Action Rules in Consensus Reaching Process Support

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***Abstract*—We discuss a conceptually new extension of our previous works in which we proposed a concept of a consensus reaching support system based on a new, gradual notion of consensus devised in the framework of fuzzy preference relations and a fuzzy majority. Here, first of all, we propose the use of action rules as a tool to generate some advice as to the further running of discussion in the group. Moreover, we propose to employ intuitionistic fuzzy preference relations to better model individual preferences and to obtain data more suitable for the action rules based analysis.**

I. Introduction

We study how to effectively and efficiently support consensus reaching. We assume that there is a set of individuals (experts, decision-makers, ...) and a set of alternatives (options, variants, decisions, issues, ...). The individuals provide their testimonies concerning alternatives in question as *fuzzy preference relations*. A decision is to be taken by agreement of all individuals, i.e., after a *consensus* is reached. Usually, the individuals are far from consensus, and then, a discussion is carried out in the group to clarify points of view, exchange information, etc., and – possibly after some iterations – a consensus is attained. We wish to provide tools for this.

It is important to provide the group with hints as to most promising directions of further discussion, focusing discussion in group, cf. [1], [2], and our earlier papers [3]. Here we propose to employ so-called *action rules*, introduced in [4], and also a more flexible and human consistent representation of preferences via linguistic intuitionistic fuzzy preference relations which offer more flexibility and better fit into the action rules mining paradigm.

II. A Group Decision Making Model with Fuzzy Linguistic Preferences and a Consensus Reaching Process

A. Preferences Modelling

There is a set of $N \geq 2$ alternatives, $S = \{s_1, \ldots, s_N\}$, and a set of $M \geq 2$ individuals, $E = \{e_1, \ldots, e_M\}$. Each individual $e_m \in E$ expresses his/her preferences as individual *fuzzy preference relation* R_m in $S \times S$, and μ_{R_m} may be meant as that $\mu_{R_m}(s_i, s_j) > 0.5$ denotes the preference degree of an s_i over s_j, $\mu_{R_m}(s_i, s_j) < 0.5$ denotes the preference degree of s_j over s_i,and $\mu_{R_m}(s_i, s_j) = 0.5$ denotes the indifference between s_i and s_j. Usually R_m is assumed *reciprocal*, i.e.,

$$\mu_{R_m}(s_i, s_j) + \mu_{R_m}(s_j, s_i) = 1 \quad (1)$$

holds.

We extend the basic fuzzy preference modelling approach in the following way:

- first, we adopt a bipolar view of preferences [5], and in particular their modelling via *Atanassov's intuitionistic fuzzy sets* [6], called $\mathbb{F}$-sets;
- second, instead of using numeric membership (and non-membership) degrees we use *linguistic terms*.

An $\mathbb{F}$-set X is represented by a pair of membership, μ_X, and non-membership, ν_X, functions,such that:

$$\mu_X(x) + \nu_X(x) \leq 1 \quad (2)$$

An $\mathbb{F}$ *preference relation* $R^{\mathbb{F}}_m$, of individual e_m, is an $\mathbb{F}$-set in $S \times S$, which is thus defined by its membership function $\mu_{R^{\mathbb{F}}_m}(s_i, s_j)$ and non-membership function $\nu_{R^{\mathbb{F}}_m}(s_i, s_j)$. The former is meant as the degree of preference (intensity) of s_i over s_j, and latter as the degree to which s_i is *not* preferred over s_j, here. To simplify, this may be interpreted as the intensity of preference of s_j over s_i which implies:

$$\mu_{R^{\mathbb{F}}_m}(s_i, s_j) = \nu_{R^{\mathbb{F}}_m}(s_j, s_i) \quad (3)$$

The use of $\mathbb{F}$-sets provides for a more flexible representation, notably for taking into account pro and con arguments while determining the preferences which is a widely adopted approach (cf., e.g., [7], [8]). We will omit the superscript $\mathbb{F}$ in $R^{\mathbb{F}}_m$, for brevity. For more details on the $\mathbb{F}$ preference relations and their use in group decision making and consensus reaching, cf. Szmidt and Kacprzyk [9], [10].

In many practical scenarios the precision attained by using $[0, 1]$ to express preference intensities is not necessary, and the use of linguistic terms will be enough. Here we assume the ordinal linguistic approach [11], with an ordered set of linguistic labels. Due to (3) we assume that an individual is specifying for each pair (s_i, s_j) only the membership degrees $\mu_R(s_i, s_j)$ and $\mu_R(s_j, s_i)$ using the following linguistic terms set $\mathcal{L}$:

$$definitely \succ strongly \succ moderately \succ weakly \succ not\_at\_all \quad (4)$$

which in general will be denoted as:

$$\mathcal{L} = \{l_T, l_{T-1}, \ldots, l_1, l_0\} \quad (5)$$

$$l_T \succ l_{T-1} \succ \ldots \succ l_1 \succ l_0 \quad (6)$$

where T is an even number.

Moreover, an *antonym* operator ant is assumed on $\mathcal{L}$:

$$ant : \mathcal{L} \longrightarrow \mathcal{L} \qquad ant(l_k) = l_{T-k} \quad (7)$$

The use of such an operator effectively assumes that the linguistic terms form an interval scale. This assumption will be also employed later when comparing two linguistic membership degrees. It makes operations on the linguistic terms indices, notably the difference, meaningful.

To observe the property of IF-sets (2) we need the following restriction (notice that due to (3) we have $\nu_R(s_i, s_j) = \mu_R(s_j, s_i)$):

$$(\mu_R(s_i, s_j) = l_u) \wedge (\mu_R(s_j, s_i) = l_w) \Rightarrow \neg(l_w \succ ant(l_u)) \quad (8)$$

Thus, for example (referring to the linguistic terms set (4)) , for $\mu_R(s_i, s_j) = definitely$ the value of $\mu_R(s_j, s_i)$ is determined to be not_at_all, what means that alternative s_i is definitely (fully) preferred to the alternative s_j. Other two interesting cases are where $\mu_R(s_i, s_j) = \mu_R(s_j, s_i) = moderately$ and $\mu_R(s_i, s_j) = \mu_R(s_j, s_i) = not\_at\_all$. In both cases an individual may be seen as indifferent to the choice between the alternatives s_i and s_j, but due to the semantics of the IF-sets the first case may be seen as a real indiferrence, while the second corresponds to the situation where an individual is unable to make a choice due to, e.g., lack of information.

Basically, the semantics of these examples of IFpreference relations may be motivated by the following scenario. Let an individual consider a set of criteria while comparing two alternatives. Then, if all criteria, support the choice of s_i over s_j, then it may be reasonable to express the preference as $\mu_R(s_i, s_j) = definitely$;in fact, it may be enough if, e.g., most of the important criteria support this. On the other hand if there is a more or less equal number of criteria supporting both alternatives, then it may be reasonable to have $\mu_R(s_i, s_j) = \mu_R(s_i, s_j) = moderately$. Finally, if all criteria are inconclusive (e.g., due to the lack of the information on the value of the alternatives' attributes relevant for these criteria), then the preferences may be reasonably expressed as $\mu_R(s_i, s_j) = \mu_R(s_i, s_j) = not\_at\_all$.

This also shows a need for a "linguistic counterpart" of the *hesitation margin* $\pi(x)$, which is defined in IFsets theory for regular "numerical" membership degrees as $\pi(x) = 1 - \mu(x) - \nu(x)$. Its counterpart for linguistic degrees is proposed as, for $\mu(x) = l_u$, $\nu(x) = l_w$ and $ant(l_u) = l_t$:

$$\pi(x) = l_z \quad (9)$$

$$z = t - w \quad (10)$$

Thus, in the context considered here it holds that $\pi_R(s_i, s_j) = l_z$, where $z = t - w$ and $ant(\mu_R(s_i, s_j)) = l_t$, and $\mu_R(s_j, s_i) = l_w$. Notice that due to (8) it holds that $\neg(l_w \succ l_t)$ and thus $t \geq w$ and thus $z \geq 0$.

The hesitation margin π expresses the degree to which an individual is unable to decide (e.g., due to the lack of information) on his or her preferences. For example, for the set (4), the term "not at all" as the value of π denotes a lack of hesitation of an individual, while "definitely" denotes a complete inability to decide.

Both, the alternatives and individuals are assigned weights of importance (competence, confidence, relevance, ...) modeled as fuzzy subsets I and B of S and E, respectively (e.g $\mu_B(e_k) \in [0, 1]$ denotes a degree of importance of individual e_k). Also here a set of linguistic labels may be used, for instance:

very important $\succ$ *important* $\succ$ *medium important* $\succ$ $\succ$ *less important* $\succ$ *weakly important* (11)

A number of approaches were proposed for linguistic modeling of preferences, cf. notably Herrera-Viedma et al. [12], [13], [2], [14], [15], [16].

B. Consensus Reaching and a Soft Consensus Measure

Decision making via obtaining consensus boils down to running a discussion in the group to make individual preferences as close as possible. It is almost always unrealistic to reach a classically meant consensus, i.e., full agreement of all individuals as to all their preferences, and an operational definition of consensus is needed, accounting for a satisfactory agreement in a group. One of the first such operational definitions was proposed by Kacprzyk and Fedrizzi [17], [18], [19], and then extended by the authors [20], and is adopted also here. It treats consensus as a gradual notion, via a *measure of consensus*.

Thus, the group may comprise a small or large number of members, in one location or even around the world, and their discussion in asynchronous. The individuals express their preferences which are represented by IFpreference relations. A user interface hiding the technicalities of such an representation is assumed. A preliminary idea of such an interface and a more comprehensive vision of the whole decision support system is presented in our earlier paper [7]. The preferences of the individuals usually initially differ and the session is started which, through an exchange of information, rational argument, discussion, creative thinking, clarification of positions, etc., is expected to get the preferences closer one to another.

A *moderator*, in charge of running the session, tries to focus the discussion on the issues which may resolve the conflict of opinions in the group. We concentrate on a new technique which may be helpful in pointing out such crucial points with respect to raw IFpreference relations. In [7] we consider a more sophisticated environment in which the consensus reaching process is to be run, and which provides a richer framework for preferences expression and adjustment.

The operational definition of consensus is the primary indicator of the agreement in the group. In the next section we discuss other tools, notably based on action rules, which can help the moderator to run the session. Here we briefly remind the essence of the consensus measure proposed by Kacprzyk and Fedrizzi [17], [18], [19] and show its adaptation to the case of IFpreference relations.

The operational definition of consensus used is expressed by a linguistically quantified proposition [21]:

"*Most* (Q1) of the *important* (B) individuals *agree* as to *almost all* (Q2) *relevant* (I) alternatives". (12)

where: Q1 and Q2 are linguistic quantifiers [21], e.g., "most" and "almost all", and B and I stand for fuzzy sets denoting the importance/relevance of the individuals and alternatives.

The consensus degree, for a set of IFpreference relations $\{R_k\}_{k=1,\ldots,M}$, is computed as the truth value of the linguistically quantified proposition (12). First, for each pair of individuals (e_m, e_n) and each pair of alternatives (s_i, s_j) a degree of agreement $v_{ij}(m,n)$ is derived. It is, in general, computed as a function AG of $\mu_{R_m}(s_i, s_j)$, $\mu_{R_m}(s_j, s_i)$, $\mu_{R_n}(s_i, s_j)$ and $\mu_{R_n}(s_j, s_i)$. For "numerically" expressed membership degrees such a function may be defined via a similarity measure between IFsets; cf., e.g., [22], [23]. For membership degrees expressed using linguistic terms (5) this function may be defined using the indices of these linguistic terms, i.e.:

$$AG(\mu_{R_m}(s_i, s_j), \mu_{R_m}(s_j, s_i), \mu_{R_n}(s_i, s_j), \mu_{R_n}(s_j, s_i)) = g(l_{k_1}, l_{k_2}, l_{k_3}, l_{k_4})$$

$$AG : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L} \longrightarrow [0,1]$$
$$g : [0\ldots T] \times [0\ldots T] \times [0\ldots T] \times [0\ldots T] \longrightarrow [0,1]$$

for some function g, assuming that $\mu_{R_m}(s_i, s_j) = l_{k_1}$, $\mu_{R_m}(s_j, s_i) = l_{k_2}$, $\mu_{R_n}(s_i, s_j) = l_{k_3}$, $\mu_{R_n}(s_j, s_i)) = l_{k_4}$.

Secondly, for each pair of individuals (e_m, e_n) a degree of agreement $v^B_{Q_1}(m,n)$ as to their preferences between Q_1 pairs of *relevant* alternatives is derived and, finally these degrees are aggregated to obtain a degree of agreement $con(Q_1, Q_2, I, B)$ of Q_2 pairs of *important* individuals as to their preferences between Q_1 pairs of *relevant* alternatives, and this is meant to be the *degree of consensus* sought. For details,see [17], [7]. The computations are via Zadeh's calculus of linguistically quantified propositions [21].

The consensus degree itself plays an important role in guiding the consensus reaching process but here, for our purposes, more important are some derived indicators of consensus. In [3] we introduced the following measures.

The *personal consensus degree*, $PCD(e_k)$ is defined as the truth value of:

"Preferences of expert e_k as to *most relevant* pairs of alternatives are in agreement with the preferences of *most important* experts" (13)

The *detailed personal consensus degree*, $DPCD(e_k, s_i, s_j)$ is defined as the truth value of:

"Preference of expert e_k as to a pair of options (s_i, s_j) is in agreement with the preferences of *most important* experts" (14)

And,similarly, for the alternatives, the *option consensus degree*, $OCD(s_i)$ is defined as the truth value of:

"*Most important* pairs of experts agree in their preferences with respect to the alternative s_i" (15)

These consensus indicators make it possible to point out the most controversial alternatives and/or experts isolated in their opinions. This may help in further running of the discussion in the group. In Section IV this is discussed in a more detail. Some conceptually similar indicators were proposed by Herrera, Herrera-Viedma and Verdegay in [12].

III. Action Rules

The concept of an *action rule* was proposed in [4], in the context of Pawlak's [24] *information systems*, i.e. triples $IS = \{O, A, V\}$, where O is a finite set of objects, $A = \{a\}$ is a set of its attributes and $V = \bigcup_a V_a$, with V_a being a domain of attribute a. If one of the attributes $d \in A$ is distinguished and called *the decision*, then an information system is called a *decision system*. The set of attributes A may be further partitioned into subsets of *stable* and *flexible* attributes, denoted as A_{St} and A_{Fl} [4]. Thus $A = A_{St} \cup A_{Fl} \cup \{d\}$. An intended meaning of this partitioning is closely related to action rules as the essence of an action rule is to show how a subset of flexible attributes should be changed to obtain expected change of the decision attribute for a subset of objects characterized by some values of the subset of stable attributes. For example, let objects $o \in O$ be bank customers characterized by such stable (from a bank perspective) attributes as age, profession, etc. and flexible attributes such as type of the account, reduction of the monthly fee etc., and the decision attribute is the customer's total monthly spendings. Then, an action rule may, e.g., indicate that offering a 20% reduction in monthly fee instead of 10% to a middle-aged customer is expected to increase his or her spendings from medium to high.

In order to define action rules formally, let us define first some auxiliary concepts [25]. An *atomic action term* is $(a, x \to y)$, where $a \in A$ is an attribute and $x, y \in V_a$ are values belonging to its domain. An *action term* t is a set of atomic action terms: $t = \{(a_1, x_1 \to y_1), \ldots, (a_n, x_n \to y_n)\}$, $a_i \in A$, $a_i \neq a_j$ for $i \neq j$ and $x_i, y_i \in V_{a_i}$. The *domain* of an action term t, denoted by $Dom(t)$, is a set of all attributes in t, i.e., $Dom(t) = \{a_1, \ldots, a_n\}$.

Finally, an *action rule* is $r = [t_1 \Rightarrow t_2]$, where t_1 is an action term and t_2 is an atomic action term referring to the decision attribute, i.e., $Dom(t_2) = \{d\}$. So, if the bank customers are characterized by `age`, `reduction` (monthly fee reduction) and `spendings`, then the action rule may be:

$$[\{(\texttt{age}, \mathit{middleaged} \to \mathit{middleaged}), (\texttt{reduction}, 10\% \to 20\%)\} \Rightarrow (\texttt{spendings}, \mathit{medium} \to \mathit{high})]$$

The measures of *support* (*supp*) and *confidence* (*conf*) are used to evaluate the action rules for a given information system $S = \{O, A, V\}$. For an action term $t = \{(a_1, x_1 \to y_1), \ldots, (a_n, x_n \to y_n)\}$ let us denote by $N_S(t)$ the following pair of sets:

$$N_S(t) = [X, Y] = [\bigcap_{1 \le i \le n} \{o \in O : a_i(o) = x_i\}, \bigcap_{1 \le i \le n} \{o \in O : a_i(o) = y_i\}]$$

Further, for an action rule $[t_1 \Rightarrow t_2]$, let $N_S(t_1) = [X_1, Y_1]$ and $N_S(t_2) = [X_2, Y_2]$. Then the measures of support and confidence are defined as follows:

$$\mathit{supp}(r) = \overline{\overline{X_1 \cap X_2}}$$
$$\mathit{conf}(r) = \frac{\overline{\overline{X_1 \cap X_2}}}{\overline{\overline{X_1}}} \frac{\overline{\overline{Y_1 \cap Y_2}}}{\overline{\overline{Y_1}}}$$

where $\overline{\overline{A}}$ denotes the cardinality of a set A; for denominators equal 0 the confidence measure is assumed to be zero.

These measures are usually used to mine action rules, similarly to the mining of association rules. There is also an important issue of the cost of change of an attribute value, which may be different for different attributes. The goal is thus to find the "cheapest" rules supporting the expected change of the decision attribute. If the cost is identical for all attributes then the best are shortest rules. For details on the algorithms, cf., e.g., [26], [25], [27].

IV. Monitoring the Discussion Using Action Rules

In the group decision making model assumed in Section II, the goal is to reach consensus and the main driving force of the consensus reaching process is an exchange of arguments during the discussion. Thus, a system has to provide the moderator and the whole group with some advice (feedback information) on how far the group is from consensus, what are the most controversial issues (alternatives), whose preferences are in the highest disagreement with the rest of the group, how their change would influence the consensus degree, etc. We propose to use action rules to generate such a feedback. The approaches proposed will be classified according to the form of a decision system $IS = \{O, A, V\}$ (cf. Section III) assumed.

A. Individuals Treated as Objects

If we identify the set of objects O with the set of individuals E, then the set of attributes A is composed of:

- preference degrees for particular pairs of options (three linguistic values corresponding to the membership, non-membership and hesitation margin);
- importance degree of an individual,
- the personal consensus degree PCD, defined by (13) for given individual and $DPCD$s, defined by (14), for a given individual and all pairs of the alternatives.

From the perspective of the action rule generation we will treat these three groups of attributes as the flexible, stable and decision attributes, respectively. Thus, while mining action rules we pick up one from the last group of attributes and then start one of the algorithms mentioned in [28], [4], [25], [27], [26], [29]. A typical scenario may be the following. If the consensus degree (computed as discussed in Section II-B) in the group is too low, then PCDs and DPCDs are computed. Next, we look for the rules which suggest how some individuals should change their preferences so as to change their PCD value from low to high, e.g.:

$$[\{(\texttt{importance}, \mathit{important} \to \mathit{important}), (\mu_R(s_i, s_j), \mathit{not\_at\_all} \to \mathit{moderately}) (\mu_R(s_j, s_i), \mathit{not\_at\_all} \to \mathit{not\_at\_all})\} \Rightarrow (\texttt{PCD}, \mathit{medium} \to \mathit{high})]$$

suggesting that for important individuals it is enough to change preferences as to a given pair of options to get an increase of the personal consensus degree (PCD).

We need to clarify some issues regarding the generation of the rules. First, in order to produce such rules we need to discretize the values of PCD, using, e.g., another set of linguistic terms $\{\mathit{very\ high}, \mathit{high}, \mathit{medium}, \mathit{low}, \mathit{very\ low}\}$. Second, one has to be careful while generating the action rules so as not to suggest changes in preferences violating the consistency of the IFpreference relations (8). The simplest solution is to treat both membership values $\mu_R(s_i, s_j)$ and $\mu_R(s_j, s_i)$ as one atomic value, from the point of view of the action rules. Third, the special role of the hesitation margin $\pi(x)$ should be noted. Namely its value is a function of two other degrees, thus its direct use in action rules does not make any difference. However, the hesitation margin may be used to assess the cost of given action rule since it may be assumed that the cost of changing the preference degree for which the hesitance degree is high should be lower. Also the importance may be seen as contributing to the cost evaluation: the higher importance of an individual the higher the cost of change.

Finally, it should be stressed that we mean the action rules as strictly only a recommendation Thus, the changes suggested by generated action rules are presented for consideration to the relevant individuals and they decide if and how to take them into account. We assume a highly dynamic situation in the group with respect to the individuals preferences and the automatic implementation of the suggestion provided by the action rules is not appropriate. The suggestions provided by an action rule are meant to trigger a discussion by showing some patterns in the group's preferences. It does not have to be necessarily the case that immediate implementation of these changes secures the increase of the aggreement in the group.

Notice that the partition of the attributes into flexible, stable and decision attributes, which is assumed above does not have to be strictly followed. For example, an attribute from the first group, i.e., the preference of given individual expressed for a selected pair of alternatives, may also play the role of the decision attribute. Action rules generated in such a scenario may show some dependencies between the preferences, which are valid for most of the individuals, and may encourage other individuals to re-think their preferences and either strongly motivate their choice during the discussion or accept the arguments of the majority and adjust their preferences.

Similar action rules may be generated with respect to a specific individual and specific pair of alternatives, using DPCD indicator as a decision attribute.

B. Alternatives Treated as Objects

Action rules may be also generated with respect to another information systems employed, namely, the set of objects O may be identified with the set of alternatives S. The set of attributes A is then composed of:

- preference degrees with respect to the rest of the alternatives as expressed by all individuals
- a relevance degree of an alternative,
- the option consensus degree OCD, defined by (15) for a given alternative.

The scenario for the generation and use of action rules with respect to this information system is similar to the one for individuals playing the role of objects. In particular, the attributes of the first group are treated as flexible, the second attribute is stable, and the third is a decision attribute.

V. CONCLUSION

We proposed a further extension of the concept of the consensus reaching support system, discussed in our and other authors' previous works, mainly by applying a novel technique of action rules to stimulate and support discussion in the group. Moreover, we proposed to model the preferences using the IFsets theory. This is beneficial, first, due to a higher flexibility of preferences representation attained. Moreover the notion of hesitance margin, representing the degree to which an individual is unable to make his or her decision with respect to the preferences, perfectly fits the notion of ease of changing attribute values, which is considered in the framework of the action rules generation.

In order to make the preferences elicitation more human consistent and, at the same time, make the resulting data set better suited for the action rules generation, we have introduced the concept of a linguistic IFpreference relation. We have proposed some basic consistency condition for the linguistically expressed membership degrees, but further studies of this concept are surely needed.

Further research will concern the possibility to extend the profiles of the individuals to be done in a richer framework of a group decision support taking into account the information environment of the decision making process, as proposed in [7]. However, also in the framework of the solely preference based profiles some possibilities to extend the current approach do exist. For example, some choice rules may be employed to determine a subset of alternatives which are implied as the best by given IFpreference relation. Then, such a subset may become a part of the profile.

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