International Journal of Uncertainty,
Fuzziness and Knowledge-Based Systems
Vol. 18, No. 4 (2010) 451–470

DOI: 10.1142/S0218488510006647

HOW TO SUPPORT CONSENSUS REACHING USING ACTION RULES: A NOVEL APPROACH

JANUSZ KACPRZYK* and SŁAWOMIR ZADROŻNY†

Systems Research Institute, Polish Academy of Sciences,
01-447 Warsaw, Poland
**Janusz.Kacprzyk@ibspan.waw.pl*
†Slawomir.Zadrozny@ibspan.waw.pl

ZBIGNIEW W. RAŚ

Department of Computer Science, University of North Carolina,
Charlotte, N.C. 28223, USA
and Institute of Computer Science, Polish Academy of Sciences,
01-237 Warsaw, Poland
ras@uncc.edu

Received 30 January 2010
Revised 15 May 2010
Accepted 5 June 2010

We consider a consensus reaching process in a group of individuals meant as an attempt to make preferences of the individuals more and more similar, that is, getting closer and closer to consensus. We assume a general form of intuitionistic fuzzy preferences and a soft definition of consensus that is basically meant as an agreement of a considerable (e.g., most, almost all) majority of individuals in regards to a considerable majority of alternatives. The consensus reaching process is meant to be run by a moderator who tries to get the group of individuals closer and closer to consensus by argumentation, persuasion, etc. The moderator is to be supported by some additional information, exemplified by more detailed information on which individuals are critical as, for instance, they are willing to change their testimonies or are stubborn, which pairs of options make the reaching of consensus difficult, etc. In this paper we extend this paradigm proposed and employed in our former works with the use of a novel data mining tool, so called action rules which make it possible to more clearly indicate and suggest to the moderator with which experts and with respect to which option it may be expedient to deal. We show the usefulness of this new approach.

Keywords: Consensus reaching; action rules; group decision making; soft consensus measure; fuzzy preference; intuitionistic fuzzy linguistic preference; fuzzy majority.

1. Introduction

We consider the problem of reaching consensus in a group of individuals (agents, experts, . . .). We assume the following basic setting. There is a group of individuals who present their testimonies concerning alternatives in question as preferences between the particular pairs of options. We assume fuzzy preferences, and more specifically their more general form of intuitionistic fuzzy preferences that make it possible to express some *pro* and *contra* testimonies and/or arguments. The group of individuals aims at reaching some state of a (sufficient) agreement as to their preferences, a *consensus*. We assume, first of all, a conceptual general and human-consistent framework initiated by Kacprzyk and Fedrizzi,[1–3] and Zadrożny[4] in which a new degree of consensus was proposed meant basically as a degree to which, for instance, "*most* of the *important* individuals agree in their preferences as to *almost all* of the *relevant* options". This framework was then implemented in a decision support system for group decision making and consensus reaching by Fedrizzi, Kacprzyk and Zadrożny,[5] and Kacprzyk and Zadrożny.[6,7] An important further development in the representation and analysis of consensus reaching processes in the above conceptual framework has come from the Spanish researchers, notably, alphabetically, Chiclana, Herrera, Herrera-Viedma, Martinez, Mata, etc. Refs. 8 and 9 Among most relevant further developments along the above line of reasoning one can also cite, first, Kacprzyk and Zadrożny's[10–12] works on general choice functions in group decision making, and – above all – a recent proposal by Kacprzyk and Zadrożny[13] in which the use of ontologies on the consensus reaching process and a particular domain in which the consensus reaching process is concerned are considered.

Basically, in all the above works the following paradigm has been assumed. The individuals provide their testimonies concerning alternatives in question as *fuzzy preference relations* expressing preferences given in pairwise comparisons of options. A decision is to be taken by an agreement of the individuals, i.e., after a *consensus* is reached. Usually, at the beginning the individuals are far from consensus, and a discussion is carried out in the group to clarify points of view, exchange information, advocate opinions, persuade stubborn individuals to be more flexible, etc., which is expected to trigger some changes in the preferences, leading to their unification. This all is run by a *moderator* who is responsible for an effective and efficient running of the consensus reaching process.

To facilitate the work of the moderator, and make it more effective and efficient, it is important to provide him or her, and the group of individuals, with some hints as to the most promising directions of a further discussion, how to focus discussion in the group, etc.

One of the first, and more successful approach is due to Kacprzyk and Zadrożny[14] in which linguistic data summaries were used to derive some additional measures of what is going on in the group with respect to the uniformity of their preferences. For instance, various summaries focus on the individuals and alternatives, exemplified by "Most individuals definitely prefer some alternative o_{i1} to o_{i2}, moderately prefer o_{i3} to $o_{i4}, \ldots$", "Most individuals definitely preferring o_{i1} to o_{i2} also definitely prefer o_{i3} to o_{i4}", "Most individuals choose alternatives o_{i1}, $o_{i2}, \ldots$", "Most individuals reject alternatives $o_{i1}, o_{i2}, \ldots$", "Most alternatives are dominated by alternative o_i in opinion of individual e_k", "Most alternatives are dominated by option o_i in opinion of individual e_{k1}, e_{k2}, ...", "Most alternatives dominating alternative o_i in opinion of individual e_{k1} also dominate alternative o_i in opinion of individual e_{k2}","Most alternatives are preferred by an individual e_k", etc. Notice that such summaries may help the moderator and the group members get a deeper understanding of even intricate and sophisticated relations within the individuals and their testimonies (preferences). In the derivation of those valuable summaries Kacprzyk and Zadrożny's[10–12] works on general choice functions in group decision making have played a significant role. Among other approaches one should also cite Herrera-Viedma, Mata, Martinez and Pérez.[15,16]

Notice that the above approaches assume (traditional) fuzzy preference relations and a fuzzy majority (equated with a fuzzy linguistic quantifiers like, most, almost all, etc.). In this paper we extend the fuzzy preference relations to the intuitionistic fuzzy preferences relations which specify for each pair of individuals a degree of preference of the former alternative over the latter as a pair that consists of the degree of membership and non-membership, which do not need to sum up to 1, hence a hesitation margin (the difference between 1 and the sum of them) exists. The intuitionistic fuzzy preference relation may provide a better and more flexible framework to deal with lack of information and bipolar information, and also in a sense are better suited for the use of action rules to be briefly explained below.

The main purpose of this paper is to propose the use of so called *action rules* as a tool to obtain an additional information that can be useful for running the consensus reaching process.

The concept of an *action rule* was proposed by Raś and Wieczorkowska,[17] in the context of Pawlak's[18] *information systems*, and then has been extensively further studied and developed by Raś and his collaborators.[19–24]

Essentially, the purpose of an action rule is to show how a subset of flexible attributes should be changed to obtain an expected change of the decision attribute for a subset of objects characterized by some values of the subset of stable attributes. For example, in a bank context, an action rule may, e.g., indicate that offering a 20% reduction in monthly bank account fee instead of 10% to a middle-aged customer is expected to increase his or her spendings from medium to high. There is also an important issue of the cost of change of an attribute value, which may be different for different attributes, and the very purpose is thus to find the "cheapest" rules supporting the expected change of the decision attribute. It is easy to see that one can notice at the first glance a great potential of action rules in our context of the supporting of consensus reaching processes. Namely, we can imagine that we wish to find some "concessions" to eventually be offered to some individuals so that they change their preferences, and to choose those "concessions" that would bear the lowest "cost".

The structure of the paper is the following. In Sec. 2 we present the essence of the problem considered, i.e. our model of a consensus reaching process. In particular, the representation of preferences is discussed and various consensus indicators proposed in our previous work are recalled and adapted to this more sophisticated representation. This section concludes with a brief discussion of the notion of the choice function (choice set), which is later used for the purposes of supporting the consensus reaching process. Section 3 presents the concept of an action rule, both in its basic and more elaborated form, and Sec. 4 shows how the action rules may be used to support the consensus reaching process. We conclude the paper by summarizing the proposed approach and setting directions for further research.

2. A Consensus Reaching Process Model

2.1. *Representation of preferences*

There is a set of $N \geq 2$ alternatives, $S = \{s_1, \ldots, s_N\}$, and a set of $M \geq 2$ individuals, $E = \{e_1, \ldots, e_M\}$. Each individual $e_m \in E$ expresses his/her preferences as individual *fuzzy preference relation* R_m in $S \times S$, and μ_{R_m} may be meant as that $\mu_{R_m}(s_i, s_j) > 0.5$ denotes the preference degree of an s_i over s_j, $\mu_{R_m}(s_i, s_j) < 0.5$ denotes the preference degree of s_j over s_i, and $\mu_{R_m}(s_i, s_j) = 0.5$ denotes the indifference between s_i and s_j. Usually R_m is assumed *reciprocal*, i.e.,

$$\mu_{R_m}(s_i, s_j) + \mu_{R_m}(s_j, s_i) = 1 \tag{1}$$

holds.

We extend this basic fuzzy preference modeling approach in the following way. First, we adopt a bipolar view of preferences,[31] and in particular their modeling via *Atanassov's intuitionistic fuzzy sets*,[27] called A-IFS sets. Second, instead of using numeric membership (and non-membership) degrees we use *linguistic terms*.

An A-IFS set X is represented by a pair of membership, μ_X, and non-membership, ν_X, functions, such that:

$$\mu_X(x) + \nu_X(x) \leq 1\,. \tag{2}$$

An A-IFS *preference relation* $R_m^{\text{A-IFS}}$, of individual e_m, is an A-IFS set in $S \times S$, which is thus defined by its membership function $\mu_{R_m^{\text{A-IFS}}}(s_i, s_j)$ and non-membership function $\nu_{R_m^{\text{A-IFS}}}(s_i, s_j)$. The former is meant as the degree of preference (intensity) of s_i over s_j, and latter as the degree to which s_i is *not* preferred over s_j, here. To simplify, the latter may be interpreted in the spirit of the reciprocity (1) as the intensity of preference of s_j over s_i which implies:

$$\nu_{R_m^{\text{A-IFS}}}(s_i, s_j) = \mu_{R_m^{\text{A-IFS}}}(s_j, s_i)\,. \tag{3}$$

The use of A-IFS sets provides for a more flexible representation, notably for taking into account pro and con arguments while determining the preferences which is a widely advocated approach (cf., e.g., Refs. 13 and 32). For more details on the A-IFS preference relations and their use in group decision making and consensus reaching, cf. Szmidt and Kacprzyk.[36,37]

For our purposes a discretization of the universe in which degrees of preference are expressed is needed. Moreover such a discretization is also advantageous from the practical point of view. Hence, instead of using the interval $[0, 1]$ we adopt the ordinal linguistic approach,[30] and represent preference degrees using an ordered set of linguistic terms (linguistic labels). Due to (3) we assume that an individual is specifying for each pair (s_i, s_j) only the membership degrees $\mu_R(s_i, s_j)$ and $\mu_R(s_j, s_i)$ using the following linguistic terms set $\mathcal{L}$:

$$definitely \succ \mathit{strongly} \succ \mathit{moderately} \succ \mathit{weakly} \succ \mathit{not\_at\_all} \tag{4}$$

which in general will be denoted as:

$$\mathcal{L} = \{l_T, l_{T-1}, \ldots, l_1, l_0\} \tag{5}$$

$$l_T \succ l_{T-1} \succ \ldots \succ l_1 \succ l_0 \tag{6}$$

where T is an even number.

Due to the assumed order (6), the infimum and supremum operators may be applied to sets as well as to multisets of linguistic terms:

$$\inf(\{l_{k_1}, l_{k_2}, \ldots, l_{k_n}\}) = l_{k_i} \text{ if } k_i \in \{k_1, \ldots, k_n\} \wedge \neg\exists_{j\in\{k_1,\ldots,k_n\}} l_j \succ l_{k_i} \tag{7}$$

$$\sup(\{l_{k_1}, l_{k_2}, \ldots, l_{k_n}\}) = l_{k_i} \text{ if } k_i \in \{k_1, \ldots, k_n\} \wedge \neg\exists_{j\in\{k_1,\ldots,k_n\}} l_j \succ l_{k_i} \tag{8}$$

Moreover, an *antonym* operator *ant*, which is an involutive order-reversing operator, is assumed on $\mathcal{L}$:

$$ant : \mathcal{L} \longrightarrow \mathcal{L} \qquad ant(l_k) = l_{T-k}\,. \tag{9}$$

The use of such an operator effectively assumes that the linguistic terms form an interval scale. This assumption will be also employed later when comparing two linguistic membership degrees. It makes operations on the linguistic terms indices, notably the difference, meaningful.

An A-IFS preference relation, generalized using linguistic terms instead of numerical membership and non-membership degrees, is an *Atanassov's intuitionistic L-fuzzy set*,[28] to be denoted as A-ILFS set, and has to satisfy a counterpart of the property (2), which is expressed as:

$$\neg(\mu_R(s_i, s_j) \succ ant(\nu_R(s_i, s_j))) \tag{10}$$

what can be rewritten as:

$$\neg(\mu_R(s_i, s_j) \succ ant(\mu_R(s_j, s_i))) \tag{11}$$

as due to (3) we have $\nu_R(s_i, s_j) = \mu_R(s_j, s_i)$. We will refer to such an R as A-ILFS preference relation, in what follows.

Thus, for example (referring to the linguistic terms set (4)), for $\mu_R(s_i, s_j) = definitely$ the value of $\mu_R(s_j, s_i)$ is determined to be *not_at_all*, what means that alternative s_i is definitely (fully) preferred to the alternative s_j. Other two interesting cases are where $\mu_R(s_i, s_j) = \mu_R(s_j, s_i) = \mathit{moderately}$ and $\mu_R(s_i, s_j) = \mu_R(s_j, s_i) = \mathit{not\_at\_all}$. In both cases an individual may be seen as indifferent to the choice between the alternatives s_i and s_j, but due to the semantics of the A-IFS/A-ILFS sets the first case may be seen as a genuine indifference, while the second corresponds to the situation where an individual is undecided, unable to make a choice due to, e.g., lack of information.

Basically, the semantics of these examples of A-ILFS preference relations may be motivated by the following scenario. Let an individual consider a set of criteria while comparing two alternatives. Then, if all criteria support the choice of s_i over s_j, then it may be reasonable to express the preference as $\mu_R(s_i, s_j) = definitely$; in fact, it may be enough if, e.g., most of the important criteria support this. On the other hand if there is a more or less equal number of criteria supporting both alternatives, then it may be reasonable to have $\mu_R(s_i, s_j) = \mu_R(s_j, s_i) = moderately$. Finally, if all criteria are inconclusive (e.g., due to the lack of the information on the value of the alternatives' attributes relevant for these criteria), then the preferences may be reasonably expressed as $\mu_R(s_i, s_j) = \mu_R(s_j, s_i) = not\_at\_all$.

This also shows a need for a "linguistic counterpart" of the *hesitation margin* $\pi(x)$, which is defined in A-IFS sets theory for regular "numerical" membership degrees as

$$\pi(x) = 1 - \mu(x) - \nu(x) \tag{12}$$

In case of a simple lattice defined by (5)-(8), its counterpart for A-ILFS sets may be defined in the following way. Let

$$\pi(x) = l_z, \text{ where } z = t - w\,. \tag{13}$$

Thus, in the context considered here it holds that $\pi_R(s_i, s_j) = l_z$, where $z = t - w$ and $ant(\mu_R(s_i, s_j)) = l_t$, and $\mu_R(s_j, s_i) = l_w$. Notice that due to (11) it holds that $\neg(l_w \succ l_t)$ and thus $t \geq w$ and thus $z \geq 0$ is well defined.

The hesitation margin π expresses the degree to which an individual is unable to decide (e.g., due to the lack of information) on his or her preferences. For example, for the set (4), the term "not at all" as the value of π denotes a lack of hesitation of an individual, while "definitely" denotes a complete inability to decide.

In the proposed model of the consensus reaching process, the alternatives and the individuals are assigned degrees of relevance and weights of importance, respectively. These are usually modeled as fuzzy subsets I and B of S and E, respectively (e.g, $\mu_B(e_k) \in [0, 1]$ denotes a degree of importance of individual e_k). Here, we assume that I and B are A-ILFS sets, with a simple lattice, this time based on a different set of linguistic terms, exemplified by:

$$\begin{aligned} &\textit{very important} \succ \textit{important} \succ \textit{medium important} \\ &\qquad \succ \textit{less important} \succ \textit{weakly important} \end{aligned} \tag{14}$$

and similarly for the relevance: *very relevant*, *relevant* etc.

In the definitions of various consensus indicators discussed in this paper also relevance degree of *pairs* of alternatives and the importance weights of *pairs* of individuals are used. These may be derived from I and B, respectively, in many different ways. For example, considering importance weights given as linguistic terms

(14) one may assume for a pair of individuals a lower of the weights assigned to these individuals.

It should be noted that in the literature there are many approaches based on linguistic representation of preferences. In particular, many important ideas were proposed in this area by Herrera and Herrera-Viedma with collaborators.[8,16,26,29,33,34]

2.2. *Consensus reaching and measuring*

Decision making via obtaining consensus boils down to running a discussion in the group to make individual preferences as close as possible. It is almost always unrealistic to reach a classically meant consensus, i.e., full agreement of all individuals as to all their preferences, and an operational definition of consensus is needed, accounting for a satisfactory agreement in a group. One of the first such operational definitions was proposed by Kacprzyk and Fedrizzi,[1,2,35] and then extended by the current authors.[5] It is also a starting point in this work. Consensus is treated as a gradual notion, which may be evaluated by a *measure of consensus*.

Thus, the group may comprise a small or large number of members, in one location or even around the world, and the discussion among them may be asynchronous. The individuals express their preferences which are represented by A-ILFS preference relations. A user interface hiding the technicalities of such a representation is assumed. A preliminary idea of such an interface and a more comprehensive vision of the whole decision support system is presented in our earlier paper.[13] The preferences of the individuals usually initially differ and the session is started which, through an exchange of information, rational argument, discussion, creative thinking, clarification of positions, etc., is expected to get the preferences closer one to another.

A *moderator*, in charge of running the session, tries to focus the discussion on the issues which may resolve the conflict of opinions in the group. We concentrate on a new technique which may be helpful in pointing out such crucial points with respect to raw A-ILFS preference relations. In Ref. 13 we consider a more sophisticated environment in which the consensus reaching process is to be run, and which provides a richer framework for preferences expression and adjustment.

The operational definition of consensus, mentioned earlier, is the primary indicator of the agreement in the group. In what follows, we recall it briefly and adapt to the new preferences representation provided by A-ILFS preference relations. In the next section we discuss other tools, notably based on action rules, which can help the moderator to run the session.

Our starting point is the consensus measure proposed by Kacprzyk and Fedrizzi.[1–3,35] This consensus measure is based on an operational definition of consensus expressed by a linguistically quantified proposition:[40]

"*Most* ($Q1$) of the *important* (B) individuals *agree* as to *almost all* ($Q2$) *relevant* (I) alternatives". (15)

where: $Q1$ and $Q2$ are linguistic quantifiers,[40] e.g., "most" and "almost all", and

B and I stand for fuzzy sets denoting the importance/relevance of the individuals and alternatives.

The consensus degree, for a set of A-ILFS preference relations $\{R_k\}_{k=1,\dots,M}$, is computed as the truth value of the linguistically quantified proposition (15). First, for each pair of individuals (e_m, e_n) and each pair of alternatives (s_i, s_j) a degree of agreement $v_{ij}(m,n)$ is derived. It is, in general, computed as a function AG of $\mu_{R_m}(s_i,s_j)$, $\mu_{R_m}(s_j,s_i)$, $\mu_{R_n}(s_i,s_j)$ and $\mu_{R_n}(s_j,s_i)$. For "numerically" expressed membership degrees such a function may be defined via a similarity measure between A-IFS sets; cf., e.g., Refs. 38 and 39. For our A-ILFS based representation with membership degrees expressed using linguistic terms (5) this function may be defined using the indices of these linguistic terms, i.e.:

$$AG(\mu_{R_m}(s_i,s_j), \mu_{R_m}(s_j,s_i), \mu_{R_n}(s_i,s_j), \mu_{R_n}(s_j,s_i)) = g(l_{k_1}, l_{k_2}, l_{k_3}, l_{k_4})$$

$$AG : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L} \longrightarrow [0,1]$$
$$g : [0\dots T] \times [0\dots T] \times [0\dots T] \times [0\dots T] \longrightarrow [0,1]$$

for some function g, assuming that $\mu_{R_m}(s_i,s_j) = l_{k_1}$, $\mu_{R_m}(s_j,s_i) = l_{k_2}$, $\mu_{R_n}(s_i,s_j) = l_{k_3}$, $\mu_{R_n}(s_j,s_i)) = l_{k_4}$.

Further on we are dealing again only with numbers and the same scheme to derive the overall consensus degree may be applied as in our previous work.[13,35] Hence, for each pair of individuals (e_m, e_n) a degree of agreement $v^B_{Q_1}(m,n)$ as to their preferences between Q_1 pairs of *relevant* alternatives is derived and, finally these degrees are aggregated to obtain a degree of agreement $con(Q_1, Q_2, I, B)$ of Q_2 pairs of *important* individuals as to their preferences between Q_1 pairs of *relevant* alternatives, and this is meant to be the *degree of consensus* sought. For details, see Refs. 13, 35. The computations are via Zadeh's calculus of linguistically quantified propositions.[40]

The consensus degree itself plays an important role in guiding the consensus reaching process but here, for our purposes, more important are some derived indicators of consensus. In Ref. 4 we introduced the following measures.

The *personal consensus degree*, $PCD(e_k)$ is defined as the truth value of:

"Preferences of individual e_k as to *most relevant* pairs of alternatives are in agreement with the preferences of *most important* individuals" (16)

The *detailed personal consensus degree*, $DPCD(e_k, s_i, s_j)$ is defined as the truth value of:

"Preference of individual e_k as to a pair of alternatives (s_i, s_j) is in agreement with the preferences of *most important* individuals" (17)

Similarly, for the alternatives, the *option consensus degree*, $OCD(s_i)$ is defined as the truth value of a linguistically quantified proposition:

"*Most important* pairs of individuals agree in their preferences with respect to the alternative s_i" (18)

These consensus indicators make it possible to point out the most controversial alternatives and/or individuals isolated in their opinions. This may help in further running of the discussion in the group. In Sec. 4 this is discussed in a more detail. Some conceptually similar indicators were proposed by Herrera, Herrera-Viedma and Verdegay in Ref. 33.

2.3. *Beyond the plain preference relations: Choosing 'best' alternatives*

The aim of the session is assumed here to primarily consist in thoroughly discussing a set of available alternatives, clarifying all their pros and cons with respect to the goal of the community represented by the group of individuals. This should lead ultimately to arriving at the agreement among the individuals signaled by a high consensus measure computed for the preference relations expressed by them. Usually, the next step will be the choice of some of the alternatives which should be seen as best matching the goals of the community, in view of the individual preference relations. This choice may be, in an abstract way, meant as defined by a *choice function* (cf., e.g., Refs. 7, 9, 10, 11, 41). Let us briefly remind the basics of this concept.

A general form of the choice function $C(S, R)$ may be defined as follows:

$$C(S, R) = S_0, \qquad S_0 \subseteq S\,. \tag{19}$$

It may take the following particular form:

$$C(S, R) = \{s_i \in S : \forall_{i \neq j} R(s_i, s_j)\} \tag{20}$$

where R denotes a classical crisp preference relation. In what follows we will call S_0 a *choice set*. In case the preference relation R is fuzzy, the choice set S_0 is assumed to be fuzzy, and we get the following counterpart of (20):

$$\mu_{C(S,R)}(s_i) = \min_j \mu_R(s_i, s_j)\,. \tag{21}$$

We have to further extend the classical definition as in our approach the preferences are modeled using A-ILFS preference relations (sets). Thus, now the choice set becomes also an A-ILFS set and is defined as follows:

$$\mu_{C(S,R)}(s_i) = \inf_j \mu_R(s_i, s_j) \tag{22}$$

$$\nu_{C(S,R)}(s_i) = \sup_j \nu_R(s_i, s_j) = \sup_j \mu_R(s_j, s_i) \tag{23}$$

employing the definition of the intersection of A-ILFS sets.[28]

In order to arrive at the final choice of alternatives preferred by the group first the individual preference relations are aggregated to form a *group preference relation*, representing the preferences of the whole group. Many approaches has been proposed in the literature on how to arrive at such a group preference relation in case of regular fuzzy preference relations[35] and they were adapted to the case of

A-IFS preference relations,[36,37] and may be easily further extended to the case of A-ILFS preference relations considered in this paper. In case the discussion ended with all individuals having the same preference relation (what is rather unlikely, but possible), the aggregation is of course not needed. Then, a choice function is applied to the resulting group preference relation and the whole set of alternatives as arguments, and an A-ILFS set of alternatives preferred by the group is obtained. The ranking of the alternatives may be then obtained using an approach to ordering of an A-IFS (A-ILFS) set, exploiting, e.g., the correspondence with the interval valued fuzzy sets and many existing approaches to ordering intervals.

In the framework of consensus reaching support we are more interested in applying a choice function to each individual preference relation and thus obtain A-ILFS sets of alternatives preferred (to different extent) by particular individuals. A collection of these choice sets may be very helpful for the discussion guidance. Firstly, it provides for another way of measuring consensus meant as the compatibility between these A-ILFS sets (cf., e.g., Ref. 6). Secondly, and what is more important here, it may be used to generate advice for the individuals regarding their preferences changes, as it is discussed in Sec. 4.

3. Action Rules

The concept of an *action rule* was proposed in Ref. 17, in the context of Pawlak's[18] *information systems*, i.e. triples $IS = \{O, A, V\}$, where O is a finite set of objects, A is a set of its attributes and $V = \bigcup_{a \in A} V_a$, with V_a being a domain of attribute a. If one of the attributes $d \in A$ is distinguished and called *the decision*, then an information system is called a *decision system*. The set of attributes A may be further partitioned into subsets of *stable* and *flexible* attributes, denoted as A_{St} and A_{Fl}.[17] Thus $A = A_{St} \cup A_{Fl} \cup \{d\}$. An intended meaning of this partitioning is closely related to action rules as the essence of an action rule is to show how a subset of flexible attributes should be changed to obtain expected change of the decision attribute for a subset of objects characterized by some values of the subset of stable attributes. For example, let objects $o \in O$ be bank customers characterized by such stable (from a bank perspective) attributes as age, profession, etc. and flexible attributes such as type of the account, reduction of the monthly fee etc., and the decision attribute is the customer's total monthly spendings. Then, an action rule may, e.g., indicate that offering a 20% reduction in monthly fee instead of 10% to a middle-aged customer is expected to increase his or her spendings from medium to high.

In order to define action rules formally, let us define first some auxiliary concepts.[20] An *atomic action term* is $(a, x \rightarrow y)$, where $a \in A$ is an attribute and $x, y \in V_a$ are values belonging to its domain. An *action term* t is a set of atomic action terms: $t = \{(a_1, x_1 \rightarrow y_1), \ldots, (a_n, x_n \rightarrow y_n)\}$, $a_i \in A$, $a_i \neq a_j$ for $i \neq j$ and $x_i, y_i \in V_{a_i}$. The *domain* of an action term t, denoted by $Dom(t)$, is a set of all attributes in t, i.e., $Dom(t) = \{a_1, \ldots, a_n\}$.

Finally, an *action rule* is $r = [t_1 \Rightarrow t_2]$, where t_1 is an action term and t_2 is an atomic action term referring to the decision attribute, i.e., $Dom(t_2) = \{d\}$. So, if the bank customers are characterized by `age`, `reduction` (monthly fee reduction) and `spendings`, then an action rule may take the following form:

$$[\{(\texttt{age}, \mathit{middleaged} \rightarrow \mathit{middleaged}), (\texttt{reduction}, 10\% \rightarrow 20\%)\} \Rightarrow (\texttt{spendings}, \mathit{medium} \rightarrow \mathit{high})]$$

The measures of *support* (*supp*) and *confidence* (*conf*) are used to evaluate the objective interestingness of action rules for a given information system $IS = \{O, A, V\}$. For an action term $t = \{(a_1, x_1 \rightarrow y_1), \ldots, (a_n, x_n \rightarrow y_n)\}$ let us denote by $N_S(t)$ the following pair of sets:

$$N_S(t) = [X, Y] = \Big[\bigcap_{1 \le i \le n} \{o \in O : a_i(o) = x_i\}, \bigcap_{1 \le i \le n} \{o \in O : a_i(o) = y_i\}\Big] .$$

Further, for an action rule $[t_1 \Rightarrow t_2]$, let $N_S(t_1) = [X_1, Y_1]$ and $N_S(t_2) = [X_2, Y_2]$. Then the measures of support and confidence are defined as follows:

$$supp(r) = card(X_1 \cap X_2) \tag{24}$$

$$conf(r) = \frac{card(X_1 \cap X_2)}{card(X_1)} \frac{card(Y_1 \cap Y_2)}{card(Y_1)} \tag{25}$$

where $card(A)$ denotes the cardinality of a set A; for denominators equal 0 the confidence measure is assumed to be zero.

These measures are usually used to mine action rules, similarly to the mining of classification or association rules. There is also an important issue of the cost of change of an attribute value, which may be different for different attributes. The goal is thus to find the "cheapest" rules supporting the expected change of the decision attribute. Assuming that there are no correlations between classification attributes listed in an action rule and the cost is identical for all these attributes then the best are shortest rules.

Now, we recall the notion of an association action rule, introduce the concept of a simple association action rule, and give a strategy to construct simple association action rules of lowest cost.

By an *association action rule* we mean any expression $[t_1 \Rightarrow t_2]$, where t_1 and t_2 are action terms.

Let $(a, x_1 \rightarrow y_1)$ is an atomic action term. We assume that the cost of changing attribute a from x_1 to y_1 is denoted by $cost_{IS}((a, x_1 \rightarrow y_1))$ as introduced in Ref. 23. For simplicity reason, the subscript IS is usually omitted if this does not lead to a confusion. Let $t_1 = (a, x_1 \rightarrow y_1)$, $t_2 = (b, x_2 \rightarrow y_2)$ be two atomic action terms. We say that t_1, t_2 are positively correlated if change represented by t_1 implies change represented by t_2 and vice versa. Similarly, t_1 and t_2 are negatively correlated if change represented by t_1, i.e., $x_1 \to y_1$ implies change opposite to the one represented by t_2, i.e., $y_2 \to x_2$ and vice versa.

Now, assume that action term t is constructed from atomic action terms $T = \{t_1, t_2, \ldots, t_m\}$. We introduce a binary relation $\simeq$ on T defined as: $t_i \simeq t_j$ iff t_i and t_j are positively correlated.

Relation $\simeq$ is an equivalence relation and it partitions T into m equivalence classes ($T = T_1 \cup T_2 \cup \cdots \cup T_m$), for some m. Now, in each equivalence class T_i, an atomic action term $a(T_i)$ of the lowest cost is identified. The cost of t is defined as: $cost(t) = \sum\{cost(a(T_i)) : 1 \le i \le m\}$.

Now, assume that $r = [t_1 \Rightarrow t]$ is an association action rule. We say that r is simple if $cost(t_1 \cup t) = cost(t_1)$. The cost of r is defined as $cost(t_1)$.

We assume that user gives three threshold values, λ_1 - minimum support, λ_2 – minimum confidence, λ_3 - maximum cost. Let t be a frequent action set in S and t_1 is its subset. Any association action rule $r \in AAR_S(\lambda_1, \lambda_2)$ is called association action rule of acceptable cost if $cost(r) \le \lambda_3$. Similarly, frequent action set t is called a frequent action set of acceptable cost if $cost(t) \le \lambda_3$.

Now, in order to construct simple association action rules of a lowest cost, we build frequent action sets of acceptable cost following classical Agrawal's strategy (see Ref. 25) for generating frequent sets enhanced by additional constraint which requires to verify the cost of frequent action sets being produced. Any frequent action set which cost is higher than λ_3, is removed. Now, if t is a frequent action set of acceptable cost and $\{a(T_i) : i \le m\}$ is a collection of atomic action sets constructed by this strategy, then $\bigcup\{a(T_i) : i \le m\} \Rightarrow [t - \{a(T_i) : i \le m\}]$ is a simple association action rule of acceptable cost assuming that its confidence is not greater than λ_2.

For details related to action rules discovery, see Refs. 19, 20, 21.

4. Discussion Guidance using Action Rules

In the group decision making model assumed in Sec. 2, the goal is to reach consensus and the main driving force of the consensus reaching process is an exchange of arguments during the discussion. Thus, a system has to provide the moderator and the whole group with some advice (feedback information) on how far the group is from consensus, what are the most controversial issues (alternatives), whose preferences are in the highest disagreement with the rest of the group, how their change would influence the consensus degree, etc. We propose to use action rules to generate such a feedback. The approaches proposed will be classified according to the form of a decision system $IS = \{O, A, V\}$ (cf. Sec. 3) assumed.

4.1. *Individuals treated as objects*

If we identify the set of objects O with the set of individuals E, then the set of attributes A is composed of:

(1) preference degrees (membership and non-membership expressed using linguistic terms) of given individual for particular pairs of alternatives,
(2) importance degree of an individual,
(3) the personal consensus degree PCD, defined by (16) for given individual and $DPCD$s, defined by (17), for a given individual and all pairs of the alternatives.
(4) the choice set implied by the preference relation of given individual via (22)-(23).

From the perspective of the action rule generation we will treat these groups of attributes as flexible (group (1)), stable (group (2)) and decision attributes (groups (3)-(4)). Thus, while mining action rules we pick up one from the last group of attributes and then start one of the algorithms mentioned in.[17,19–22,24] A typical scenario may be the following. If the consensus degree (computed as discussed in Sec. 2.2) in the group is too low, then PCDs and DPCDs are computed. Next, we look for the rules which suggest how some individuals should change their preferences so as to change their PCD value from low to high, e.g.:

$$[\{(\texttt{importance}, \mathit{important} \rightarrow \mathit{important}), \\ (\mu_R(s_i, s_j), \mathit{not\_at\_all} \rightarrow \mathit{moderately}) \\ (\mu_R(s_j, s_i), \mathit{not\_at\_all} \rightarrow \mathit{not\_at\_all})\} \Rightarrow \\ (\texttt{PCD}, \mathit{medium} \rightarrow \mathit{high})]$$

suggesting that for important individuals it is enough to change preferences as to a given pair of alternatives to get an increase of the personal consensus degree (PCD).

We need to clarify some issues regarding the generation of the rules. First, in order to produce such rules we need to discretize the values of PCD, using, e.g., another set of linguistic terms $\{\mathit{very\ high}, \mathit{high}, \mathit{medium}, \mathit{low}, \mathit{very\ low}\}$.

Second, one has to be careful while generating the action rules so as not to suggest changes in preferences violating the consistency of the A-ILFS preference relations (11). The simplest solution is to treat both membership values $\mu_R(s_i, s_j)$ and $\mu_R(s_j, s_i)$ as one atomic value, from the point of view of the action rules.

Third, the special role of the hesitation margin $\pi(x)$ should be noted. Namely its value is a function of two other degrees, thus its direct use as an attribute in the description of an individual and further use in action rules does not make any difference. However, the hesitation margin may be used to assess the cost of given action rule since it may be assumed that the cost of changing the preference degree for which the hesitation margin is high should be lower. Also the importance may be seen as contributing to the cost evaluation: the higher importance of an individual the higher the cost of change. We consider these aspects of the action rules generation in a more detailed way in Sec. 4.3.

Finally, it should be stressed that we mean the action rules as a recommendation only. Thus, the changes suggested by action rules generated are presented to the relevant individuals for consideration, and they decide if and how to take them into account. We assume a highly dynamic situation in the group with respect to the individual preferences and the automatic implementation of the suggestion provided by the action rules is not appropriate. Moreover,one should bear in mind that the model of dynamics of the consensus reaching process is unknown. The suggestions provided by an action rule are therefore meant to trigger a discussion by showing some patterns in the group's preferences. Clearly, an immediate and direct implementation of changes suggested by the action rules generated does not guarantee in general that an increase of the agreement in the group will occur. However, one can expect that this should happen thanks to the very semantics of the action rules.

Notice that the partition of the attributes into flexible, stable and decision attributes, which is assumed above does not have to be strictly followed. For example, an attribute from the first group, i.e., the preference of given individual expressed for a selected pair of alternatives, may also play the role of the decision attribute. Action rules generated in such a scenario may show some dependencies between the preferences, which are valid for most of the individuals, and may encourage other individuals to re-think their preferences and either strongly motivate their choice during the discussion or accept the arguments of the majority and adjust their preferences.

Similar action rules may be generated with respect to a specific individual and specific pair of alternatives, using DPCD indicator as a decision attribute.

The fourth group of attributes which may be associated with an individual consists of the membership and non-membership degrees of particular alternatives to a choice set induced by the preference relation of this individual. These attributes may appear in actions rules of the following type:

$$
\begin{aligned}
&[\{(\texttt{importance}, \mathit{very\ important} \rightarrow \mathit{very\ important}),\\
&\qquad (\mu_R(s_i, s_j), \mathit{not\_at\_all} \rightarrow \mathit{moderately})\\
&\qquad (\mu_R(s_i, s_k), \mathit{not\_at\_all} \rightarrow \mathit{strongly})\} \Rightarrow\\
&\qquad\qquad (\mu_{C(S,R)}(s_i), \mathit{weakly} \rightarrow \mathit{strongly})]
\end{aligned}
$$

stating that for very important individuals if they change preferences as to an alternative s_i with respect to the alternatives s_j and s_k in the shown way then the membership of s_i to the choice sets of these individuals should be promoted from "weakly" to "strongly".

Such an action rule, again, should be seen first of all as providing possibly interesting information for the individuals. Namely, in the example given above, the individuals learn that there is some tight relation between the overall status of the alternative s_i (represented by its membership degree to the individual's choice sets) and its standing against the s_j and s_k alternatives. This may trigger a further discussion and lead to better clarifying positions of particular individuals and to deeper study the intricacies of the decision problem under consideration.

Finally, it should be observed that the association action rules, mentioned in Sec. 3 may provide even more valuable information as they point out a set of changes that are expected to occur simultaneously provided that a number of preferences degrees changes are implemented.

4.2. *Alternatives treated as objects*

Action rules may be also generated with respect to another information systems employed, namely, the set of objects O may be identified with the set of alternatives S. The set of attributes A is then composed of:

(1) preference degrees with respect to the rest of the alternatives as expressed by all individuals
(2) a relevance degree of an alternative,
(3) the option consensus degree OCD, defined by (18) for a given alternative,
(4) the choice sets membership and non-membership degrees of an alternative (22)-(23)

The scenario for the generation and use of action rules with respect to this information system is similar to the one for individuals playing the role of objects. In particular, the attributes of the first group are treated as flexible, and may be expressed for the s_i alternative as a sequence of pairs of the membership and non-membership degrees of the A-ILFS preference relation:

$$\begin{gathered}
\mu_{R_1}(s_1, s_i), \nu_{R_1}(s_1, s_i), \ldots, \mu_{R_1}(s_i, s_N), \nu_{R_1}(s_i, s_N), \\
\mu_{R_2}(s_1, s_i), \nu_{R_2}(s_1, s_i), \ldots, \mu_{R_2}(s_i, s_N), \nu_{R_2}(s_i, s_N), \\
\cdots \\
\mu_{R_M}(s_1, s_i), \nu_{R_M}(s_1, s_i), \ldots, \mu_{R_M}(s_i, s_N), \nu_{R_M}(s_i, s_N),
\end{gathered}$$

assuming $1 < i < N$. The attributes in this group are thus indexed by a number of an individual and by a number of an alternative, different from s_i The second attribute is stable, and the third and fourth serve as decision attributes.

Thus, for example, we can obtain action rules stating that for relevant alternatives, if individuals e_k and e_m change their preferences (membership degrees) from "weakly" to "strongly" then the option consensus degree (OCD) change from "low" to "high" (we assume, as in case of PCD's, that the range of values of OCD is discretized using a set of linguistic terms).

The fourth group of attributes which may be associated with the alternatives consists of the membership and non-membership degrees of given alternative to the choice sets induced by the preference relations of particular individuals. For the alternative s_i these form a vector:

$$[\mu_{C(S,R_1)}(s_i), \nu_{C(S,R_1)}(s_i), \ldots, \mu_{C(S,R_M)}(s_i), \nu_{C(S,R_M)}(s_i)] \tag{26}$$

Based on these attributes an action rule of the following type may be generated:

$$[\{(\texttt{relevance}, \mathit{relevant} \to \mathit{relevant}),\\ (\mu_{R_k}(\cdot, s_j), \mathit{not\_at\_all} \to \mathit{strongly})\} \Rightarrow\\ (\mu_{C(S,R_k)}, \mathit{weakly} \to \mathit{strongly})]$$

stating that for a relevant alternative if the preferences of an individual e_k concerning this alternative and alternative s_j change from lack of preference to strong preference then the membership of this alternative to the choice set of individual R_k is expected to change from weak to strong.

4.3. *Interpretation of the cost of an action rule*

In Sec. 3 it has been pointed out that changes of the attribute values may incur different cost and thus the rules incurring the least cost are the most appealing. In the context of the group decision making this cost may be interpreted and quantified in a few different ways. Basically, the cost may be related to the ease of implementing changes recommended by a given action rules. Another words, the concept of the cost may be replaced for our purposes with the notion of the possibility that recommended changes will be accepted by particular individuals. That is, the higher possibility of the acceptance, the lower cost is associated with given rule. Thus, rules suggesting changes highly unlikely to be adopted by given individual should be treated as costly and used for the generation of recommendations as a last resort. However, some other interpretations of the cost are also conceivable, and will be briefly discussed in what follows.

The following assumptions may be reasonable, depending on the specificity of a particular decision making task:

(1) the less decided an individual is regarding his or her preferences for given pair of alternatives the higher possibility that he or she accepts a suggestion to change them,
(2) the less essential the preferences concerning given pair of alternatives are the higher possibility the change of them is accepted,
(3) the higher importance of a given individual the lower his or her acceptance of the recommendation might be,
(4) the lower the relevance of an alternative the higher possibility that a recommendation concerning it is accepted by an individual.

Some of the above mentioned assumptions seems to be obvious (cf., e.g., the second one), but their formalization may be not that trivial. In what follows we discuss particular assumptions more in-depth and show how they may be formalized.

The assumption (1) seems to be fairly obvious. If an individual is not decided in his preferences then he or she should be open for a recommendation to change them.

In the model of preferences assumed in our approach it is fairly easy to implement this assumption. Namely, the hesitation margin (12)-(13) represents exactly the degree to which an individual is undecided. Also from the more technical point of view a high hesitation margin makes the changes to preferences easier as there is a room for such a change, without a risk to violate the consistency condition (11) (cf., also (2)) for the A-ILFS based preferences representation. It is easier to show for the case of an A-IFS preference relation, but the same applies to the A-ILFS preference relations. Namely, if $\pi_R(s_i, s_j) >> 0$ then $\mu_R(s_i, s_j) + \nu_R(s_i, s_j) << 1$, and thus even a large change to one of the membership or non-membership function's value may not lead to the consistency condition violation.

The assumption (2) is also obvious but the question is how to determine/define if preferences with respect to a given pair of alternatives are essential. Here the notion of the choice set may provide required formal means. Namely, if the changes recommended to an individual e_k concern the preferences with respect to the alternatives s_i and s_j which both rather not belong to the choice set $C(S, R_k)$, i.e., $\mu_{C(S,R_k)}(s_i)$ is low and $\nu_{C(S,R_k)}(s_i)$ is high, and the same for s_j, then both alternatives stand low in the preferences of the individual e_k and he or she can easily accept proposed changes, what in turn may lead to the increase of the overall consensus measure for the group. Obviously, more detailed rules may be elaborated, taking into account various combinations of the membership and non-membership degrees of the compared alternatives to a respective choice set.

The assumption (3) may be motivated in several ways. The importance weight of an individual is related to its formal position in the group or to its competence and thus an individual with high importance weight associated may be more reluctant to change it than an individual of a lower importance. Moreover, in particular if the importance weight is related to the competence of an individual then his or her preferences may be seen as "correct", properly reflecting the utility of particular alternatives, and thus it is more reasonable to recommend changes to less competent individuals.

Finally, the assumption (4) is also reasonable if we assume that the relevance of an alternative is related to its in advance known applicability as a solution for the decision problem in question. If it is low then the individuals should be expected to easier accept a recommendation to change preferences with respect to it.

This completes our brief discussion of possible interpretation of the notion of cost, which is very important when a set of action rules is generated and a decision has to be made in which order they should be used to propose some recommendation to the individuals. As many different aspects should be taken into account, including those listed above, then some aggregation has to be executed to come up with a scalar evaluation of the cost of an action rules. An alternative approach may consist in using some multi-criteria decision making techniques. Anyway, this subject definitely requires further studies and experiments confirming practical value of the above identified aspects of the cost.

5. Conclusion

We considered a consensus reaching process in a group of individuals meant as an attempt to make preferences of the individuals more and more similar. We assumed a more general form of intuitionistic fuzzy preferences and a soft (fuzzy) majority that implies a soft definition of consensus which is basically meant as an agreement of a considerable (e.g., most, almost all) majority of individuals in regards to a considerable majority of alternatives. The consensus reaching process was meant to be run by a moderator who tried to get the group of individuals closer and closer to consensus by argumentation, persuading, etc. For providing the moderator with additional information that could be useful for him or her to more effectively and efficiently run the consensus reaching session, we proposed to use a novel data mining tool, the so called action rules, to stimulate and support the discussion in the group. We showed examples of some classes of action rules which might be useful for the support of consensus reaching. We discussed possible interpretations of the concept of the (general) cost of an action rule what is very relevant from the practical point of view. Moreover, we used a very flexible preferences representation using Atanassov's intuitionistic L-fuzzy preference relations which, in addition, could make the implementation of the action rules easier. Some previously proposed concepts of flexible measures dealing with various aspects of consensus, were extended for the case of this more sophisticated preferences representation and the use of the action rules.

As to further research directions, first, the usefulness of the action rules has to be verified in a more comprehensive experimental setting. A more elaborate and specific scheme for their use during the session has to be proposed. Moreover, profiles of the individuals, and possibly also alternatives, should be extended to provide for a more meaningful and intuitive action rules. This may be done in a richer framework of a group decision support taking into account the information environment of the decision making process, as proposed in the recent paper by Kacprzyk and Zadrożny.[13]

Acknowledgment

This work was partially supported by the Ministry of Science and Higher Education under Grants N N519 404734 and N N519 384936.

References

1. J. Kacprzyk and M. Fedrizzi, 'Soft' consensus measures for monitoring real consensus reaching processes under fuzzy preferences, *Control and Cybernetics* **15** (1986) 309–323.
2. J. Kacprzyk and M. Fedrizzi, A 'soft' measure of consensus in the setting of partial (fuzzy) preferences, *European Journal of Operational Research* **34** (1988) 315–325.
3. J. Kacprzyk and M. Fedrizzi, A 'human consistent' degree of consensus based on fuzzy logic with linguistic quantifiers, *Mathematical Social Sciences* **18** (1989) 275–290.

4. S. Zadrożny, An approach to the consensus reaching support in fuzzy environment, *Consensus Under Fuzziness*, eds. J. Kacprzyk, H. Nurmi, and M. Fedrizzi (Kluwer, Boston, 1997) pp. 83–109.
5. M. Fedrizzi, J. Kacprzyk and S. Zadrożny, An interactive multi-user decision support system for consensus reaching processes using fuzzy logic with linguistic quantifiers, *Decision Support Systems* **4**(3) (1988) 313–327.
6. J. Kacprzyk and S. Zadrożny, A group decision support system based on linguistic multicriteria assessments, *Technologies for Constructing Intelligent Systems 1*, eds. B. Bouchon-Meunier, J. Gutierrez-Rios, L. Magdalena and R. R. Yager (Physica-Verlag, Heidelberg, 2002) pp. 139–152.
7. J. Kacprzyk and S. Zadrożny, An Internet-based group decision support system, *Management* **VII**(28) (2003) 2–10.
8. F. Mata, L. Martínez and E. Herrera-Viedma, An adaptive consensus support model for group decision-making problems in a multigranular fuzzy linguistic context, *IEEE T. Fuzzy Systems* **17**(2) (2009) 279–290.
9. F. Herrera and E. Herrera-Viedma, Choice Functions for Linguistic Preference Relations, *6th Int. Conf. on Information Processing and Management of Uncertainty in Knowledge-Bases Systems. IPMU'98*, Paris, France, 1998, pp. 152–157.
10. J. Kacprzyk, and S. Zadrożny, Computing with words in decision making through individual and collective linguistic choice rules, *Int. J. Uncertain. Fuzziness Knowl.-Based Syst.* **9** (Supplement) (2001) 89–102.
11. S. Zadrożny, Collective choice rules in group decision making under fuzzy preferences and fuzzy majority: a unified OWA operator based approach, *Control and Cybernetics*, **31** (2002) 937–948.
12. J. Kacprzyk and S. Zadrożny, Dealing with imprecise knowledge on preferences and majority in group decision making: towards a unified characterization of individual and collective choice functions, *Bulletin of the Polish Academy of Sciences* (*Tech. Sci.*) **51** (2003) 279–302.
13. J. Kacprzyk and S. Zadrożny, Soft computing and web intelligence for supporting consensus reaching, *Soft Computing* **14** (8) (2010) 833–846.
14. J. Kacprzyk and S. Zadrożny, Linguistically quantified propositions for consensus reaching support, *Proc. IEEE Int. Conf. on Fuzzy Systems*, Budapest, Hungary, 25–29 July 2004, pp. 1135–1140.
15. E. Herrera-Viedma, F. Mata, L. Martinez and L. G. Pérez, An Adaptive Module for the Consensus Reaching Process in Group Decision Making Problems, *Proc. MDAI*, 2005, pp. 89–98.
16. E. Herrera-Viedma, L. Martinez, F. Mata and F. Chiclana, A consensus support system model for group decision-making problems with multi-granular linguistic preference relations, *IEEE Trans. on Fuzzy Systems* **13**(5) (2005) 644–658.
17. Z. W. Raś and A. Wieczorkowska, Action-Rules: How to increase profit of a company, *LNAI* **1910** (2000) 587–592.
18. Z. Pawlak, Information systems theoretical foundations, *Inf. Syst. Journal* **6**(3) (1981) 205–218.
19. A. Dardzińska and Z. W. Raś, Extracting Rules from Incomplete Decision Systems: System ERID, *Foundations and Novel Approaches in Data Mining*, Studies in Computational Intelligence, Vol. 9 (Springer, 2006), pp. 143–153.
20. Z. W. Raś and A. Dardzińska, Action rules discovery without pre-existing classification rules, *LNAI* **5306** (2008) 181–190.
21. Z. W. Raś, L.-S. Tsay and A. Dardzińska, Tree-based algorithms for action rules discovery, *Mining Complex Data*, Studies in Computational Intelligence, Vol. 165 (Springer, 2009), pp. 153–163.

22. A. Tzacheva and Z. W. Raś, Action rules mining, *Int. J. Intelligent Systems* **20**(7) (2005) 719–736.
23. A. Tzacheva and Z. W. Raś, Constraint based action rule discovery with single classification rules, *LNAI* **4482** (2007) 322–329.
24. L.-S. Tsay, Z.W. Raś, and A. Wieczorkowska, Tree-based algorithm for discovering extended action-rules (System DEAR2), *Intelligent Information Systems* (2004), pp. 459–464.
25. R. Agrawal and R. Srikant, Fast algorithms for mining association rules, *Proc. 20th Int. Conf. on Very Large Databases*, eds. J. B. Bocca, M. Jarke and C. Zaniolo (Morgan Kaufmann Publishers Inc., San Francisco, 1994), pp. 487–499.
26. S. Alonso, F. J. Cabrerizo, F. Chiclana, F. Herrera and E. Herrera-Viedma, Group decision making with incomplete fuzzy linguistic preference relations, *Int. J. Intell. Syst.* **24**(2) (2009) 201–222.
27. K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **20** (1986) 87–96.
28. K. T. Atanassov, *Intuitionistic Fuzzy Sets* (Physica-Verlag, Heidelberg/New York, 1999).
29. F. J. Cabrerizo, S. Alonso and E. Herrera-Viedma, A Consensus Model for Group Decision Making Problems with Unbalanced Fuzzy Linguistic Information, *Int. J. Information Technology and Decision Making* **8**(1) (2009) 109–131.
30. M. Delgado, J. L. Verdegay and M. A. Vila, On aggregation operations of linguistic labels, *IJIS* **8** (1993) 351–370.
31. D. Dubois and H. Prade, An introduction to bipolar representations of information and preference, *Int. J. Intelligent Systems* **23**(8) (2008) 866–877.
32. J.-F. Bonnefon, D. Dubois, H. Fargier and S. Leblois, Qualitative heuristics for balancing the pros and cons, *Theory and Decision* **65** (2008) 71–95.
33. F. Herrera, E. Herrera-Viedma and J. L. Verdegay, A model of consensus in group decision making under linguistic assessments, *Fuzzy Sets and Systems* **78** (1996) 73–88.
34. F. Herrera and E. Herrera-Viedma, Choice functions and mechanisms for linguistic preference relations, *European J. Operational Research* **120** (2000) 144–161.
35. J. Kacprzyk, On some fuzzy cores and “soft” consensus measures in group decision making, *The Analysis of Fuzzy Information*, eds. J. C. Bezdek, Vol. 2 (CRC Press, Boca Raton, 1987), pp. 119–130.
36. E. Szmidt and J. Kacprzyk, Using intuitionistic fuzzy sets in group decision making, *Control and Cybernetics* **31**(4) (2002).
37. E. Szmidt and J. Kacprzyk, A consensus-reaching process under intuitionistic fuzzy preference relations, *Int. J. Intelligent Systems* **18**(7) (2003) 837–852.
38. E. Szmidt and J. Kacprzyk, A new concept of a similarity measure for intuitionistic fuzzy sets and its use in group decision making, *LNCS* **3558** (2005) 272–282.
39. Z. Xu and R. R. Yager, Intuitionistic and interval-valued intuitionistic fuzzy preference relations and their measures of similarity for the evaluation of agreement within a group, *Fuzzy Optimization and Decision Making* **8** (2009) 123–139.
40. L. A. Zadeh, A computational approach to fuzzy quantifiers in natural languages, *Computers and Mathematics with Applications* **9** (1983) 149–184.
41. J. Kacprzyk and S. Zadrożny, Towards a general and unified characterization of individual and collective choice functions under fuzzy and nonfuzzy preferences and majority via the ordered weighted average operators, *Int. J. Intelligent Systems* **24** (1) (2009) 4–26.