

Skolemization –

Process of converting any predicate calculus wff formula to a set of clauses.

Example:

$$(\forall x)[P(x) \rightarrow ((\forall y)[P(y) \rightarrow P(f(x,y))]) \wedge (\forall y)[Q(x,y) \rightarrow P(y)]$$

Eliminate implication symbols

$$(\forall x)[\neg P(x) \vee ((\forall y)[\neg P(y) \vee P(f(x,y))]) \wedge (\forall y)[\neg Q(x,y) \vee P(y)]$$

Reduce scopes of negation symbols (can be applied only to atomic formulas)

$$(\forall x)[\neg P(x) \vee ((\forall y)[\neg P(y) \vee P(f(x,y))]) \wedge (\forall y)[\neg Q(x,y) \vee P(y)]$$

Standardize variables

$$(\forall x)[\neg P(x) \vee ((\forall y)[\neg P(y) \vee P(f(x,y))]) \wedge (\forall z)[\neg Q(w,z) \vee P(z)]$$

Eliminate existential quantifiers (introduce Skolem functions)

$$(\forall x)[\neg P(x) \vee ((\forall y)[\neg P(y) \vee P(f(x,y))]) \wedge (\forall z)[\neg Q(w,z) \vee P(z)]$$

Move universal quantifiers to the front

$$(\forall x)(\forall y)(\forall z)[\neg P(x) \vee (\neg P(y) \vee P(f(x,y)))] \wedge [\neg Q(w,z) \vee P(z)]$$

Eliminate universal quantifiers

$$[\neg P(x) \vee (\neg P(y) \vee P(f(x,y)))] \wedge [\neg Q(w,z) \vee P(z)]$$

Convert to conjunctive normal form

$$[\neg P(x) \vee \neg P(y) \vee P(f(x,y))] \wedge [\neg Q(w,z) \vee P(z)]$$

Eliminate “ $\wedge$ ” symbol

$$\neg P(x) \vee \neg P(y) \vee P(f(x,y)), \neg Q(w,z) \vee P(z)$$

Rename variables

$$\neg P(x) \vee \neg P(y) \vee P(f(x,y)), \neg Q(w,z) \vee P(z)$$

