Chase

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Algorithm Chase

<u>GIVEN:</u>

- Incomplete Information System (*IIS*)
- Constraints (functional dependencies,..) which *IIS* satisfies

[$Dept \rightarrow Chair, Chair * Dept \rightarrow Faculty\text{-}Name, \ldots \ Dept(x_1) = Dept(x_2)$]

X	***Faculty-Name***	***Dept.***	***Chair***
x_1	Bob		
x_2	John		Jones
x_3	Mike		
x_4		EE	
x_5	Tom	EE	

Tableau System for *IIS* –

information system with null values replaced by variables

X	***Faculty Name***	***Department***	***Chair***
x_1	Bob	v_d	n_1
x_2	John	v_d	Jones
x_3	Mike	n_2	n_3
x_4	v_E	EE	n_4
x_5	Tom	EE	n_5

X	*Faculty Name*	*Department*	*Chair*
x_1	Bob	v_d	n_1
x_2	John	v_d	Jones
x_3	Mike	n_2	n_3
x_4	v_E	EE	n_4
x_5	Tom	EE	n_5

Functional Dependencies:

[*Department* → *Chair*]

[*Department* **Chair* → *Faculty Name*]

X	*Faculty Name*	*Department*	*Chair*
x_1	Bob	v_d	Jones
x_2	John	v_d	Jones
x_3	Mike	n_2	n_3
x_4	*Tom*	EE	n_4
x_5	Tom	EE	n_4

Algorithm Chase

Input: tableaux system S and set of functional dependencies F

Output: tableaux system $CHASE_F(S)$

Begin

$S_1 := S$;

while there are $t_1, t_2 \in S_1$ and $(B \rightarrow b) \in F$

such that $t_1[B] = t_2[B]$ and $t_1[b] < t_2[b]$

do change all the occurrences of the value

$t_2[b]$ in S_1 to $t_1[b]$

$CHASE_F(S) := S_1$

End

Algorithm Chase

Input: tableaux system S and set of functional dependencies F

Output: tableaux system $CHASE_F(S)$

Begin

$S_1 := S$;

while there are $t_1, t_2 \in S_1$ and $(B \rightarrow b) \in F$

such that $t_1[B] = t_2[B]$ and $t_1[b] < t_2[b]$

do change all the occurrences of the value

$t_2[b]$ in S_1 to $t_1[b]$

$CHASE_F(S) := S_1$

End

The algorithm always terminates if applied to a finite tableaux system. If one execution of the algorithm generates a tableaux system satisfying F, then every execution of the algorithm generates the same tableaux system.

Algorithm Chase 1

Chase supported by rules extracted from *IS* (*Chase 1*)

1. *Chase 1* identifies all incomplete attributes (their values are called *concepts*) in *IS* .
2. Main Algorithm
   - Extraction of rules from *IS* describing these *concepts*,
   - Null values in *IS* are replaced by values suggested by these rules.
3. These two steps are repeated till fixpoint is reached.

Example (*Chase1*)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

$S = (X, A, V)$

$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$

$A = \{b, c, d, e, f, g\}$

<u>Attribute ***b***</u>

$e_2 \rightarrow b_1$ (support 2),
$c_1 * f_1 \rightarrow b_1$ (support 2),
$g_2 \rightarrow b_2$ (support 2),
$c_3 \rightarrow b_3$ (support 1),
$c_2 \rightarrow b_2$ (support 2),
$g_3 * d_2 \rightarrow b_2$ (support 1),
$e_3 * d_3 \rightarrow b_3$ (support 1),
$f_2 * d_2 \rightarrow b_2$ (support 1).

Example (*Chase1*)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

Attribute ***b***

Two null values in S: $b(x_6)$, $b(x_8)$

$b(x_6)$:

$e_2 \rightarrow b_1$ (support 2),
$c_1 * f_1 \rightarrow b_1$ (support 2),
$g_2 \rightarrow b_2$ (support 2),
$c_3 \rightarrow b_3$ (support 1),
$c_2 \rightarrow b_2$ (support 2),
$g_3 * d_2 \rightarrow b_2$ (support 1),
$e_3 * d_3 \rightarrow b_3$ (support 1),
$f_2 * d_2 \rightarrow b_2$ (support 1).

Example (*Chase1*)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

Attribute ***b***

Two null values in S: $b(x_6)$, $b(x_8)$

$b(x_6)$:

$e_2 \rightarrow b_1$ (support 2),
$c_1 * f_1 \rightarrow b_1$ (support 2),
$g_2 \rightarrow b_2$ (support 2),
$c_3 \rightarrow b_3$ (support 1),
$c_2 \rightarrow b_2$ (support 2),
$g_3 * d_2 \rightarrow b_2$ (support 1),
$e_3 * d_3 \rightarrow b_3$ (support 1),
$f_2 * d_2 \rightarrow b_2$ (support 1).

Example (*Chase1*)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

$b(x_6) = b_2$

Attribute ***b***

Two null values in S: $b(x_6)$, $b(x_8)$

$b(x_8)$:

$e_2 \to b_1$ (support 2),
$c_1 * f_1 \to b_1$ (support 2),
$g_2 \to b_2$ (support 2),
$c_3 \to b_3$ (support 1),
$c_2 \to b_2$ (support 2),
$g_3 * d_2 \to b_2$ (support 1),
$e_3 * d_3 \to b_3$ (support 1),
$f_2 * d_2 \to b_2$ (support 1).

Example (*Chase1*)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

$b(x_6) = b_2$

Two null values in *S*: $c(x_7)$, $c(x_8)$.

$c(x_7)$:

$b_1 \to c_1$ (support 2),
$e_2 \to c_1$ (support 1),
$f_4 \to c_1$ (support 1),
$g_1 \to c_1$ (support 2),
$b_2 * d_2 \to c_2$ (support 1),
$b_2 * e_1 \to c_2$ (support 1),
$b_2 * f_2 \to c_2$ (support 1),
$b_2 * g_3 \to c_2$ (support 1),
$d_2 * e_1 \to c_2$ (support 1),
$d_2 * g_3 \to c_2$ (support 1).

Example (*Chase1*)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

$b(x_6) = b_2$

Two null values in S: $c(x_7)$, $c(x_8)$.

$c(x_7)$:

$b_1 \to c_1$ (support 2),
$e_2 \to c_1$ (support 1),
$f_4 \to c_1$ (support 1),
$g_1 \to c_1$ (support 2),
$b_2 * d_2 \to c_2$ (support 1),
$b_2 * e_1 \to c_2$ (support 1),
$b_2 * f_2 \to c_2$ (support 1),
$b_2 * g_3 \to c_2$ (support 1),
$d_2 * e_1 \to c_2$ (support 1),
$d_2 * g_3 \to c_2$ (support 1).

Example (*Chase1*)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

$b(x_6) = b_2$, $c(x_7) = c_1$

Two null values in *S*: $c(x_7)$, $c(x_8)$.

$c(x_8)$:

$b_1 \rightarrow c_1$ (support 2),

$e_2 \rightarrow c_1$ (support 1),

$f_4 \rightarrow c_1$ (support 1),

$g_1 \rightarrow c_1$ (support 2),

$b_2 * d_2 \rightarrow c_2$ (support 1),

$b_2 * e_1 \rightarrow c_2$ (support 1),

$b_2 * f_2 \rightarrow c_2$ (support 1),

$b_2 * g_3 \rightarrow c_2$ (support 1),

$d_2 * e_1 \rightarrow c_2$ (support 1),

$d_2 * g_3 \rightarrow c_2$ (support 1).

Algorithm *Chase1*(S, $In(A)$, $L(D)$)

Input: System $S=(X, A, V)$
Set of incomplete attributes $In(A)=\{a_1, a_2, ..., a_k\}$
Set of rules $L(D)$

Output: System $Chase1(S)$

```
begin
  j:=1;   while j ≤ k do begin
                 S_j:=S
                 for all v∈V_aj do
                        while
                           there is x∈X and rule (t→v)∈L(D)
                           such that x∈N_Sj(t) and card(a_j(x))≠1
                        begin
                           a(x):=v;
                        end
               j:=j+1
             end
  S:=∪{S_j:1 ≤ j ≤ k},  Chase1 (S, In(A), L(D))
end
```

Rules Discovery from partially Incomplete Information Systems

Data (Incomplete)

Information System $S = (X, A, V)$

X - finite set of objects,

A - finite set of attributes,

$V = \bigcup \{ V_a : a \in A \}$ - set of their values.

Assumption

1. For any $a \in A$, $x \in X$

$$a(x) = \{(a_i, p_i) : i \in J_{a(x)} \wedge a_i \in V_a \wedge \sum_{i \in J_{a(x)}} p_i = 1\}$$

2. For any $a \neq b$

$$V_a \cap V_b = \emptyset$$

Example

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Algorithm *ERID* for Extracting Rules from partially Incomplete Information System

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	$(b_1,\frac{1}{3})$ $(b_2,\frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{2})$ $(c_3,\frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1,\frac{2}{3})$ $(e_2,\frac{1}{3})$
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

<u>Goal:</u>

Describe ***e*** in terms of {***a***,***b***,***c***,***d***}

$$a_1{}^* = \{(x_1,\tfrac{1}{3}),(x_3,1),(x_5,\tfrac{2}{3})\}$$

$$a_2{}^* = \{(x_1,\tfrac{2}{3}),(x_2,\tfrac{1}{4}),(x_5,\tfrac{1}{3}),(x_6,1),(x_7,1)\}$$

$$a_3{}^* = \{(x_2,\tfrac{3}{4}),(x_4,1),(x_8,1)\}$$

$$b_1{}^* = \{(x_1,\tfrac{1}{3}),(x_2,\tfrac{1}{3}),(x_4,\tfrac{1}{2}),(x_5,1),(x_7,\tfrac{1}{4})\}$$

$$b_2{}^* = \{(x_1,\tfrac{1}{3}),(x_2,\tfrac{2}{3}),(x_3,1),(x_4,\tfrac{1}{2}),(x_6,1),(x_7,\tfrac{3}{4}),(x_8,1)\}$$

Algorithm *ERID* for Extracting Rules from partially Incomplete Information System

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	$(b_1,\frac{1}{3})$ $(b_2,\frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{2})$ $(c_3,\frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1,\frac{2}{3})$ $(e_2,\frac{1}{3})$
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

<u>Goal:</u>

Describe ***e*** in terms of {***a***,***b***,***c***,***d***}

$$c_1^* = \{(x_1,1),(x_2,\tfrac{1}{3}),(x_3,\tfrac{1}{2}),(x_7,\tfrac{1}{3}),(x_8,1)\}$$

$$c_2^* = \{(x_2,\tfrac{1}{3}),(x_4,1),(x_5,1),(x_2,\tfrac{2}{3})\}$$

$$c_3^* = \{(x_2,\tfrac{1}{3}),(x_3,\tfrac{1}{2}),(x_6,1)\}$$

$$d_1^* = \{(x_1,1),(x_4,1),(x_5,\tfrac{1}{2}),(x_8,1)\}$$

$$d_2^* = \{(x_2,1),(x_3,1),(x_5,\tfrac{1}{2}),(x_6,1),(x_7,1)\}$$

Algorithm *ERID*

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe ***e*** in terms of {***a***,***b***,***c***,***d***}

For the values of the decision attribute we have:

$$e_1{}^* = \{(x_1, \tfrac{1}{2}), (x_2, 1), (x_4, \tfrac{2}{3}), (x_5, 1)\}$$

$$e_2{}^* = \{(x_1, \tfrac{1}{2}), (x_4, \tfrac{1}{3}), (x_6, \tfrac{1}{3}), (x_7, 1)\}$$

$$e_3{}^* = \{(x_3, 1), (x_6, \tfrac{2}{3}), (x_8, 1)\}$$

Algorithm *ERID*

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	$(b_1,\frac{1}{3})$ $(b_2,\frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{2})$ $(c_3,\frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1,\frac{2}{3})$ $(e_2,\frac{1}{3})$
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe ***e*** in terms of {***a***,***b***,***c***,***d***}.

2. Check the relationship “ $\prec$ ” between values of classification attributes {***a***,***b***,***c***,***d***} and values of decision attribute ***e***

Algorithm *ERID*

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	$(b_1,\frac{1}{3})$ $(b_2,\frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{2})$ $(c_3,\frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1,\frac{2}{3})$ $(e_2,\frac{1}{3})$
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe ***e*** in terms of {***a***,***b***,***c***,***d***}

Let $c_i^* = \{(x_i, p_i)\}_{i\in N}, e_j^* = \{(y_j, q_j)\}_{j\in N}$.

We say that:

$c_i^* \prec e_j^*$ iff support

and confidence of the rule $c_i \to e_j$

are above some threshold values.

Algorithm *ERID*

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	$(b_1,\frac{1}{3})$ $(b_2,\frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{2})$ $(c_3,\frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1,\frac{2}{3})$ $(e_2,\frac{1}{3})$
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe ***e*** in terms of {***a***,***b***,***c***,***d***}

Let $c_i^* = \{(x_i, p_i)\}_{i\in N}$, $e_j^* = \{(y_j, q_j)\}_{j\in N}$

We say that:

$c_i^* \prec e_j^*$ iff support

and confidence of the rule $c_i \rightarrow e_j$

are above some threshold values.

How to define support and confidence of a rule $c_i \rightarrow e_j$?

Definition of Support and Confidence (by example)

To define support and confidence of the rule $a_1 \to e_3$ we compute:

$$a_1{}^* = \{(x_1, \tfrac{1}{3}), (x_3, 1), (x_5, \tfrac{2}{3})\}$$

$$e_3{}^* = \{(x_3, 1), (x_6, \tfrac{2}{3}), (x_8, 1)\}$$

Support of the rule: $\sup(a_1 \to e_3) = \tfrac{1}{3} \cdot 0 + 1 \cdot 1 + \tfrac{2}{3} \cdot 0 = 1$

Support of the term a_1: $\sup(a_1) = \tfrac{1}{3} + 1 + \tfrac{2}{3} = 2$

Confidence of the rule: $conf(a_1 \to e_3) = \dfrac{\sup(a_1 \to e_3)}{\sup(a_1)}$

Extracting Rules from partially Incomplete Information System (Algorithm *ERID*(λ1, λ2))

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	$(b_1,\frac{1}{3})$ $(b_2,\frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{2})$ $(c_3,\frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1,\frac{2}{3})$ $(e_2,\frac{1}{3})$
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe ***e*** in terms of {***a***,***b***,***c***,***d***}

Thresholds (provided by user):

Minimal support ($\lambda_1 = 1$)

Minimal confidence ($\lambda_2 = ½$)

$a_1^* \prec e_1^*$ $(\sup = \frac{5}{6} < 1)$ - marked negative

$a_1^* \prec e_2^*$ $(\sup = \frac{1}{6} < 1)$ - marked negative

$a_1^* \prec e_3^*$ $(\sup = 1 \geq 1)$ $(conf = 0.5)$

- marked positive

Algorithm *ERID*(λ1, λ2)

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$a_3^* \prec e_3^*$ $(\sup = 1 \geq 1)$ but $(conf = 0.36)$

$b_2^* \prec e_1^*$ $(\sup = \frac{7}{6} \geq 1)$ but $(conf = 0.22)$

$b_2^* \prec e_2^*$ $(\sup = \frac{17}{12} \geq 1)$ but $(conf = 0.27)$

$c_1^* \prec e_3^*$ $(\sup = \frac{3}{2} \geq 1)$ but $(conf = 0.47)$

$c_2^* \prec e_2^*$ $(\sup = 1 \geq 1)$ but $(conf = 0.33)$

$d_1^* \prec e_1^*$ $(\sup = \frac{5}{3} \geq 1)$ but $(conf = 0.48)$

$d_1^* \prec e_3^*$ $(\sup = 1 \geq 1)$ but $(conf = 0.28)$

$d_2^* \prec e_1^*$ $(\sup = \frac{3}{2} \geq 1)$ but $(conf = 0.33)$

$d_2^* \prec e_3^*$ $(\sup = \frac{5}{3} \geq 1)$ but $(conf = 0.37)$

Algorithm *ERID*(λ1, λ2)

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	$(b_1,\frac{1}{3})$ $(b_2,\frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{2})$ $(c_3,\frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1,\frac{2}{3})$ $(e_2,\frac{1}{3})$
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$a_3^* \prec e_3^*$ $(sup = 1 \geq 1)$ but $(conf = 0.36)$

$b_2^* \prec e_1^*$ $(sup = \frac{7}{6} \geq 1)$ but $(conf = 0.22)$

$b_2^* \prec e_2^*$ $(sup = \frac{17}{12} \geq 1)$ but $(conf = 0.27)$

$c_1^* \prec e_3^*$ $(sup = \frac{3}{2} \geq 1)$ but $(conf = 0.47)$

$c_2^* \prec e_2^*$ $(sup = 1 \geq 1)$ but $(conf = 0.33)$

$d_1^* \prec e_1^*$ $(sup = \frac{5}{3} \geq 1)$ but $(conf = 0.48)$

$d_1^* \prec e_3^*$ $(sup = 1 \geq 1)$ but $(conf = 0.28)$

$d_2^* \prec e_1^*$ $(sup = \frac{3}{2} \geq 1)$ but $(conf = 0.33)$

$d_2^* \prec e_3^*$ $(sup = \frac{5}{3} \geq 1)$ but $(conf = 0.37)$

They all are not marked

Algorithm *ERID*(λ1, λ2)

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$(a_3 \cdot c_1)^* \prec e_3^* \, (\sup = 1 \geq 1)$ and $(conf = 0.8)$

$(a_3 \cdot d_1)^* \prec e_3^* \, (\sup = 1 \geq 1)$ and $(conf = 0.5)$

$(a_3 \cdot d_2)^* \prec e_3^* \quad (\sup = 0 < 1)$

$(b_2 \cdot d_2)^* \prec e_1^* \quad (\sup = \frac{2}{3} < 1)$

$(b_2 \cdot c_2)^* \prec e_2^* \quad (\sup = \frac{1}{2} < 1)$

Algorithm *ERID*(λ1, λ2)

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	$(b_1,\frac{1}{3})$ $(b_2,\frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{2})$ $(c_3,\frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1,\frac{2}{3})$ $(e_2,\frac{1}{3})$
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$(a_3 \cdot c_1)^* \prec e_3^* \, (\sup = 1 \geq 1)$ and $(conf = 0.8)$

$(a_3 \cdot d_1)^* \prec e_3^* \, (\sup = 1 \geq 1)$ and $(conf = 0.5)$

They all are marked positive.

$(a_3 \cdot d_2)^* \prec e_3^* \quad (\sup = 0 < 1)$

$(b_2 \cdot d_2)^* \prec e_1^* \quad (\sup = \frac{2}{3} < 1)$

$(b_2 \cdot c_2)^* \prec e_2^* \quad (\sup = \frac{1}{2} < 1)$

Algorithm *ERID*(λ1, λ2)

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$(a_3 \cdot c_1)^* \prec e_3{}^* (\sup = 1 \geq 1)$ and $(conf = 0.8)$

$(a_3 \cdot d_1)^* \prec e_3{}^* (\sup = 1 \geq 1)$ and $(conf = 0.5)$

They all are marked positive.

$(a_3 \cdot d_2)^* \prec e_3{}^* \quad (\sup = 0 < 1)$

$(b_2 \cdot d_2)^* \prec e_1{}^* \quad (\sup = \frac{2}{3} < 1)$

$(b_2 \cdot c_2)^* \prec e_2{}^* \quad (\sup = \frac{1}{2} < 1)$

They all are marked negative.

Algorithm *ERID*(λ1, λ2)

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

The algorithm continues for terms of length 3, 4, … till all of them have either positive or negative marks.

Rules are automatically constructed from relations marked positive.

Algorithm Chase 2
(for Partially *IIS*)

Algorithm Chase 2

$S = (X, A, V)$ - partially incomplete information system of type λ, if S is incomplete and the following three conditions hold:

- $a_S(x)$ is defined for any $x \in X$, $a \in A$
- $(\forall x \in X)(\forall a \in A)[(a_S(x) = \{(a_i, p_i) : 1 \le i \le m\}) \rightarrow \sum p_i = 1]$
- $(\forall x \in X)(\forall a \in A)[(a_S(x) = \{(a_i, p_i) : 1 \le i \le m\}) \rightarrow (\forall i)(p_i \ge \lambda)]$

Algorithm Chase 2

S_1, S_2 - partially incomplete, both of type λ and both classifying the same sets of objects (from X) using the same sets of attributes (A)

Let $a_{S_1}(x) = \{a_{1i}, p_{1i}) : 1 \le i \le m_1\}$ and $a_{S_2}(x) = \{(a_{2i}, p_{2i}) : 1 \le i \le m_2\}$.

The pair (S_1, S_2) satisfies containment relation Ψ (or $\Psi(S_1) = S_2$) if:

- $(\forall x \in X)(\forall a \in A)[card(a_{S_1}(x)) > card(a_{S_2}(x))]$ or

$$[[card(a_{S_1}(x)) = card(a_{S_2}(x))] \rightarrow [\sum_{i \neq j} | p_{2i} - p_{2j} | > \sum_{i \neq j} | p_{1i} - p_{1j} |]]$$

We also denote that fact by $(\forall x \in X)(\forall a \in A)[\Psi(a_{S_1}(x)) = a_{S_2}(x)]$

System S_1

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	$(b_1,\frac{1}{3})$ $(b_2,\frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{2})$ $(c_3,\frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1,\frac{2}{3})$ $(e_2,\frac{1}{3})$
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_2
x_8	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_2	c_1	d_1	e_3

System S_2

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	b_2	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{3})$ $(c_3,\frac{2}{3})$	d_2	e_3
x_4	a_3		c_2	d_1	e_2
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	c_2	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Incomplete Information System *S* of type $\lambda = 0.3$

ERID(λ_1, λ_2) $\lambda_1=1, \lambda_2=0.5$

$r_1 = [a_1 \rightarrow e_3]$ $\sup(r_1) = 1, conf(r_1) = 0.5$

$r_2 = [a_2 \rightarrow e_2]$ $\sup(r_2) = \frac{5}{3}, conf(r_2) = 0.51$

$r_4 = [b_1 \rightarrow e_1]$ $\sup(r_4) = 2, conf(r_4) = 0.72$

$r_5 = [b_2 \rightarrow e_3]$ $\sup(r_5) = \frac{8}{3}, conf(r_5) = 0.51$

$r_{10} = [c_1 \cdot d_1 \rightarrow e_3]$ $\sup(r_{10}) = 1, conf(r_{10}) = 0.5$

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Incomplete Information System *S* of type $\lambda = 0.3$

ERID(λ_1, λ_2) $\lambda_1=1, \lambda_2=0.5$

Algorithm *Chase* 2 will try to replace

$$e(x_1) = \{(e_1, \tfrac{1}{2}), (e_2, \tfrac{1}{2})\}$$

by $e_{new}(x_1) = \{(e_1,), (e_2,), (e_3,)\}$.

We will show that $\Psi(e(x_1)) = e_{new}(x_1)$ (the value $e(x_1)$ will be changed by *Chase* 2).

X	a	b	c	d	e
x_1	$(a_1,\frac{1}{3})$ $(a_2,\frac{2}{3})$	$(b_1,\frac{2}{3})$ $(b_2,\frac{1}{3})$	c_1	d_1	$(e_1,\frac{1}{2})$ $(e_2,\frac{1}{2})$
x_2	$(a_2,\frac{1}{4})$ $(a_3,\frac{3}{4})$	$(b_1,\frac{1}{3})$ $(b_2,\frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1,\frac{1}{2})$ $(c_3,\frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1,\frac{2}{3})$ $(e_2,\frac{1}{3})$
x_5	$(a_1,\frac{2}{3})$ $(a_2,\frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2,\frac{1}{3})$ $(e_3,\frac{2}{3})$
x_7	a_2	$(b_1,\frac{1}{4})$ $(b_2,\frac{3}{4})$	$(c_1,\frac{1}{3})$ $(c_2,\frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Incomplete Information System *S* of type λ = 0.3

For x1:

$$(e_3, (\frac{1}{3}\cdot 1\cdot \frac{1}{2} + \frac{1}{3}\cdot \frac{8}{3}\cdot \frac{51}{100} + 1\cdot 1\cdot \frac{1}{2})/(14/3)) = (e_3, 0.24)$$

$$(e_2, (\frac{2}{3}\cdot \frac{5}{3}\cdot \frac{51}{100})/(5/3)) = (e_2, 0.97)$$

$$(e_1, (\frac{2}{3}\cdot 2\cdot \frac{72}{100})/2) = (e_1, 0.48)$$

we have:

$$e(x_1) = \{(e_1, 0.48), (e_2, 0.97), (e_3, 0.24)\}$$

Because the confidence assigned to e_3 is below the threshold λ, then only two values remain:

$(e_1, 0.48), (e_2, 0.97)$.

The value of attribute *e* assigned to x_1 is:

$\{(e_1, 0.33), (e_2, 0.67)\}$.

THANK YOU!