

Sample Midterm Exam problems.

Problem 1

Reduce to CNF the following formulas:

- $p \rightarrow (q \wedge r)$
- $(p \vee q) \rightarrow r$
- $\neg(\neg p \vee q) \vee (r \rightarrow \neg s)$
- $\neg((p \rightarrow (q \rightarrow r))) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$
- $p \vee (\neg q \wedge (r \rightarrow \neg p))$
- $\neg((((a \rightarrow b)) \rightarrow a) \rightarrow a)$
- $\neg(a \vee (a \rightarrow b))$

Solutions:

$$[1] p \rightarrow (q \wedge r) = \neg p \vee (q \wedge r) = (\neg p \vee q) \wedge (\neg p \vee r)$$

$$[2] (p \vee q) \rightarrow r = \neg(p \vee q) \vee r = (\neg p \wedge \neg q) \vee r = (\neg p \vee r) \wedge (\neg q \vee r)$$

$$[3] \neg(\neg p \vee q) \vee (r \rightarrow \neg s) = (p \wedge \neg q) \vee (\neg r \vee \neg s) = (p \vee \neg r \vee \neg s) \wedge (\neg q \vee \neg r \vee \neg s)$$

$$[5] p \vee (\neg q \wedge (r \rightarrow \neg p)) = p \vee (\neg q \wedge (\neg r \vee \neg p)) = (p \vee \neg q) \wedge (p \vee \neg r \vee \neg p)$$

Problem 2

Anyone whom Mary loves is a football star. Any student who does not pass does not play. John is a student. Any student who does not study does not pass. Anyone who does not play is not a football star.

Predicates: $M(x)$ – [Mary loves x], $F(x)$ – [x is a football star], $P(x)$ – [x passed the exam], $G(x)$ – [x play football], $S(x)$ – [x is a student], $St(x)$ – [x studies].

Prove that: If John does not study, then Mary does not love John.

Formulas:

$$M(x) \rightarrow F(x)$$

$$[S(x) \wedge \neg P(x)] \rightarrow \neg G(x)$$

$$S(J)$$

$$[S(x) \wedge \neg St(x)] \rightarrow \neg P(x)$$

$$\neg G(x) \rightarrow \neg F(x)$$

$$\text{Prove that: } \neg St(J) \rightarrow \neg M(J)$$

Convert to set of clauses.

$$\mathbf{KB} = \{\neg M(x) \vee F(x), \sim S(x) \vee P(x) \vee \sim G(x), S(J), \sim S(x) \vee St(x) \vee \sim P(x), G(x) \vee \sim F(x)\}$$

$$\text{Negation of the goal: } \sim[St(J) \vee \sim M(J)] = \sim St(J) \wedge M(J)$$

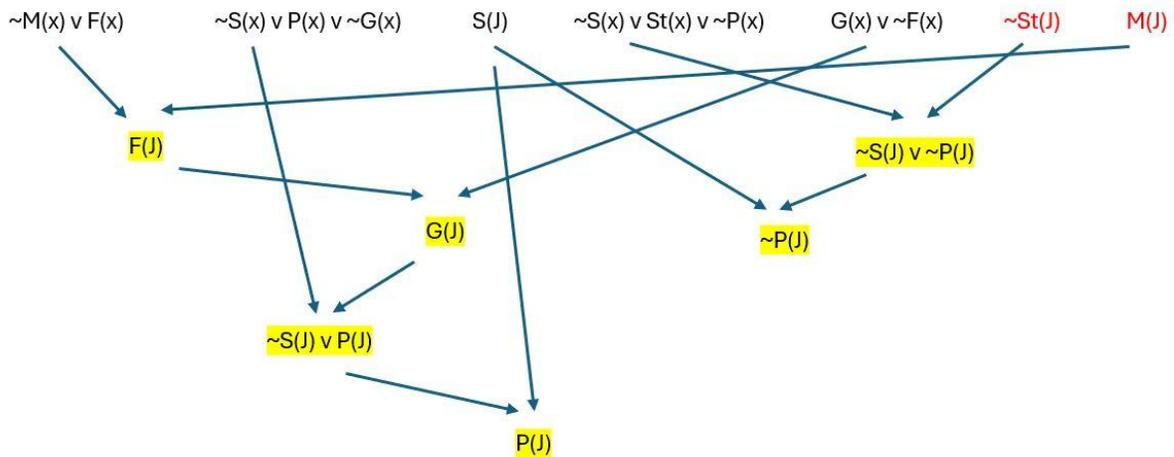
$$\mathbf{KB} \cup \{\sim St(J), M(J)\} \vdash \text{NIL ?}$$

$$\mathbf{KB} = \{\neg M(x) \vee F(x), \sim S(x) \vee P(x) \vee \sim G(x), S(J), \sim S(x) \vee St(x) \vee \sim P(x), G(x) \vee \sim F(x)\}$$

$$\text{Negation of the goal: } \sim[St(J) \vee \sim M(J)] = \sim St(J) \wedge M(J)$$

$$\mathbf{KB} \cup \{\sim St(J), M(J)\} \vdash \text{NIL ?}$$

Set-of-support



Problem 3

Convert the following formulas to CNF and check if they are *satisfiable*:

(1) $\neg(((a \rightarrow b) \rightarrow a) \rightarrow a)$

(2) $\neg((p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)))$

(3) $(\exists x)(\forall y)[[p \wedge q \wedge \neg r(x,y) \wedge s(x)] \rightarrow (\neg q \wedge s(x))]$

Solution (1).

$$\begin{aligned} \sim[((a \rightarrow b) \rightarrow a) \rightarrow a] &= \sim[\sim((\sim a \vee b) \rightarrow a) \rightarrow a] = \sim[\sim(\sim(\sim a \vee b) \vee a) \rightarrow a] = \sim[\sim(\sim(\sim a \vee b) \vee a) \vee a] = \\ & \sim\sim(\sim(\sim a \vee b) \vee a) \wedge \sim a = (\sim(\sim a \vee b) \vee a) \wedge \sim a = ((\sim\sim a \wedge \sim b) \vee a) \wedge \sim a = ((a \wedge \sim b) \vee a) \wedge \sim a = \\ & ((a \vee a) \wedge (\sim b \vee a)) \wedge \sim a = (a \wedge (\sim b \vee a)) \wedge \sim a \quad \text{NO (False for all valuations)} \end{aligned}$$

Solution (3).

$$(\forall y)[[p \wedge q \wedge \neg r(A,y) \wedge s(A)] \rightarrow (\neg q \wedge s(A))] =$$

$$[p \wedge q \wedge \neg r(A,y) \wedge s(A)] \rightarrow (\neg q \wedge s(A)) =$$

$$[p \wedge q \wedge \neg r(A,y) \wedge s(A)] \rightarrow (\neg q \wedge s(A)) =$$

$$\sim[p \wedge q \wedge \sim r(A,y) \wedge s(A)] \vee (\sim q \wedge s(A)) =$$

$$[\sim p \vee \sim q \vee r(A,y) \vee \sim s(A)] \vee (\sim q \wedge s(A)) =$$

$$[\sim p \vee \sim q \vee r(A,y) \vee \sim s(A) \vee \sim q] \wedge [\sim p \vee \sim q \vee r(A,y) \vee \sim s(A) \vee s(A)] \quad \text{YES (for instance when } p = \text{false)}$$

Problem 4

Translate into symbols the following statements, using quantifiers, variables and predicate symbols:

- Tony, Mike, and John belong to the Alpine club.
- Every member of the Alpine club is either a skier or a mountain climber or both,
- Mountain climbers do not like rain and anyone who does not like snow is not a skier.
- Mike dislikes whatever Tony likes and likes whatever Tony dislikes.
- Tony likes rain and snow.

Use resolution to show that:

- There is a member of the Alpine club who is a mountain climber but not a skier

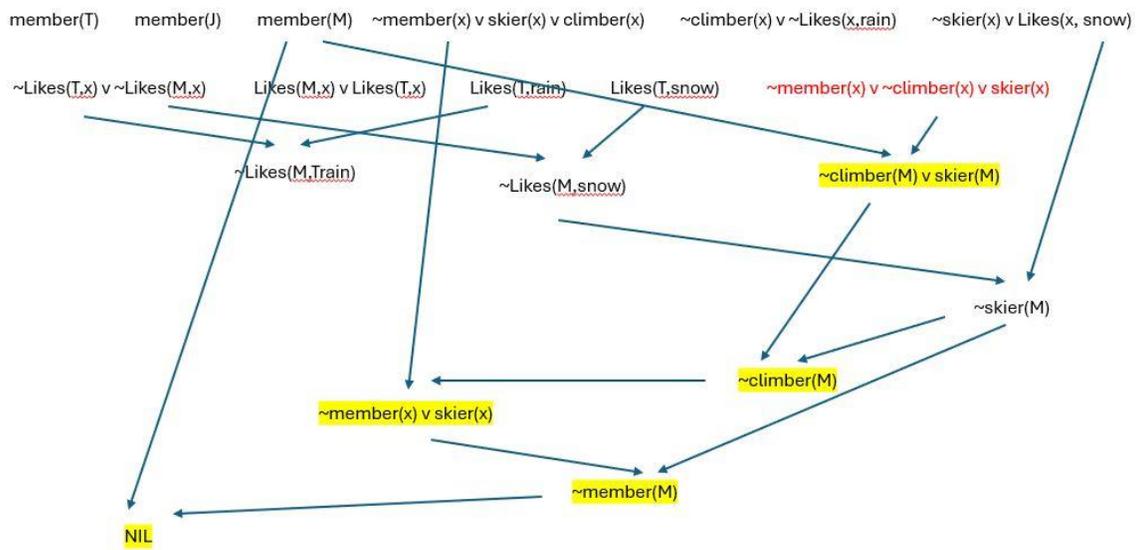
Solution:

member(x): x is a member of the Alpine club. T – Tony, J – John, M - Mike

KB = {member(T), member(J), member(M), member(x) \rightarrow skier(x) \vee climber(x), climber(x) \rightarrow \sim Likes(x,rain), skier(x) \rightarrow Likes(x, snow), Likes(T,x) \rightarrow \sim Likes(M,x), \sim Likes(M,x) \rightarrow Likes(T,x), Likes(T,rain), Likes(T,snow) } |- $(\exists x)[\text{member}(x) \wedge \text{climber}(x) \wedge \sim \text{skier}(x)]$.

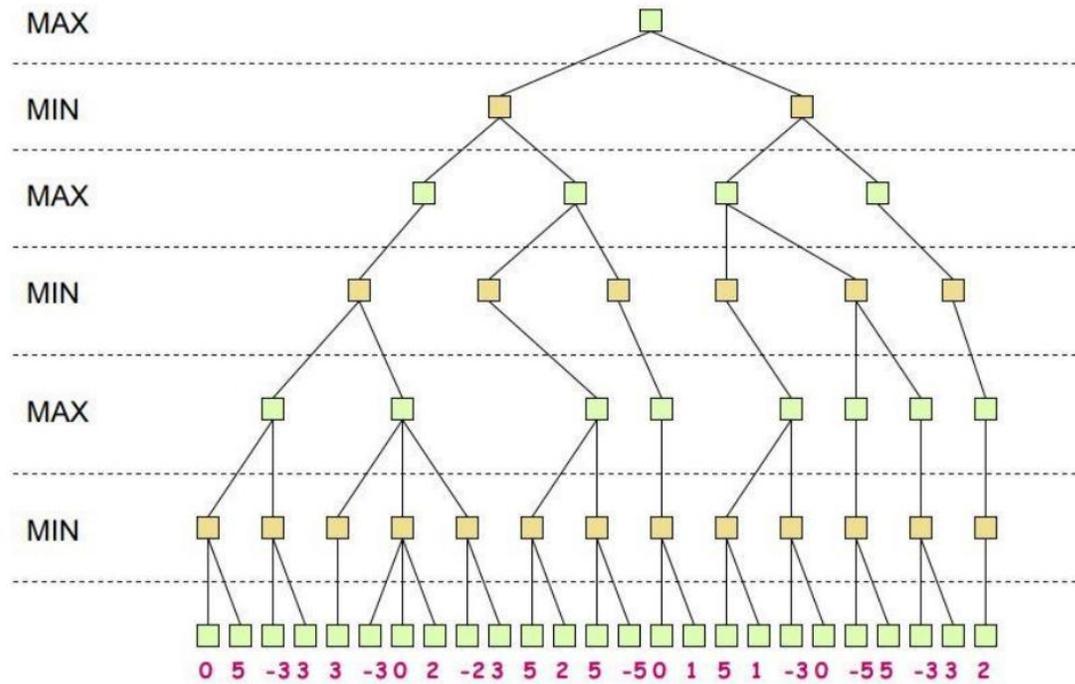
KB = {

- (1) member(T),
- (2) member(J),
- (3) member(M),
- (4) \sim member(x) \vee skier(x) \vee climber(x),
- (5) \sim climber(x) \vee \sim Likes(x,rain),
- (6) \sim skier(x) \vee Likes(x, snow),
- (7) \sim Likes(T,x) \vee \sim Likes(M,x),
- (8) Likes(M,x) \vee Likes(T,x),
- (9) Likes(T,rain),
- (10) Likes(T,snow),
- (11) \sim member(x) \vee \sim climber(x) \vee skier(x) } |- NIL ?



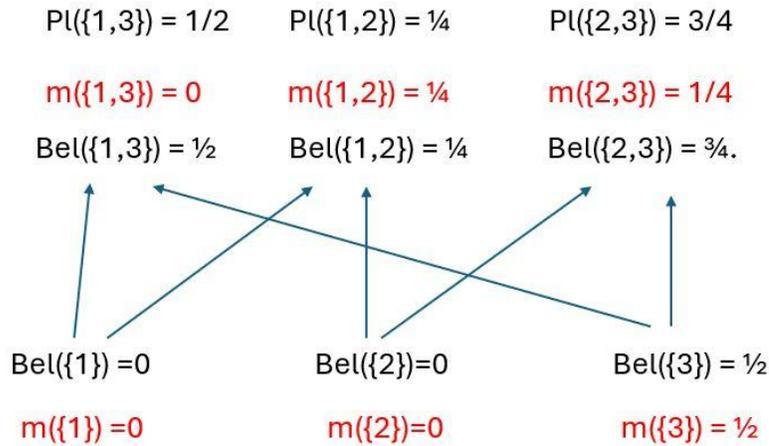
Problem 5

Follow α - β strategy for the tree below:



Problem 6

For the function $Bel: 2^X \rightarrow [0, 1]$ find the basic probability assignment $m: 2^X \rightarrow [0, 1]$ and the plausibility function $Pl: 2^X \rightarrow [0, 1]$ where $X = \{1, 2, 3\}$ and $Bel(\{1\}) = Bel(\{2\}) = 0$, $Bel(\{3\}) = Bel(\{1, 3\}) = 1/2$, $Bel(\{1, 2\}) = 1/4$, $Bel(\{2, 3\}) = 3/4$.



Information needed to solve this problem.

Basic probability assignment – m

$m: 2^X \rightarrow [0, 1]$ such that: (1) $m(\emptyset) = 0$, (2) $\sum [m(Y) : Y \subseteq X] = 1$ /total belief/.

$X = \{x_1, x_2, x_3, x_4, x_5\}$

$m(x_1, x_2, x_3) = 1/2$, $m(x_1, x_2) = 1/4$, $m(x_2, x_4) = 1/4$

Belief function: $Bel: 2^X \rightarrow [0, 1]$ such that:

$Bel(Y) = \sum [m(Z) : Z \subseteq Y]$.

$Bel(\{x_1, x_2, x_3, x_5\}) = 1/2 + 1/4 = 3/4$,

Focal Element and Core

$Y \subseteq X$ is called focal element iff $m(Y) > 0$.

Core – the union of all focal elements.

Doubt Function - $Dou: 2^X \rightarrow [0, 1]$, $Y \subseteq X$

$Dou(Y) = Bel(\neg Y)$.

Plausibility Function – $Pl(Y) = 1 - Dou(Y)$

$Pl(Y) = \sum [m(Z) : Z \cap Y \neq \emptyset]$

Problem 7.

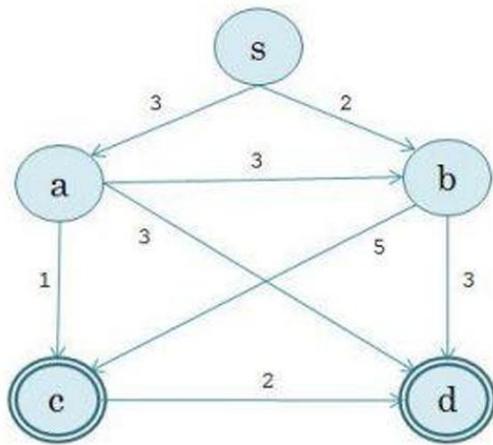
$X=\{a,b,c\}$, and two basic probability assignments, m and n are given below:

	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}	{a,b,c}
m	0.3	0	0.2	0.3	0	0.1	0.1
n	0	0	0.2	0.2	0.3	0.2	0.1

Assuming independence of both pieces of evidence, find their orthogonal sum $m \oplus n$.

Problem 8.

In what order A*-algorithm will visit the nodes in the graph below ($h(x)$ is the heuristic estimation of the distance from node x to one of the final nodes).



$h(s)$	$h(a)$	$h(b)$	$h(c)$	$h(d)$
1	3	3	0	0

$f(n)=g(n)+h(n)$ / $g(n)$ – distance to node n from the initial node/

