

Problem 1. Assume that fuzzy relations R, S ($R \subseteq X, S \subseteq Y$) are defined below:

$R=[0.9, 0.2, 0.8, 0.2, 0.4], S=[0.1, 0.7, 0.2, 0.3, 0.1, 0.7]$.

- Find the join $R \times S$ of R and S .
- Find the composition $R \circ S$ of R and S .

Problem 2. Assume that fuzzy relations R, S ($R \subseteq X, S \subseteq Y$) are defined below:

$R=[0.9, 0.2, 0.8, 0.2, 0.4]$,

$S=$

0.1	0.2	0.2	0.3	0.9	0.4
0.2	0.1	0.8	0.2	0.1	0.3
0.1	0.3	0.2	0.8	0.1	0.1
0.2	0.7	0.1	0.5	0.4	0.3
0.1	0.9	0.2	0.3	0.1	0.7

- Find the join $R \times S$ of R and S .
- Find the composition $R \circ S$ of R and S .

Problem 3. Assume that S is an information system given below:

X	E	F	G	C
x1	e1	f1	$\{(g1,1/2),(g2,1/2)\}$	c2
x2	e2	f1	g2	c1
x3	$\{(e1,1/2), (e2,1/2)\}$	f1	g1	c2
x4	e2	$\{(f1,1/2),(f2,1/2)\}$	g2	c1
x5	e1	f2	g1	c2
x6	e2	f2	$\{(g1,1/2),(g2,1/2)\}$	c2

Expert-based semantics of attribute C (values $c1, c2$) is based on the following set of rules:

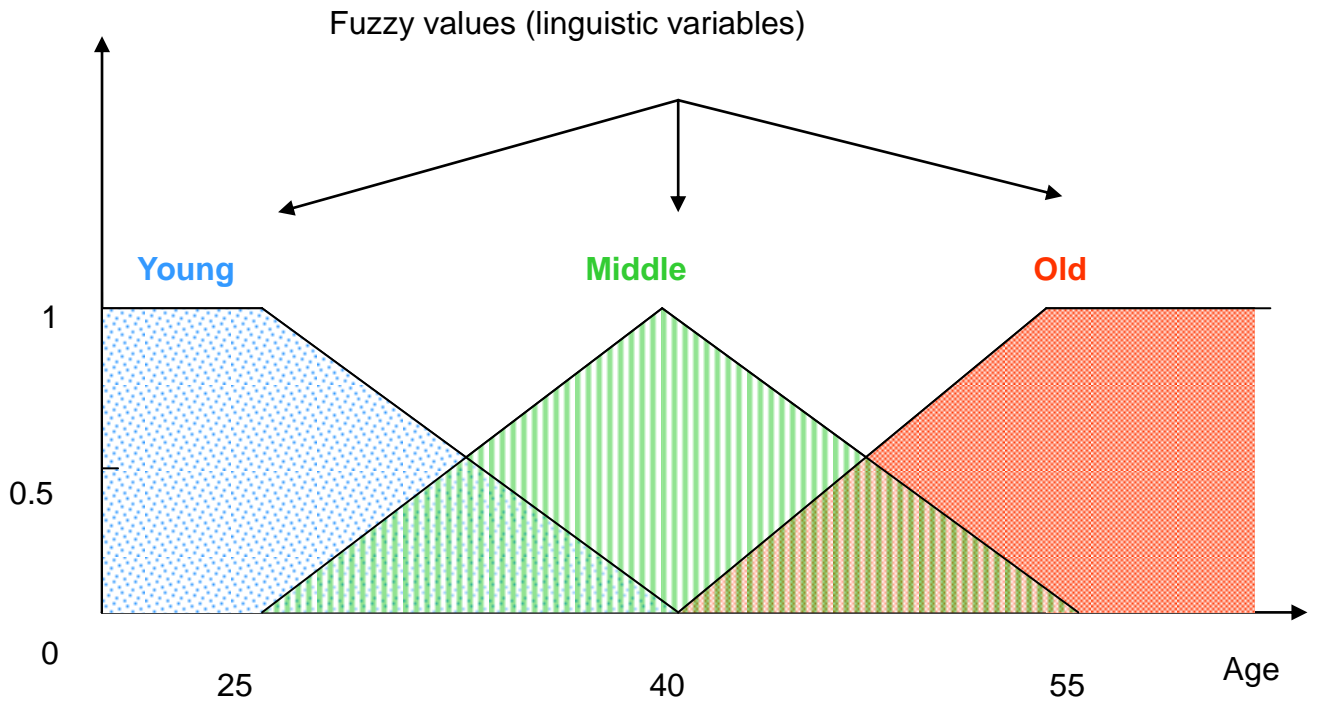
- $e1 \rightarrow c2$ sup=2, conf=1
- $f2 \rightarrow c2$ sup=5/2, conf= 5/6
- $g1 \rightarrow c2$ sup=2, conf=2/3
- $g2 \rightarrow c1$ sup=2, conf = 2/3
- $e2.f1 \rightarrow c1$ sup=3/2, conf=1
- $e2.g2 \rightarrow c1$ sup=2, conf=4/5

$$f1.g2 \rightarrow c1 \quad \text{sup}=3/2, \text{ conf}=3/4$$

$$f2.g1 \rightarrow c2 \quad \text{sup}=3/2, \text{ conf}=1$$

Find precision and recall of user semantics with respect to attribute C.

Problem 4. Assume that Young, Middle, Old are defined by membership functions given below:



- Find the membership function representing (Middle and not Old).
- Find membership function $\mu_R(x,y)$ where $R = [\text{Middle} \rightarrow \text{not Old}]$ or equivalently $R = [\text{if } x \text{ is Middle then } y \text{ is not Old}]$. Take propositional calculus interpretation of R defined as $A \rightarrow B = \neg A \cup (A \cap B)$. Other interpretations of R:

- 1) Material implication: $A \rightarrow B = \neg A \cup B$
- 2) Extended propositional calculus: $(\neg A \cap \neg B) \cup B$
- 3) Generalization of modus ponens:

$$\mu_R(x,y) = \mu_{[A \rightarrow B]}(x,y) = [1 \text{ if } \mu_A(x) \leq \mu_B(y), \text{ else } \mu_B(y)].$$

