# REDUCTS IN INCOMPLETE INFORMATION SYSTEMS

Zbigniew Ras

# Information Systems

S = (X, AT) is an information system, where

- **X** - objects,
- **AT-**attributes **(**partial functions from X into $2^{V_a} \cup \{*\}$**)**,
- $\mathbf{V_a}$ - set of values of attribute a.

# Example 1:

S = ({1,2,3,4,5,6}, {Price, Mileage, Size, Accident}) defined below:

| **Car** | **Price** | **Mileage** | **Size** | **Accident** |
|---|---|---|---|---|
| 1 | {high} | {high} | {full} | {doors, engine} |
| 2 | {low} | * | {full, compact} | {engine} |
| 3 | * | * | {compact} | {doors} |
| 4 | {high} | * | {full} | {doors} |
| 5 | * | * | {full} | {doors} |
| 6 | {low} | {high} | {full} | * |

- Let $A \subseteq AT$. By similarity relation based on A we mean:

$SIM(A) = \{(x,y) \in X \times X: (\forall a \in A)[a(x) \cap a(y) \neq \emptyset \text{ or } a(x) = * \text{ or } a(y) = *]\}$.

SIM(A) is a **tolerance relation** (reflexive, symmetric).

- Let $I_A(x) = \{y \in X: (x,y) \in SIM(A)\}$ - *tolerance class for x* with regard to A.

$X/SIM(A) = \{I_A(x) : x \in X\}$ – not a partition of X in general.

# Definition:

$A \subseteq AT$ is a reduct of information system $S = (X, AT)$ iff

- $SIM(A) = SIM(AT)$ and
- $(\forall B \subset A)[SIM(B) \neq SIM(A)]$.

$A \subseteq AT$ is a reduct of information system $S=(X, AT)$ for $x$ iff

- $I_A(x) = I_{AT}(x)$ and
- $(\forall B \subset A)[I_B(x) \neq I_A(x)]$.

| **Car** | **Price** | **Mileage** | **Size** | **Accident** |
|---|---|---|---|---|
| 1 | {high} | {high} | {full} | {doors, engine} |
| 2 | {low} | * | {full, compact} | {engine} |
| 3 | * | * | {compact} | {doors} |
| 4 | {high} | * | {full} | {doors} |
| 5 | * | * | {full} | {doors} |
| 6 | {low} | {high} | {full} | * |

Removing M will not change discernibility matrix

Reduct={P, S, A} ??

| x/y | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| **1** | - | | | | | |
| **2** | P | - | | | | |
| **3** | S | A | - | | | |
| **4** | NIL | P, A | S | - | | |
| **5** | NIL | A | S | NIL | - | |
| **6** | P | NIL | S | P | NIL | - |

# Decision Systems

S = (X, AT $\cup$ {d}) decision system, where

X - objects,

AT - classification attributes,

d - decision attribute,

where d(x) $\in V_d$ (value is certain).

| Car | Price | Size | Accident | d |
|---|---|---|---|---|
| 1 | {high} | {full} | {doors, engine} | good |
| 2 | {low} | {full, compact} | {engine} | good |
| 3 | * | {compact} | {doors} | poor |
| 4 | {high} | {full} | {doors} | good |
| 5 | * | {full} | {doors} | excel |
| 6 | {low} | {full} | * | good |

T(1)={1,4,5}
T(2)={2,6}
T(3)={3}
T(4)={4,1,5}
T(5)={5,1,4,6}
T(6)={6,5,2}

System S has to be consistent!

| Car | Price | Size | Accident | d |
| --- | --- | --- | --- | --- |
| 1 | {high} | {full} | {doors, engine} | good |
| 2 | {low} | {full, compact} | {engine} | good |
| 3 | * | {compact} | {doors} | poor |
| 4 | {high} | {full} | {doors} | good |
| 5 | * | {full} | {doors} | excel |
| 6 | {low} | {full} | * | good |

T(1)={1,4,5}, T(2)={2,6}, T(3)={3}, T(4)={4,1,5}, T(5)={5,1,4,6}, T(6)={6,5,2}

| Car | Price | Size | Accident | d | $\delta_{AT}$ |
| --- | --- | --- | --- | --- | --- |
| 1 | {high} | {full} | {doors, engine} | good | {good, excel} |
| 2 | {low} | {full, compact} | {engine} | good | {good} |
| 3 | * | {compact} | {doors} | poor | {poor} |
| 4 | {high} | {full} | {doors} | good | {good, excel} |
| 5 | * | {full} | {doors} | excel | {good, excel} |
| 6 | {low} | {full} | * | good | {good, excel} |

# Example 2:

Decision System S with "*generalized decision*" as the extra feature.

| **Car** | **Price** | **Size** | **Accident** | $\delta_{AT}$ |
|---|---|---|---|---|
| 1 | {high} | {full} | {doors, engine} | {good, excel} |
| 2 | {low} | {full, compact} | {engine} | {good} |
| 3 | * | {compact} | {doors} | {poor} |
| 4 | {high} | {full} | {doors} | {good, excel} |
| 5 | * | {full} | {doors} | {good, excel} |
| 6 | {low} | {full} | * | {good, excel} |

**Definition:**

Set $A \subseteq AT$ is a reduct of S (relative reduct or d-reduct), iff
$\delta_A = \delta_{AT}$ and $(\forall B \subseteq A)[\ \delta_B \neq \delta_A\ ]$.

Set $A \subseteq AT$ is a reduct of S for $x \in X$ (relative reduct for x or d-reduct for x) iff
$\delta_A(x) = \delta_{AT}(x)$ and $(\forall B \subseteq A)[\ \delta_B(x) \neq \delta_A(x)\ ]$.

# Computation of $\delta_{AT}$ -Reducts:

**Discernibility table**

| x/y | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **1** | | | | | | |
| **2** | P | | | | | |
| **3** | S | A | | | | |
| **4** | - | P, A | S | | | |
| **5** | - | A | S | | - | |
| **6** | - | NIL | S | P | - | - |

| Car | Price | Size | Accident | $\delta_{AT}$ |
|---|---|---|---|---|
| 1 | {high} | {full} | {doors, engine} | {good, excel} |
| 2 | {low} | {full, compact} | {engine} | {good} |
| 3 | * | {compact} | {doors} | {poor} |
| 4 | {high} | {full} | {doors} | {good, excel} |
| 5 | * | {full} | {doors} | {good, excel} |
| 6 | {low} | {full} | * | {good, excel} |

**Discernibility Function:**

$F(P,S,A) = P\wedge S\wedge A\wedge(P\vee A) \wedge NIL = NIL$

Reducts for objects: $F(1)=P\wedge S$, $F(2)=NIL$, $F(3)=S\wedge A$, $F(4)=(P\vee A)\wedge S\wedge P$, $F(5)=S\wedge A$, $F(6)=NIL$.

| Car | Price | Size | Accident | $\delta_{AT}$ |
|---|---|---|---|---|
| 1 | {high} | {full} | {doors, engine} | {good, excel} |
| 2 | {low} | {full, compact} | {engine} | {good} |
| 3 | * | {compact} | {doors} | {poor} |
| 4 | {high} | {full} | {doors} | {good, excel} |
| 5 | * | {full} | {doors} | {good, excel} |
| 6 | {low} | {full} | * | {good, excel} |

Reducts for objects:
F(1)=$P \wedge S$, F(2)=NIL, F(3)=$S \wedge A$, F(4)=$(P \vee A) \wedge S \wedge P = P \wedge S$, F(5)=$S \wedge A$, F(6)=NIL.

**Rules:**

(Price, high) $\wedge$ (Size, full) -> good $\vee$ excel   sup = 2
(Size, compact) $\wedge$ (Accident, doors) -> poor   sup = 1
(Price, high) $\wedge$ (Size, full) -> good $\vee$ excel
(Size, full) $\wedge$ (Accident, doors) -> good $\vee$ excel   sup = 1

# Thank You !