

Chase

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Algorithm Chase

GIVEN:

- Incomplete Information System (*IIS*)
- Constraints (functional dependencies,..) which *IIS* satisfies

[$Dept \rightarrow Chair$, $Chair * Dept \rightarrow Faculty-Name$, ... $Dept(x_1) = Dept(x_2)$]

<i>X</i>	<i>Faculty-Name</i>	<i>Dept.</i>	<i>Chair</i>
x_1	Bob		
x_2	John		Jones
x_3	Mike		
x_4		EE	
x_5	Tom	EE	

Tableau System for *IIS* –

information system with null values replaced by variables

X	<i>Faculty Name</i>	<i>Department</i>	<i>Chair</i>
x_1	Bob	v_d	n_1
x_2	John	v_d	Jones
x_3	Mike	n_2	n_3
x_4	v_E	EE	n_4
x_5	Tom	EE	n_5

X	<i>Faculty Name</i>	<i>Department</i>	<i>Chair</i>
x_1	Bob	v_d	n_1
x_2	John	v_d	Jones
x_3	Mike	n_2	n_3
x_4	v_E	EE	n_4
x_5	Tom	EE	n_5

Functional Dependencies:

$[Department \rightarrow Chair]$

$[Department * Chair \rightarrow Faculty Name]$

X	<i>Faculty Name</i>	<i>Department</i>	<i>Chair</i>
x_1	Bob	v_d	Jones
x_2	John	v_d	Jones
x_3	Mike	n_2	n_3
x_4	Tom	EE	n_4
x_5	Tom	EE	n_4

Algorithm Chase

Input: tableaux system S and set of functional dependencies F

Output: tableaux system $CHASE_F(S)$

Begin

$S_1 := S;$

while there are $t_1, t_2 \in S_1$ and $(B \rightarrow b) \in F$
such that $t_1[B] = t_2[B]$ and $t_1[b] < t_2[b]$

do change all the occurrences of the value
 $t_2[b]$ in S_1 to $t_1[b]$

$CHASE_F(S) := S_1$

End

Algorithm Chase

Input: tableaux system S and set of functional dependencies F

Output: tableaux system $CHASE_F(S)$

Begin

$S_1 := S;$

while there are $t_1, t_2 \in S_1$ and $(B \rightarrow b) \in F$
such that $t_1[B] = t_2[B]$ and $t_1[b] < t_2[b]$

do change all the occurrences of the value
 $t_2[b]$ in S_1 to $t_1[b]$

$CHASE_F(S) := S_1$

End

The algorithm always terminates if applied to a finite tableaux system. If one execution of the algorithm generates a tableaux system satisfying F , then every execution of the algorithm generates the same tableaux system.



Algorithm Chase 1

Chase supported by rules extracted from *IS* (*Chase 1*)

1. *Chase 1* identifies all incomplete attributes (their values are called *concepts*) in *IS* .
2. Main Algorithm
 - Extraction of rules from *IS* describing these *concepts*,
 - Null values in *IS* are replaced by values suggested by these rules.
3. These two steps are repeated till fixpoint is reached.

Example (Chase1)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

$$S = (X, A, V)$$

$$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$$

$$A = \{b, c, d, e, f, g\}$$

Attribute b

- $e_2 \rightarrow b_1$ (support 2),
- $c_1 * f_1 \rightarrow b_1$ (support 2),
- $g_2 \rightarrow b_2$ (support 2),
- $c_3 \rightarrow b_3$ (support 1),
- $c_2 \rightarrow b_2$ (support 2),
- $g_3 * d_2 \rightarrow b_2$ (support 1),
- $e_3 * d_3 \rightarrow b_3$ (support 1),
- $f_2 * d_2 \rightarrow b_2$ (support 1).

Example (*Chase1*)

<i>X</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

Attribute *b*

Two null values in S: $b(x_6)$, $b(x_8)$

$b(x_6)$:

- $e_2 \rightarrow b_1$ (support 2),
- $c_1 * f_1 \rightarrow b_1$ (support 2),
- $g_2 \rightarrow b_2$ (support 2),
- $c_3 \rightarrow b_3$ (support 1),
- $c_2 \rightarrow b_2$ (support 2),
- $g_3 * d_2 \rightarrow b_2$ (support 1),
- $e_3 * d_3 \rightarrow b_3$ (support 1),
- $f_2 * d_2 \rightarrow b_2$ (support 1).

Example (Chase1)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

Attribute b

Two null values in S : $b(x_6)$, $b(x_8)$

$b(x_6)$:

$e_2 \rightarrow b_1$ (support 2),

$c_1 * f_1 \rightarrow b_1$ (support 2),

$g_2 \rightarrow b_2$ (support 2),

$c_3 \rightarrow b_3$ (support 1),

$c_2 \rightarrow b_2$ (support 2),

$g_3 * d_2 \rightarrow b_2$ (support 1),

$e_3 * d_3 \rightarrow b_3$ (support 1),

$f_2 * d_2 \rightarrow b_2$ (support 1).

Example (Chase1)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

$$b(x_6) = b_2$$

Attribute b

Two null values in S : $b(x_6)$, $b(x_8)$

$b(x_8)$:

$$e_2 \rightarrow b_1 \quad (\text{support } 2),$$

$$c_1 * f_1 \rightarrow b_1 \quad (\text{support } 2),$$

$$g_2 \rightarrow b_2 \quad (\text{support } 2),$$

$$c_3 \rightarrow b_3 \quad (\text{support } 1),$$

$$c_2 \rightarrow b_2 \quad (\text{support } 2),$$

$$g_3 * d_2 \rightarrow b_2 \quad (\text{support } 1),$$

$$e_3 * d_3 \rightarrow b_3 \quad (\text{support } 1),$$

$$f_2 * d_2 \rightarrow b_2 \quad (\text{support } 1).$$

Example (Chase1)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

$$b(x_6) = b_2$$

Two null values in S : $c(x_7)$, $c(x_8)$.

$c(x_7)$:

$b_1 \rightarrow c_1$ (support 2),
 $e_2 \rightarrow c_1$ (support 1),
 $f_4 \rightarrow c_1$ (support 1),
 $g_1 \rightarrow c_1$ (support 2),
 $b_2 * d_2 \rightarrow c_2$ (support 1),
 $b_2 * e_1 \rightarrow c_2$ (support 1),
 $b_2 * f_2 \rightarrow c_2$ (support 1),
 $b_2 * g_3 \rightarrow c_2$ (support 1),
 $d_2 * e_1 \rightarrow c_2$ (support 1),
 $d_2 * g_3 \rightarrow c_2$ (support 1).

Example (*Chase1*)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

$$b(x_6) = b_2$$

Two null values in S : $c(x_7)$, $c(x_8)$.

$c(x_7)$:

$b_1 \rightarrow c_1$ (support 2),

$e_2 \rightarrow c_1$ (support 1),

$f_4 \rightarrow c_1$ (support 1),

$g_1 \rightarrow c_1$ (support 2),

$b_2 * d_2 \rightarrow c_2$ (support 1),

$b_2 * e_1 \rightarrow c_2$ (support 1),

$b_2 * f_2 \rightarrow c_2$ (support 1),

$b_2 * g_3 \rightarrow c_2$ (support 1),

$d_2 * e_1 \rightarrow c_2$ (support 1),

$d_2 * g_3 \rightarrow c_2$ (support 1).

Example (Chase1)

X	b	c	d	e	f	g
x_1	b_1	c_1		e_2	f_1	
x_2	b_2	c_2	d_2	e_1	f_2	g_3
x_3	b_1	c_1	d_3	e_1	f_1	g_1
x_4	b_3	c_3	d_3	e_3	f_1	g_3
x_5	b_2	c_2		e_3	f_1	g_2
x_6		c_1	d_2		f_2	g_1
x_7	b_1		d_2	e_2	f_4	g_1
x_8			d_2	e_2	f_2	g_3
x_9	b_3	c_1	d_1		f_2	
x_{10}	b_2	c_1		e_3	f_4	g_2

$$b(x_6) = b_2, \quad c(x_7) = c_1$$

Two null values in S : $c(x_7)$, $c(x_8)$.

$c(x_8)$:

$$b_1 \rightarrow c_1 \quad (\text{support } 2),$$

$$e_2 \rightarrow c_1 \quad (\text{support } 1),$$

$$f_4 \rightarrow c_1 \quad (\text{support } 1),$$

$$g_1 \rightarrow c_1 \quad (\text{support } 2),$$

$$b_2 * d_2 \rightarrow c_2 \quad (\text{support } 1),$$

$$b_2 * e_1 \rightarrow c_2 \quad (\text{support } 1),$$

$$b_2 * f_2 \rightarrow c_2 \quad (\text{support } 1),$$

$$b_2 * g_3 \rightarrow c_2 \quad (\text{support } 1),$$

$$d_2 * e_1 \rightarrow c_2 \quad (\text{support } 1),$$

$$d_2 * g_3 \rightarrow c_2 \quad (\text{support } 1).$$

Algorithm *Chase1*($S, In(A), L(D)$)

Input: System $S=(X, A, V)$

Set of incomplete attributes $In(A)=\{a_1, a_2, \dots, a_k\}$

Set of rules $L(D)$

Output: System *Chase1*(S)

begin

$j:=1$; **while** $j \leq k$ **do begin**

$S_j:=S$

for all $v \in V_{a_j}$ **do**

while

there is $x \in X$ and rule $(t \rightarrow v) \in L(D)$

such that $x \in N_{S_j}(t)$ and $\text{card}(a_{j(x)}) \neq 1$

begin

$a(x):=v$;

end

$j:=j+1$

end

$S:=\cup\{S_j: 1 \leq j \leq k\}$, *Chase1* ($S, In(A), L(D)$)

end

Rules Discovery
from partially
Incomplete Information Systems

Data (Incomplete)

Information System $S = (X, A, V)$

X - finite set of objects,

A - finite set of attributes,

$V = \bigcup \{ V_a : a \in A \}$ - set of their values.

Assumption

1. For any $a \in A$, $x \in X$

$$a(x) = \{ (a_i, p_i) : i \in J_{a(x)} \wedge a_i \in V_a \wedge \sum_{i \in J_{a(x)}} p_i = 1 \}$$

2. For any $a \neq b$

$$V_a \cap V_b = \emptyset$$

Example

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Algorithm *ERID* for Extracting Rules from partially Incomplete Information System

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe e in terms of $\{a, b, c, d\}$

$$a_1^* = \{(x_1, \frac{1}{3}), (x_3, 1), (x_5, \frac{2}{3})\}$$

$$a_2^* = \{(x_1, \frac{2}{3}), (x_2, \frac{1}{4}), (x_5, \frac{1}{3}), (x_6, 1), (x_7, 1)\}$$

$$a_3^* = \{(x_2, \frac{3}{4}), (x_4, 1), (x_8, 1)\}$$

$$b_1^* = \{(x_1, \frac{1}{3}), (x_2, \frac{1}{3}), (x_4, \frac{1}{2}), (x_5, 1), (x_7, \frac{1}{4})\}$$

$$b_2^* = \{(x_1, \frac{1}{3}), (x_2, \frac{2}{3}), (x_3, 1), (x_4, \frac{1}{2}), (x_6, 1), (x_7, \frac{3}{4}), (x_8, 1)\}$$

Algorithm *ERID* for Extracting Rules from partially Incomplete Information System

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe e in terms of $\{a, b, c, d\}$

$$c_1^* = \{(x_1, 1), (x_2, \frac{1}{3}), (x_3, \frac{1}{2}), (x_7, \frac{1}{3}), (x_8, 1)\}$$

$$c_2^* = \{(x_2, \frac{1}{3}), (x_4, 1), (x_5, 1), (x_2, \frac{2}{3})\}$$

$$c_3^* = \{(x_2, \frac{1}{3}), (x_3, \frac{1}{2}), (x_6, 1)\}$$

$$d_1^* = \{(x_1, 1), (x_4, 1), (x_5, \frac{1}{2}), (x_8, 1)\}$$

$$d_2^* = \{(x_2, 1), (x_3, 1), (x_5, \frac{1}{2}), (x_6, 1), (x_7, 1)\}$$

Algorithm *ERID*

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe e in terms of $\{a, b, c, d\}$

For the values of the decision attribute we have:

$$e_1^* = \{(x_1, \frac{1}{2}), (x_2, 1), (x_4, \frac{2}{3}), (x_5, 1)\}$$

$$e_2^* = \{(x_1, \frac{1}{2}), (x_4, \frac{1}{3}), (x_6, \frac{1}{3}), (x_7, 1)\}$$

$$e_3^* = \{(x_3, 1), (x_6, \frac{2}{3}), (x_8, 1)\}$$

Algorithm *ERID*

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe e in terms of $\{a, b, c, d\}$.

- Check the relationship “ \prec ” between values of classification attributes $\{a, b, c, d\}$ and values of decision attribute e

Algorithm *ERID*

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe e in terms of $\{a, b, c, d\}$

Let $c_i^* = \{(x_i, p_i)\}_{i \in N}$, $e_j^* = \{(y_j, q_j)\}_{j \in N}$.

We say that:

$c_i^* \prec e_j^*$ iff support

and confidence of the rule $c_i \rightarrow e_j$

are above some threshold values.

Algorithm *ERID*

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe e in terms of $\{a, b, c, d\}$

Let $c_i^* = \{(x_i, p_i)\}_{i \in N}$ $e_j^* = \{(y_j, q_j)\}_{j \in N}$

We say that:

$c_i^* \prec e_j^*$ iff support

and confidence of the rule $c_i \rightarrow e_j$

are above some threshold values.

*How to define
support and confidence
of a rule $c_i \rightarrow e_j$?*

Definition of Support and Confidence (by example)

To define support and confidence of the rule $a_1 \rightarrow e_3$ we compute:

$$a_1^* = \{(x_1, \frac{1}{3}), (x_3, 1), (x_5, \frac{2}{3})\}$$

$$e_3^* = \{(x_3, 1), (x_6, \frac{2}{3}), (x_8, 1)\}$$

Support of the rule: $\sup(a_1 \rightarrow e_3) = \frac{1}{3} \cdot 0 + 1 \cdot 1 + \frac{2}{3} \cdot 0 = 1$

Support of the term a_1 : $\sup(a_1) = \frac{1}{3} + 1 + \frac{2}{3} = 2$

Confidence of the rule: $\text{conf}(a_1 \rightarrow e_3) = \frac{\sup(a_1 \rightarrow e_3)}{\sup(a_1)}$

Extracting Rules from partially Incomplete Information System (Algorithm $ERID(\lambda_1, \lambda_2)$)

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Goal:

Describe e in terms of $\{a, b, c, d\}$

Thresholds (provided by user):

Minimal support ($\lambda_1 = 1$)

Minimal confidence ($\lambda_2 = 1/2$)

$a_1^* \prec e_1^*$ ($\text{sup} = \frac{5}{6} < 1$) - marked negative

$a_1^* \prec e_2^*$ ($\text{sup} = \frac{1}{6} < 1$) - marked negative

$a_1^* \prec e_3^*$ ($\text{sup} = 1 \geq 1$) ($\text{conf} = 0.5$)

- marked positive

Algorithm $ERID(\lambda_1, \lambda_2)$

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$$a_3^* \prec e_3^* \text{ (sup} = 1 \geq 1) \text{ but (conf} = 0.36)$$

$$b_2^* \prec e_1^* \text{ (sup} = \frac{7}{6} \geq 1) \text{ but (conf} = 0.22)$$

$$b_2^* \prec e_2^* \text{ (sup} = \frac{17}{12} \geq 1) \text{ but (conf} = 0.27)$$

$$c_1^* \prec e_3^* \text{ (sup} = \frac{3}{2} \geq 1) \text{ but (conf} = 0.47)$$

$$c_2^* \prec e_2^* \text{ (sup} = 1 \geq 1) \text{ but (conf} = 0.33)$$

$$d_1^* \prec e_1^* \text{ (sup} = \frac{5}{3} \geq 1) \text{ but (conf} = 0.48)$$

$$d_1^* \prec e_3^* \text{ (sup} = 1 \geq 1) \text{ but (conf} = 0.28)$$

$$d_2^* \prec e_1^* \text{ (sup} = \frac{3}{2} \geq 1) \text{ but (conf} = 0.33)$$

$$d_2^* \prec e_3^* \text{ (sup} = \frac{5}{3} \geq 1) \text{ but (conf} = 0.37)$$

Algorithm *ERID*(λ_1, λ_2)

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$$a_3^* \prec e_3^* \text{ (sup} = 1 \geq 1) \text{ but (conf} = 0.36)$$

$$b_2^* \prec e_1^* \text{ (sup} = \frac{7}{6} \geq 1) \text{ but (conf} = 0.22)$$

$$b_2^* \prec e_2^* \text{ (sup} = \frac{17}{12} \geq 1) \text{ but (conf} = 0.27)$$

$$c_1^* \prec e_3^* \text{ (sup} = \frac{3}{2} \geq 1) \text{ but (conf} = 0.47)$$

$$c_2^* \prec e_2^* \text{ (sup} = 1 \geq 1) \text{ but (conf} = 0.33)$$

$$d_1^* \prec e_1^* \text{ (sup} = \frac{5}{3} \geq 1) \text{ but (conf} = 0.48)$$

$$d_1^* \prec e_3^* \text{ (sup} = 1 \geq 1) \text{ but (conf} = 0.28)$$

$$d_2^* \prec e_1^* \text{ (sup} = \frac{3}{2} \geq 1) \text{ but (conf} = 0.33)$$

$$d_2^* \prec e_3^* \text{ (sup} = \frac{5}{3} \geq 1) \text{ but (conf} = 0.37)$$

They all are not marked

Algorithm $ERID(\lambda_1, \lambda_2)$

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$(a_3 \cdot c_1)^* \prec e_3^*$ (sup = 1 \geq 1) and (conf = 0.8)

$(a_3 \cdot d_1)^* \prec e_3^*$ (sup = 1 \geq 1) and (conf = 0.5)

$(a_3 \cdot d_2)^* \prec e_3^*$ (sup = 0 < 1)

$(b_2 \cdot d_2)^* \prec e_1^*$ (sup = $\frac{2}{3}$ < 1)

$(b_2 \cdot c_2)^* \prec e_2^*$ (sup = $\frac{1}{2}$ < 1)

Algorithm $ERID(\lambda_1, \lambda_2)$

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$(a_3 \cdot c_1)^* \prec e_3^*$ (sup = 1 \geq 1) and (conf = 0.8)

$(a_3 \cdot d_1)^* \prec e_3^*$ (sup = 1 \geq 1) and (conf = 0.5)

They all are marked positive.

$(a_3 \cdot d_2)^* \prec e_3^*$ (sup = 0 < 1)

$(b_2 \cdot d_2)^* \prec e_1^*$ (sup = $\frac{2}{3}$ < 1)

$(b_2 \cdot c_2)^* \prec e_2^*$ (sup = $\frac{1}{2}$ < 1)

Algorithm *ERID*(λ_1, λ_2)

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$(a_3 \cdot c_1)^* \prec e_3^*$ (sup = 1 \geq 1) and (conf = 0.8)

$(a_3 \cdot d_1)^* \prec e_3^*$ (sup = 1 \geq 1) and (conf = 0.5)

They all are marked positive.

$(a_3 \cdot d_2)^* \prec e_3^*$ (sup = 0 < 1)

$(b_2 \cdot d_2)^* \prec e_1^*$ (sup = $\frac{2}{3}$ < 1)

$(b_2 \cdot c_2)^* \prec e_2^*$ (sup = $\frac{1}{2}$ < 1)

They all are marked negative.

Algorithm $ERID(\lambda_1, \lambda_2)$

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

The algorithm continues for terms of length 3, 4, ... till all of them have either positive or negative marks.

Rules are automatically constructed from relations marked positive.



Algorithm Chase 2
(for Partially *IIS*)

Algorithm Chase 2

$S = (X, A, V)$ - partially incomplete information system of type λ , if S is incomplete and the following three conditions hold:

- $a_S(x)$ is defined for any $x \in X, a \in A$
- $(\forall x \in X)(\forall a \in A) [(a_S(x) = \{(a_i, p_i) : 1 \leq i \leq m\}) \rightarrow \sum p_i = 1]$
- $(\forall x \in X)(\forall a \in A) [(a_S(x) = \{(a_i, p_i) : 1 \leq i \leq m\}) \rightarrow (\forall i)(p_i \geq \lambda)]$

Algorithm Chase 2

S_1, S_2 - partially incomplete, both of type λ and both classifying the same sets of objects (from X) using the same sets of attributes (A)

Let $a_{S_1}(x) = \{(a_{1i}, p_{1i}) : 1 \leq i \leq m_1\}$ and $a_{S_2}(x) = \{(a_{2i}, p_{2i}) : 1 \leq i \leq m_2\}$.

The pair (S_1, S_2) satisfies containment relation Ψ (or $\Psi(S_1) = S_2$) if:

□ $(\forall x \in X)(\forall a \in A)[card(a_{S_1}(x)) > card(a_{S_2}(x))] \text{ or}$

$$[[card(a_{S_1}(x)) = card(a_{S_2}(x))] \rightarrow [\sum_{i \neq j} |p_{2i} - p_{2j}| > \sum_{i \neq j} |p_{1i} - p_{1j}|]]$$

We also denote that fact by $(\forall x \in X)(\forall a \in A)[\Psi(a_{S_1}(x)) = a_{S_2}(x)]$

System S_1

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_2	c_1	d_1	e_3

System S_2

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	b_2	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{3})$ $(c_3, \frac{2}{3})$	d_2	e_3
x_4	a_3		c_2	d_1	e_2
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	c_2	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$$ERID(\lambda_1, \lambda_2) \quad \lambda_1=1, \lambda_2=0.5$$

$$r_1 = [a_1 \rightarrow e_3] \quad \sup(r_1) = 1, \text{conf}(r_1) = 0.5$$

$$r_2 = [a_2 \rightarrow e_2] \quad \sup(r_2) = \frac{5}{3}, \text{conf}(r_2) = 0.51$$

$$r_4 = [b_1 \rightarrow e_1] \quad \sup(r_4) = 2, \text{conf}(r_4) = 0.72$$

$$r_5 = [b_2 \rightarrow e_3] \quad \sup(r_5) = \frac{8}{3}, \text{conf}(r_5) = 0.51$$

$$r_{10} = [c_1 \cdot d_1 \rightarrow e_3] \quad \sup(r_{10}) = 1, \text{conf}(r_{10}) = 0.5$$

Incomplete Information System S
of type $\lambda = 0.3$

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

$$ERID(\lambda_1, \lambda_2) \quad \lambda_1=1, \lambda_2=0.5$$

Algorithm *Chase 2* will try to replace

$$e(x_1) = \{(e_1, \frac{1}{2}), (e_2, \frac{1}{2})\}$$

by $e_{new}(x_1) = \{(e_1,), (e_2,), (e_3,)\}$.

We will show that $\Psi(e(x_1)) = e_{new}(x_1)$ (the value $e(x_1)$ will be changed by *Chase 2*).

Incomplete Information System S
of type $\lambda = 0.3$

X	a	b	c	d	e
x_1	$(a_1, \frac{1}{3})$ $(a_2, \frac{2}{3})$	$(b_1, \frac{2}{3})$ $(b_2, \frac{1}{3})$	c_1	d_1	$(e_1, \frac{1}{2})$ $(e_2, \frac{1}{2})$
x_2	$(a_2, \frac{1}{4})$ $(a_3, \frac{3}{4})$	$(b_1, \frac{1}{3})$ $(b_2, \frac{2}{3})$		d_2	e_1
x_3	a_1	b_2	$(c_1, \frac{1}{2})$ $(c_3, \frac{1}{2})$	d_2	e_3
x_4	a_3		c_2	d_1	$(e_1, \frac{2}{3})$ $(e_2, \frac{1}{3})$
x_5	$(a_1, \frac{2}{3})$ $(a_2, \frac{1}{3})$	b_1	c_2		e_1
x_6	a_2	b_2	c_3	d_2	$(e_2, \frac{1}{3})$ $(e_3, \frac{2}{3})$
x_7	a_2	$(b_1, \frac{1}{4})$ $(b_2, \frac{3}{4})$	$(c_1, \frac{1}{3})$ $(c_2, \frac{2}{3})$	d_2	e_2
x_8	a_3	b_2	c_1	d_1	e_3

Incomplete Information System S
of type $\lambda = 0.3$

For x_1 :

$$(e_3, (\frac{1}{3} \cdot 1 \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{8}{3} \cdot \frac{51}{100} + 1 \cdot 1 \cdot \frac{1}{2}) / (14/3)) \\ = (e_3, 0.24)$$

$$(e_2, (\frac{2}{3} \cdot \frac{5}{3} \cdot \frac{51}{100}) / (5/3)) = (e_2, 0.97)$$

$$(e_1, (\frac{2}{3} \cdot 2 \cdot \frac{72}{100}) / 2) = (e_1, 0.48)$$

we have:

$$e(x_1) = \{(e_1, 0.48), (e_2, 0.97), (e_3, 0.24)\}$$

Because the confidence assigned to e_3 is below the threshold λ , then only two values remain:

$$(e_1, 0.48), (e_2, 0.97).$$

The value of attribute e assigned to x_1 is:
 $\{(e_1, 0.33), (e_2, 0.67)\}$.

THANK
YOU!

