Rule discovery strategy LERS

System LERS
*(**L**earning from **E**xamples based on **R**ough **S**ets)*

Input data is represented as a decision table.

In the decision table examples are described by values of *attributes* and characterized by a value of a *decision*.

All examples with the same value of the decision belong to the same *concept*.

This system looks for regularities in the decision table.

Algorithm LERS (Version LEM 2)

Let $S = (X, A, V)$ be the information system.

$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

X	a	b	c	d	f
x1	0	0	0	1	0
x2	0	1	1	1	1
x3	0	0	0	1	0
x4	0	1	1	1	1
x5	1	1	0	1	2
x6	1	1	0	1	2
x7	2	2	2	0	3
x8	2	2	2	0	3

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Classification attributes

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Decision attribute

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x8	2	2	2	0	3

$$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$$

The partitions of *X*, generated by single attributes are:

$$\{a\}^* = \{\{x_1, x_2, x_3, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\}$$

$$\{b\}^* = \{\{x_1, x_3\}, \{x_2, x_4, x_5, x_6\}, \{x_7, x_8\}\}$$

$$\{c\}^* = \{\{x_1, x_3, x_5, x_6\}, \{x_2, x_4\}, \{x_7, x_8\}\}$$

$$\{d\}^* = \{\{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_7, x_8\}\}$$

Let C be the set containing of one attribute {f}:

$$\{f\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\}$$

Algorithm LERS

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$$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$$

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$$\{a\}^* = \{\{x_1, x_2, x_3, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\}$$

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Let C be the set containing of one attribute {f}:

$$\{f\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\}$$

None of the sets is a subset of {f}*

Algorithm LERS

Let $S = (X, A, V)$ be the information system.

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$$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$$

forming two item sets:

$$\{a,b\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\} \subseteq \{f\}^*$$

$$\{a,c\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\} \subseteq \{f\}^*$$

$$\{a,d\}^* = \{\{x_1, x_2, x_3, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\} = \{a\}^*$$

$$\{b,c\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\} \subseteq \{f\}^*$$

$$\{b,d\}^* = b^*$$

$$\{c,d\}^* = \{\{x_1, x_3, x_5, x_6\}, \{x_2, x_4\}, \{x_7, x_8\}\} = \{c\}^*$$

$$\{f\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\}$$

Algorithm LERS

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$$X=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}$$

$\{a,b\}^*=\{\{x_1,x_3\},\{x_2,x_4\},\{x_5,x_6\},\{x_7,x_8\}\}\subseteq\{f\}^*$

$\{a,c\}^*=\{\{x_1,x_3\},\{x_2,x_4\},\{x_5,x_6\},\{x_7,x_8\}\}\subseteq\{f\}^*$

$\{a,d\}^*=\{\{x_1,x_2,x_3,x_4\},\{x_5,x_6\},\{x_7,x_8\}\}=\{a\}^*$

$\{b,c\}^*=\{\{x_1,x_3\},\{x_2,x_4\},\{x_5,x_6\},\{x_7,x_8\}\}\subseteq\{f\}^*$

$\{b,d\}^* = b^*$

$\{c,d\}^*=\{\{x_1,x_3,x_5,x_6\},\{x_2,x_4\},\{x_7,x_8\}\}=\{c\}^*$

marked

$$\{f\}^*=\{\{x_1,x_3\},\{x_2,x_4\},\{x_5,x_6\},\{x_7,x_8\}\}$$

Algorithm LERS

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$$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$$

$\{a,b\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\} \subseteq \{f\}^*$

$\{a,c\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\} \subseteq \{f\}^*$

$\{a,d\}^* = \{\{x_1, x_2, x_3, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\} = \{a\}^*$

$\{b,c\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\} \subseteq \{f\}^*$

$\{b,d\}^* = b^*$

$\{c,d\}^* = \{\{x_1, x_3, x_5, x_6\}, \{x_2, x_4\}, \{x_7, x_8\}\} = \{c\}^*$

marked, but not covering of f

$\{f\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\}$

Algorithm LERS

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$$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$$

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$$\{a,c\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\} \subseteq \{f\}^*$$

$$\{b,c\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\} \subseteq \{f\}^*$$

All of the sets are marked!

The coverings of C are:

$\{a, b\}$ $\{a, c\}$ $\{b, c\}$

How to find rules from coverings ?

Algorithm LERS

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x4	0	1	1	1	1
x5	1	1	0	1	2
x6	1	1	0	1	2
x7	2	2	2	0	3
x8	2	2	2	0	3

Covering {a,b}

$(a,0)^* = \{x_1, x_2, x_3, x_4\}$

$(a,1)^* = \{x_5, x_6\} \subseteq (f,2)^*$

$(a,2)^* = \{x_7, x_8\} \subseteq (f,3)^*$

$(b,0)^* = \{x_1, x_3\} \subseteq (f,0)^*$

$(b,1)^* = \{x_2, x_4, x_5, x_6\}$

$(b,2)^* = \{x_7, x_8\} \subseteq (f,3)^*$

marked

$\{f\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\}$

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Let $S = (X, A, V)$ be the information system.

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$(a,1)^* = \{x_5, x_6\} \subseteq (f,2)^*$

$(a,2)^* = \{x_7, x_8\} \subseteq (f,3)^*$

$(b,0)^* = \{x_1, x_3\} \subseteq (f,0)^*$

$(b,1)^* = \{x_2, x_4, x_5, x_6\}$

$(b,2)^* = \{x_7, x_8\} \subseteq (f,3)^*$

$((a,0) \wedge (b,1))^* = \{x_2, x_4\} \subseteq (f,1)^*$

marked

$\{f\}^* = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\}$

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Let $S = (X, A, V)$ be the information system.

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Covering {a,b}

Certain rules, obtained from marked items:

$$(a,1) \to (f,2)$$
$$(a,2) \to (f,3)$$
$$(b,0) \to (f,0)$$
$$(b,2) \to (f,3)$$
$$(a,0) \wedge (b,1) \to (f,1)$$

$$(a,1)^* = \{x_5, x_6\} \subseteq (f,2)^*$$
$$(a,2)^* = \{x_7, x_8\} \subseteq (f,3)^*$$
$$(b,0)^* = \{x_1, x_3\} \subseteq (f,0)^*$$
$$(b,2)^* = \{x_7, x_8\} \subseteq (f,3)^*$$
$$((a,0) \wedge (b,1))^* = \{x_2, x_4\} \subseteq (f,1)^*$$

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Covering {a,b}

Possible rules, obtained from non-marked items:

$(a,0) \rightarrow (f,0)$ with confidence ½

$(a,0) \rightarrow (f,1)$ with confidence ½

$(b,1) \rightarrow (f,1)$ with confidence ½

$(b,1) \rightarrow (f,2)$ with confidence ½

$(a,1)^* = \{x_5, x_6\} \subseteq (f,2)^*$

$(a,2)^* = \{x_7, x_8\} \subseteq (f,3)^*$

$(b,0)^* = \{x_1, x_3\} \subseteq (f,0)^*$

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