Propositional (Boolean) Logic

Syntax of allowable sentences

- atomic sentences
  - indivisible syntactic elements (can not be divided)
  - Use uppercase letters as representation
  - True and False are predefined proposition symbols

Complex sentences

Formed from symbols using connectives

- ~ (not): the negation
- ^ (and): the conjunction
- V (or): the disjunction
- => (implies): the implication
- ⇔ (if and only if): the biconditional

Backus-Naur Form (BNF)

$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathbf{True} \mid \mathbf{False} \mid \mathit{Symbol} \\
\mathit{Symbol} & \rightarrow & \mathbf{P} \mid \mathbf{Q} \mid \mathbf{R} \mid \ldots \\
\mathit{ComplexSentence} & \rightarrow & \neg\ \mathit{Sentence} \\
 & \mid & (\ \mathit{Sentence} \wedge \mathit{Sentence}\ ) \\
 & \mid & (\ \mathit{Sentence} \vee \mathit{Sentence}\ ) \\
 & \mid & (\ \mathit{Sentence} \Rightarrow \mathit{Sentence}\ ) \\
 & \mid & (\ \mathit{Sentence} \Leftrightarrow \mathit{Sentence}\ )
\end{array}
$$

Propositional (Boolean) Logic

Semantics

- given a particular model (situation), what are the rules that determine the truth of a sentence?
- use a truth table to compute the value of any sentence with respect to a model by recursive evaluation
- model – valuation assigning values {T,F} to every atomic statement

Truth table

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Figure 7.8 Truth tables for the five logical connectives. To use the table to compute, for example, the value of $P \vee Q$ when P is true and Q is false, first look on the left for the row where P is *true* and Q is *false* (the third row). Then look in that row under the $P \vee Q$ column to see the result: *true*. Another way to look at this is to think of each row as a model, and the entries under each column for that row as saying whether the corresponding sentence is true in that model.

Concepts related to entailment

logical equivalence

- a and b are logically equivalent if they are true in the same set of models…

  a⇔ b

validity (or tautology)

- a sentence that is valid in all models
  - P V ~P
  - deduction theorem: a entails b if and only if a implies b

satisfiability

- a sentence that is true in *some* model
- a entails b ⇔ (a ^ ~b) is unsatisfiable