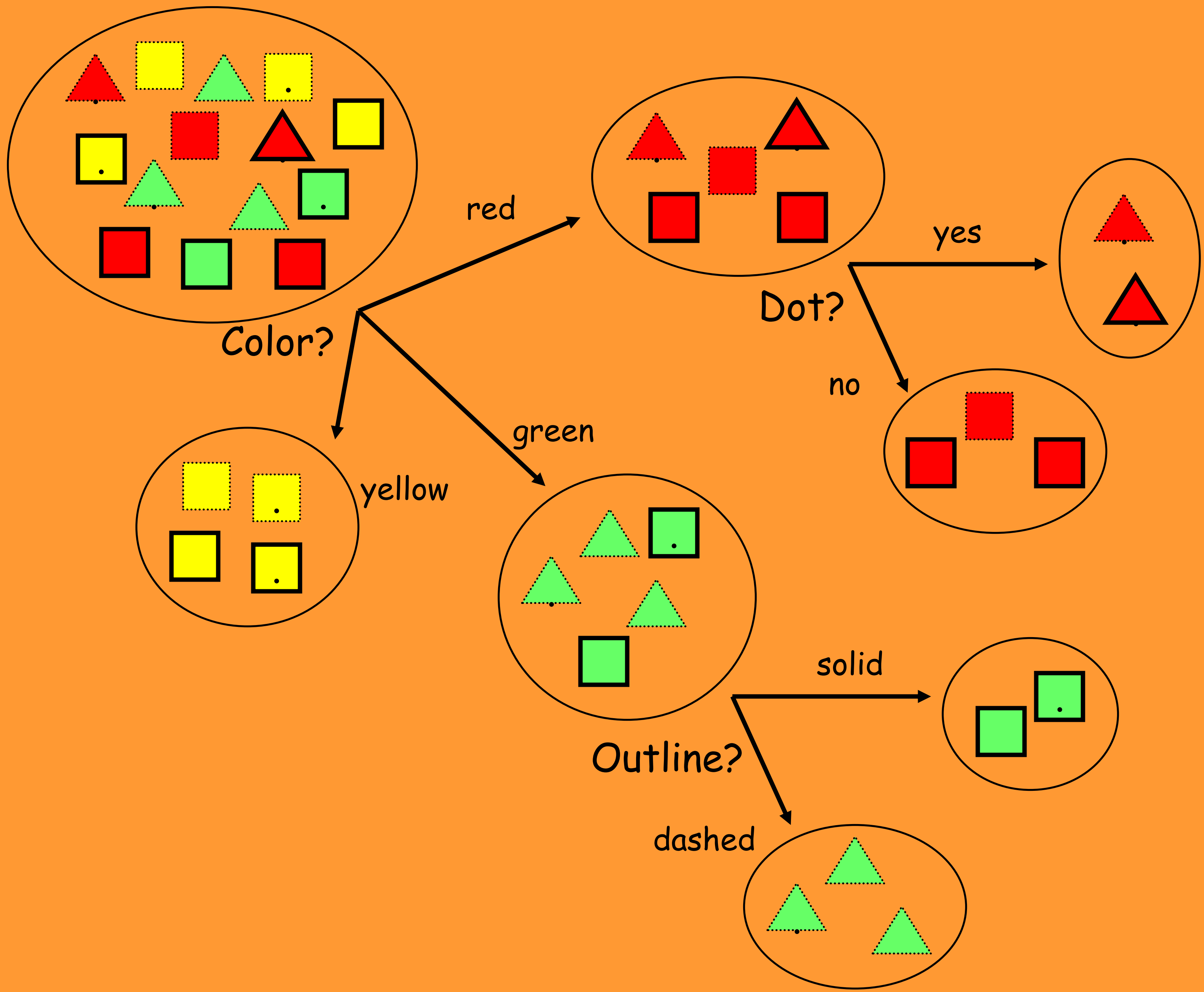
Gain(Outline) = 0.971 – 0 = 0.971 bits

Gain(Dot) = 0.971 – 0.951 = 0.020 bits

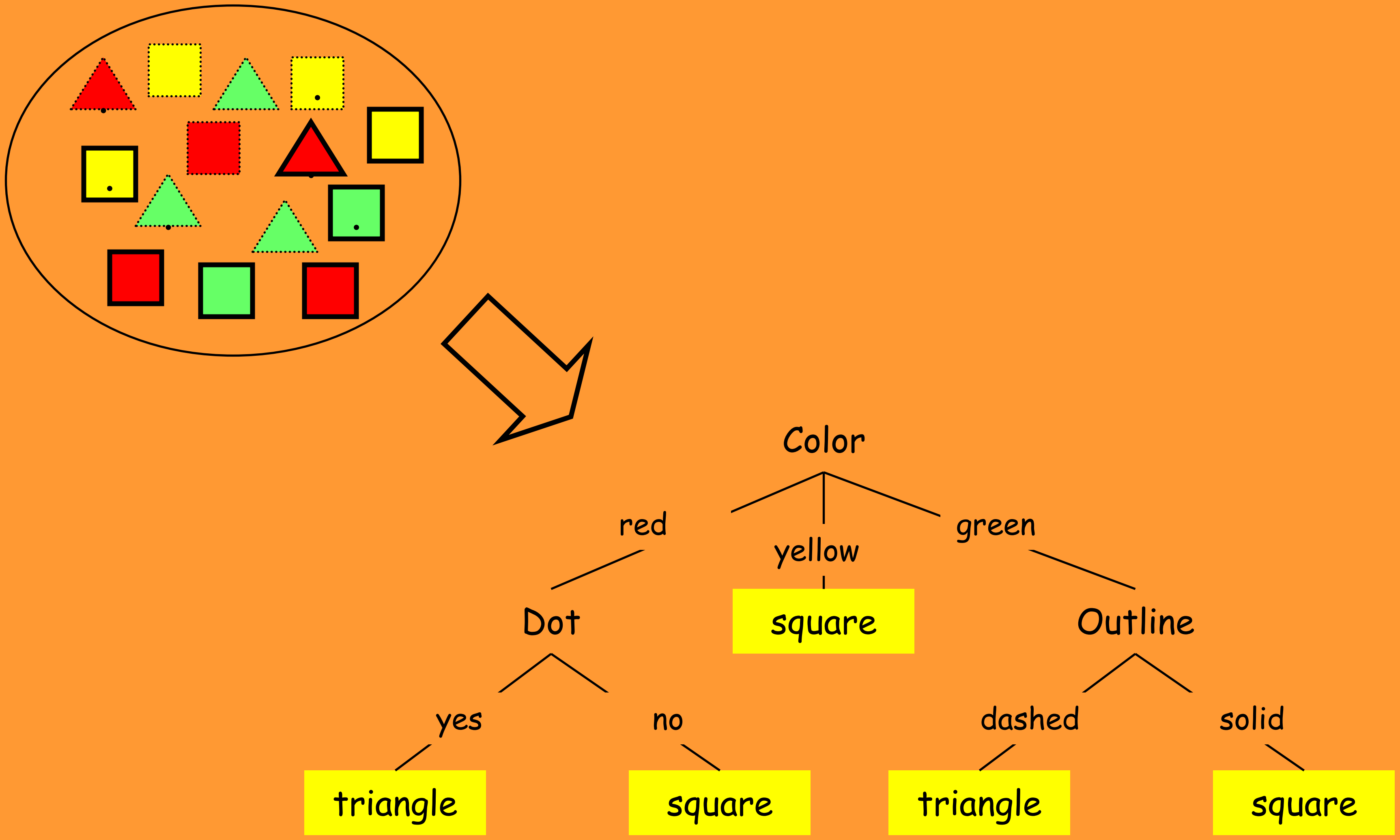


Gain(Outline) = 0.971 – 0.951 = 0.020 bits

Gain(Dot) = 0.971 – 0 = 0.971 bits


Color?
red
yellow
green
Dot?
yes
no
Outline?
solid
dashed

Decision Tree



Gini Index

- Another sensible measure of impurity (i and j are classes)

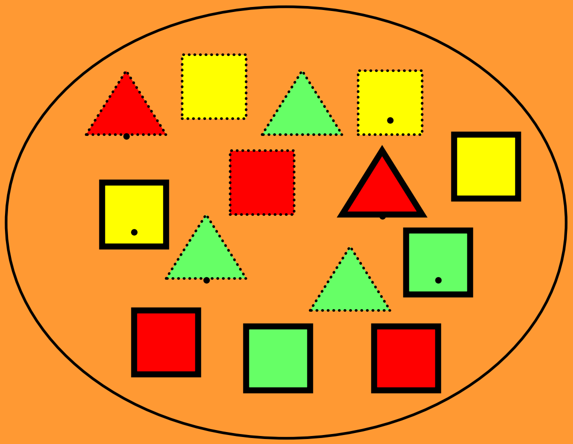
$$Gini = \sum_{i \neq j} p(i)p(j)$$

- After applying attribute A, the resulting Gini index is

$$Gini(A) = \sum_{v} p(v) \sum_{i \neq j} p(i|v)p(j|v)$$

- Gini can be interpreted as expected error rate

Gini Index



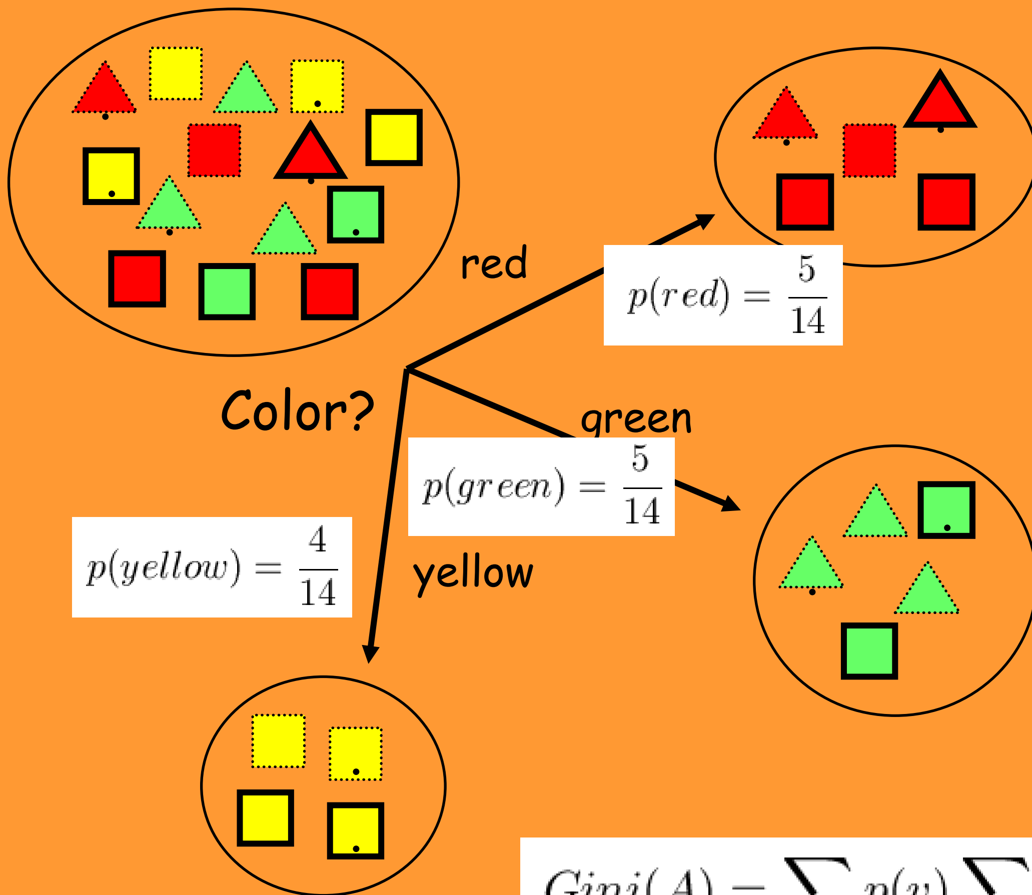
$$p(\square) = \frac{9}{14}$$

$$p(\triangle) = \frac{5}{14}$$

$$Gini = \sum_{i \neq j} p(i)p(j)$$

$$Gini = \frac{9}{14} \times \frac{5}{14} = 0.230$$

Gini Index for Color



$$Gini(A) = \sum_{v} p(v) \sum_{i \neq j} p(i|v)p(j|v)$$

$$Gini(\text{Color}) = \frac{5}{14} \times (\frac{3}{5} \times \frac{2}{5}) + \frac{5}{14} \times (\frac{2}{5} \times \frac{3}{5}) + \frac{4}{14} \times (\frac{4}{4} \times \frac{0}{4}) = 0.171$$

Gain of Gini Index

$$Gini = \frac{9}{14} \times \frac{5}{14} = 0.230$$

$$Gini(\text{Color}) = \frac{5}{14} \times (\frac{3}{5} \times \frac{2}{5}) + \frac{5}{14} \times (\frac{2}{5} \times \frac{3}{5}) + \frac{4}{14} \times (\frac{4}{4} \times \frac{0}{4}) = 0.171$$

$$\boxed{GiniGain(\text{Color}) = 0.230 - 0.171 = 0.058}$$

Three Impurity Measures

A	Gain(A)	GainRatio(A)	GiniGain(A)
Color	0.247	0.156	0.058
Outline	0.152	0.152	0.046
Dot	0.048	0.049	0.015

- These impurity measures assess the effect of a single attribute
- Criterion "most informative" that they define is local (and "myopic")
- It does not reliably predict the effect of several attributes applied jointly