Rules for Processing and Manipulating Scalar Music Theory

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Abstract

This article illustrates the application of classification rules and action rules to scalar music theory. Our intent is twofold:
(1) describe certain facts in scalar music theory using classification rules and use them to build a system for automatic indexing of music by scale, region, genre, and emotion,
(2) use action rules mining to create solutions (automatically generated hints) that permit developers to manipulate a composition by retaining the music score while simultaneously varying emotions it invokes.

1. Introduction

Western music comprises a blending of instruments emanating sounds made at a number of different frequencies. The manner in which these frequencies blend is important to the making and enjoyment (or lack thereof) of music [3]. Mathematical rules attributed to musical scales define the blend and hence genre or mood of the score. Knowing the scale of a score is necessary for music analysis and applications such as mood induction because the mode of the key is deemed to provide a specific emotional connotation [7]. The pitch distance of musical notes based on chromatic scale determines humans perception of music.

The extraction of scale from music audio is not new, (see Lemans algorithm [8] for an exception in which human tone center recognition is modelled).

Wang et al [15] asserts that the mood of a score is directly linked with the distance between consecutive musical notes where a greater distance defines more dissonance. Many who agree with this notion have adopted the spiral array model [1] which represents pitches as points on a three dimensional spiral. Cook and Fujisawa [2] disagree stating that even though some chords sound stable and resolved while others sound unstable and unresolved, this cannot be explained solely on the basis of the summation of interval dissonance among tones and their upper partials. Mere summation also does not explain why positive and negative affective valence of major and minor scales is salient both to young children and to adults from diverse cultures [2]. We, agree that the problem is not as trivial as measuring intervals and present the following analysis using classification rules and action rules approach to understand and interact with scalar music theory.

2. Tension in Music

It is well established that the difference of interval size for any three-tone combination can be taken as the basic structural unit for a model of psychological tension. Tension describes how humans perceive uneasiness or easiness towards a set of tones. Cook's tension function calculating the difference in interval sizes is Gaussian in shape. See Figure 1, where the function includes the effects of upper partials in the calculation of triadic tension [2].

$$t = v \cdot exp\left(\frac{y-x}{\alpha}\right)^2 \tag{1}$$

where t is the tension, v is the product of the relative amplitudes of the three partials, α (in our case, $\alpha =$ 0.60) is a parameter that determines the steepness of the fall from maximal tension; x and y are the lower and upper of the two intervals in each tone triplet, defined as $x = log(f_2/f_1)$ and $y = log(f_3/f_2)$, where the frequencies of the three partials satisfy the relationship: $f_1 < f_2 < f_3$ (in Hertz). As seen in Figure 1, the tension is perceived when two equivalent intervals are heard (point A), but not when the two intervals differ by one (or more) semitones (points B and C). Those are the locations on the tension curve for which there is unambiguous empirical data (in the form of traditional harmony).

Harmonious scores have a vertical dimension and a horizontal dimension. The vertical dimension encompasses the relationships among simultaneous notes. By convention, note refers to a pitch in the musical scale, and harmonic interval refers to two notes sounded simultaneously. In this

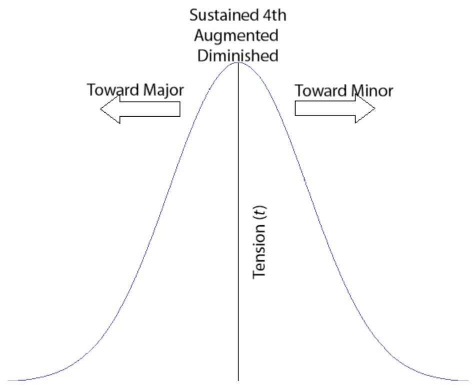


Figure 1. A Gaussian function is used because it reflects the tolerance of most listeners for slight mistuning of chords by effectively broadening the regions of perceived tension and resolution, but the exact shape of the curve remains empirically uncertain. Here and elsewhere we assume twelve-tone equitempered tuning (for further details, see Fujisawa, 2004)

paper we adhere to the simple distinction between the vertical and horizontal dimensions of harmony. We restrict use of the terms consonance and dissonance to the vertical dimension and keep the term harmony superordinate to them [13].

3. Query Answering and Action Rules

The ultimate goal of this paper is to illustrate that action rules [11] can be used as a tool to elevate a query answering system (*QAS*) into a flexible system [10], [5], [4]. It means that *QAS* will resolve user queries successfully more often. For example, a user can query an information system [9] looking for a baroque melancholy piece of music. Upon analyzing the scalar components of songs the query answering system may produce a Mozart, 40th Symphony. But then, the user may say: *"Yes but I'm sad today, play the same song but make it sadder.*" Here, action rules may suggest the smallest set of changes which are needed for some of these scalar components to guarantee that *QAS* will output mozart's 40th Symphony - like a funeral march. A listener, unbeknownst to the user's whims will still be able to identify the music score as being Mozart's 40th, but may be perplexed as to why it sounds so sad.

Now, we recall the notion of an information system used to describe a collection of related objects in terms of their features. Its definition, given here, is due to Pawlak [9]. By an information system we mean a pair $S = (U, A)$, where:

- U is a nonempty, finite set of objects (object identifiers),
- A is a nonempty, finite set of attributes (functions) i.e. $a : U \to V_a$ for $a \in A$, where V_a is called the domain of a.

We often write (a, v) instead of v, assuming that $v \in V_a$. Information systems can be seen as decision tables. In any decision table together with the set of attributes a partition of that set into conditions and decisions is given. Additionally, we assume that the set of conditions is partitioned into stable and flexible conditions [12]. Attribute $a \in A$ is called stable for the set U, if its values assigned to objects from U can not change in time. Otherwise, it is called flexible. *"Date of Birth"* is an example of a stable attribute. *"Interest rate"* on any customer account is an example of a flexible attribute. For simplicity reason, we will consider decision tables with only one decision. We adopt the following definition of a decision table:

By a decision table we mean an information system $S = (U, A_1 \cup A_2 \cup \{d\})$, where $d \notin A_1 \cup A_2$ is a distinguished attribute called decision. The elements of A_1 are called stable conditions, whereas the elements of $A_2 \cup \{d\}$ are called flexible conditions. Our goal is to suggest changes in values of attributes in A_1 for some objects from U so the values of the attribute d for these objects may change as well. A formal expression describing such a property is called an action rule [11], [14].

In order to construct action rules, first we need to extract classification rules describing relationships between attributes from A_1 and the attribute d. By $Dom(r)$ we mean all attributes listed in the IF part of a classification rule r. For example, if $r = [(a_1, 3) \wedge (a_2, 4) \to (d, 3)]$ is a rule, then $Dom(r) = \{a_1, a_2\}$. By $d(r)$ we denote the decision value of rule r. In our example $d(r) = 3$. If r_1, r_2 are rules and $B \subseteq A_1 \cup A_2$ is a set of attributes, then $r_1/B = r_2/B$ means that the conditional parts of rules r_1, r_2 restricted to attributes B are the same. For example if $r_1 = [(a_1, 3) \to (d, 3)]$, then $r_1/a_1 = r/a_1$. Assume also that $(a, v \to w)$ denotes the fact that the value of attribute a needs to be changed from v to w for some objects in U. Similarly, the term $(a, v \to w)(x)$ means that $a(x) = v$ needs to be changed to $a(x) = w$. Saying another words, the property (a, v) of an object x needs to be changed to property (a, w). Assume now that rules r_1, r_2 are extracted from S and $r_1/A_1 = r_2/A_1$, $d(r_1) = k_1$, $d(r_2) = k_2$ and $k_1 < k_2$ (k_1 is ranked lower than k_2). Also, assume that $(b_1, b_2, , b_p)$ is a list of all attributes in $Dom(r_1) \cap Dom(r_2) \cap A_2$ on which r_1, r_2 differ and $r_1(b_1) = v_1$, $r_1(b_2) = v_2$,..., $r_1(b_p) = v_p$, $r_2(b_1) = w_1$, $r_2(b_2) = w_2$,..., $r_2(b_p) = w_p$.

Figure 2. Example score of a Pentatonic Minor Scale played in the key of C

Scale	sma	ii	iii	iv	v	vi	vii	viii	ix	x	xi	xii	#
Pentatonic Minor	m	3	2	2	3								4

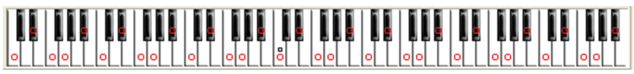
Figure 3. Representation of a Pentatonic Minor Scale

By (r_1, r_2)-action rule on $x \in U$ we mean an expression: $r = [(b_1, v_1 \to w_1) \wedge (b_2, v_2 \to w_2) \wedge ... \wedge (b_p, v_p \to w_p)](x) \to (d, k_1 \to k_2)(x)$. By the support of action rule r we mean the set of all objects in S satisfying the description $[(b_1, v_1) \wedge (b_2, v_2) \wedge ... \wedge (b_p, v_p) \wedge (d, k_1)]$. Assume now that by $r(x)$ we mean a new object obtained from x by changing its property (b_i, v_i) to (b_i, w_i) for each $1 \leq i \leq p$. If $d(r(x)) = k_2$, then x supports r.

4. KD-based Scalar Music Theory

In this section, certain properties related to music will be presented in scalar theory using classification rules. We will use action rules to describe some manipulations on them.

We start with Figure 2 showing an example of a score of a Pentatonic Minor Scale played in the key of C. We describe the scale as illustrated in Figure 3. Accordingly one plays the root, play 3 tones up, then 2 tones up then 2 tones up then 3 tones up. We said ours was in the key of C. Therefore the first note, or in musical terms, the "Root" is a C note. This results in the following notes all being in the key of the C Pentatonic Minor Scale on a piano.



And on a guitar:

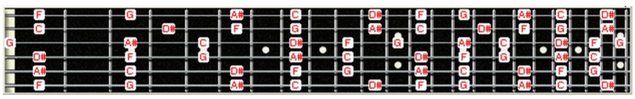


Figure 4. Visual of Pentatonic Minor Scale played in the key of C

In our example, illustrated in Figure 3, 8 notes are played in the C Pentatonic Minor Scale, they are $A\sharp, G, A\sharp, F, D\sharp, G, C$ and C. Essentially any combination of the notes $C, D\sharp, F, G, A\sharp$ can be played while remaining within the constraints of a C Pentatonic Minor Scale on the piano. However, if that score is presented to a Query Answering System (QAS), it has no idea as to the key or the scale. It can only discern the jumps between the notes and the repeated notes. Using this premise, each note, one by one, is drawn into the array of incoming signals. It will presume that the next note is the key. The segment, in our example, has notes of which the $A\sharp$ and C are repeated. Let's assume that this is also the end of the song meaning the QAS knows there are no more possibilities. Therefore we only have 5 distinct notes.

The Table 1 shows the process of reading by QAS the melody segment represented by sequence of notes in Figure 2.

Iteration 1 shows that the note $A\sharp$ is read by QAS first and placed as a root. Iteration 2.1 shows that G is read as the next note. We have two optional sequences:

[$A\sharp$, G] or [G, $A\sharp$].

In the first case, $A\sharp$ is the root. In the second, G is the root. The next note read by QAS is $A\sharp$. This note is disregarded as one which is already listed. The fourth note read by QAS is F (iteration 3). Now, we have three optional sequences:

[$A\sharp$, F, G], [G, $A\sharp$, F], or [F, G, $A\sharp$].

In the first case $A\sharp$ is the root, in the second G is the root, and in the third F. The fifth note read be QAS is $D\sharp$ (iteration 4). Now, we have four optional sequences:

[$A\sharp$, $D\sharp$, F, G], [G, $A\sharp$, $D\sharp$, F], [F, G, $A\sharp$, $D\sharp$], or [$D\sharp$, F, G, $A\sharp$].

In the first case $A\sharp$ is the root, in the second case G is the root, in the third F, and D in the last one. The sixth note read be QAS is G which is disregarded as one which is already listed. The seventh note read by QAS is C. We have five optional sequences:

[$A\sharp$, C, $D\sharp$, F, G], [G, $A\sharp$, C, $D\sharp$, F], [F, G, $A\sharp$, C, $D\sharp$], [$D\sharp$, F, G, $A\sharp$, C], or [C, $D\sharp$, F, G, $A\sharp$].

In the first case $A\sharp$ is the root, in the second case G is the root, in the third F, in the fourth $D\sharp$, and in the fifth C is the root. The last note read by QAS is C. This note is disregarded as one which is already listed. Clearly, at this point, QAS has no idea which note is the root and the same it has no idea which sequence out of the last 5 is a representative one for the input sequence of notes $A\sharp, G, A\sharp, F, D\sharp, G, C$ and C.

Iteration	Root	i	ii	iii	iv
1	A♯				
2.1	A♯	G			
2.2	G	A♯			
3.1	A♯	F	G		
3.2	G	A♯	F		
3.3	F	G	A♯		
4.1	A♯	D♯	F	G	
4.2	G	A♯	D♯	F	
4.3	F	G	A♯	D♯	
4.4	D♯	F	G	A♯	
5.1	A♯	C	D♯	F	G
5.2	G	A♯	C	D♯	F
5.3	F	G	A♯	C	D♯
5.4	D♯	F	G	A♯	C
5.4	C	D♯	F	G	A♯

Table 1. Possible Representative Jump Sequences from All Optional Roots

Root	i	ii	iii	iv
A♯	2	3	2	2
G	3	2	3	2
F	2	3	2	3
D♯	2	2	3	2
C	3	2	2	3

Table 2. Possible Representative Jump Sequences for the Input Sequence

Table 2 gives numeric representation of these last five sequences. Now, we explain the process of computing numeric representation of the first sequence from that list which is [$A\natural$, C, $D\natural$, F, G]. The score is played in the key of $A\natural$ which becomes the root. Its second note C is 2 tones up from $A\natural$. The third note $D\natural$ is 3 tones up from C. The fourth note F is two tones up from $D\natural$, and finally G is two tones up from F. This is how the sequence *[2, 3, 2, 2]* with root $A\natural$ is generated. The remaining rows in Table 2 are computed in a similar way.

Basic score classification of music is represented as an information system $S = (X, A, V)$, where $A = \{J^I, J^{II}, J^{III}, J^{IV}, J^V$, *Scale, Region, Genre, Emotion, sma*$\}$ (see Table 3). System S is built by a music expert and used by us to identify which jump sequence listed in Table 2 is seen as the most probable representation of the melody presented by Figure 2. The values of attribute sma in Table 3 listed as $\{a, s, m\}$ should be read as *arpeggio, mode, scale*. Now, for each tuple in Table 2 we search for a supporting object in Table 3. It can be easily checked that objects listed in $\{x_1, x_{17}, x_{18}, x_{19}, x_{21}\}$ are the only objects in Table 3 supporting minimum one of tuples from Table 2. Because *(A♮, G, A♮, F, D♮, G, C, C)* was the initial score sequence, then *(A♮, G, C)* are played twice in the score and *(F, D♮)* only once. It means that $A\natural$, *G, C* are preferable roots. Now, because only the note C is played twice one after the other, then C is chosen as the most probable root. It means, from Table 2, that *[3, 2, 2, 3]* is the representative jump sequence for the initial score sequence. Its score is *Pentatonic Minor*, region is *Western*, the associated emotion is *sonorous*, and it was played in *mode*.

X	J^I	J^{II}	J^{III}	J^{IV}	J^V	Scale	Region	Genre	Emotion	sma
X_1	2	2	3	2		Pentatonic Major	Western	Blues	melancholy	s
X_2	3	2	1	1	2	Blues Major	Western	Blues	depressive	s
X_3	3	2	2	3		Pentatonic Minor	Western	Jazz	melancholy	s
X_4	3	2	1	1	3	Blues Minor	Western	Blues	dramatic	s
X_5	3	1	3	1	3	Augmented	Western	Jazz	feel-good	s
X_6	2	2	2	2	2	Whole Tone	Western	Jazz	push-pull	s
X_7	1	2	4	1		Balinese	Balinese	ethnic	neutral	s
X_8	2	2	3	2		Chinese	Chinese	ethnic	neutral	s
X_9	2	3	2	3		Egyptian	Egyptian	ethnic	neutral	s
X_{10}	1	4	1	4		Iwato	Iwato	ethnic	neutral	s
X_{11}	1	4	2	1		Japanese	Japanese	Asian	neutral	s
X_{12}	2	1	4	1		Hirajoshi	Hirajoshi	ethnic	neutral	s
X_{13}	1	4	2	1		Kumoi	Japanese	Asian	neutral	s
X_{14}	2	2	3	2		Mongolian	Mongolian	ethnic	neutral	s
X_{15}	1	2	4	3		Pelog	Western	neutral	neutral	s
X_{16}	2	2	3	2		Pentatonic Majeur	Western	neutral	happy	m
X_{17}	2	3	2	3		Pentatonic 2	Western	neutral	neutral	m
X_{18}	3	2	3	2		Pentatonic 3	Western	neutral	neutral	m
X_{19}	2	3	2	2		Pentatonic 4	Western	neutral	neutral	m
X_{20}	2	2	3	3		Pentatonic Dominant	Western	neutral	neutral	m
X_{21}	3	2	2	3		Pentatonic Minor	Western	neutral	sonorous	m
X_{22}	1	3	3	2		Altered Pentatonic	Western	neutral	neutral	m
X_{23}	3	2	1	1	2	Blues	Western	Blues	depressive	m
X_{24}	4	3				Major	neutral	neutral	sonorous	a
X_{25}	3	4				Minor	neutral	neutral	sonorous	a
X_{26}	4	3	4			Major 7th Major	neutral	neutral	happy	a
X_{27}	4	3	3			Major 7th Minor	neutral	neutral	not happy	a
X_{28}	3	4	4			Minor 7th Major	neutral	neutral	happy	a
X_{29}	3	4	3			Minor 7th Minor	neutral	neutral	not happy	a
X_{30}	2	2	3	3		Major 9th	neutral	neutral	happy	a
X_{31}	2	1	4	3		Minor 9th	neutral	neutral	not happy	a
X_{32}	2	2	1	2	3	Major 11th	neutral	neutral	happy	a
X_{33}	2	1	2	2	3	Minor 11th	neutral	neutral	not happy	a
X_{34}	4	4				Augmented	neutral	neutral	happy	a
X_{35}	3	3	3			Diminished	neutral	neutral	not happy	a

Table 3. Basic Score Classification Database

Instead of using Table 3 in our investigations, we can use Table 4 which represents knowledge extracted from Table 3. Table 4 is built from Table 3 by a strategy which resembles $LERS$ (see [6]). Now, we present a general schema of this strategy, called LMC (Learning Music Concepts).

Let us assume that $S = (X, A, V)$, where $A = \{J^I, J^{II}, J^{III}, J^{IV}, J^V$, *Scale, Region, Genre, Emotion, sma*$\}$ (see Table 3), is an information system representing basic score classification of music. We also assume that attributes A_d = $\{$*Scale, Region, Genre, Emotion, sma*$\}$ jointly represent the decision attribute and our goal is to learn definitions of its values in terms of $A_C = \{J^I, J^{II}, J^{III}, J^{IV}, J^V\}$. For simplicity reason we write $J^{[i,m]}(k)$ instead of $\{[J^i(j_i) \cap J^{i+1}(j_{i+1}) \cap ... \cap J^{i+m}(j_{i+m})] : min\{j_i, j_{i+1}, j_{i+m}\} = k\}$. For instance, $J^{[2,3]}(2)$ is referring to objects which have 2 as the minimal value of attributes J^2 and J^3 in our $S = (X, A, V)$. Statement $card(J^{[1,3]}(2)) = 1$ is extracting minimal descriptions, of length 3, based on J^1, J^2, J^3, and retrieving only single objects from S under additional assumption that the minimal attribute value allowed for any of the attributes J^1, J^2, J^3 is equal to 2. In our case, $card(J^{[1,3]}(2)) = 1$ is retrieving descriptions: $[2,2,2]$, $[3,2,3]$ which identify objects x_6, x_{18}, correspondingly. Algorithm LMC applied to Table 3 is generating Table 4. For instance, the third row in Table 4 represents the rule:

$J^I_3 \cdot J^{II}_3 \longrightarrow$ *Arpeggio* $\wedge$ *(Scale, Diminished)* $\wedge$
(Region, neutral) $\wedge$ *(Genre, neutral)* $\wedge$
(Emotion, not happy)

Algorithm LMC(X, A_C, A_d)
m:=0;
while $m \leq 4$ **do**
begin
i:=1;
while $(i + m) \leq 5$ **do**
  begin
  k := 1;
  while $k \leq 4$ **do**
    begin
    if $card(J^{[i,i+m]}(k)) = 1$ **then**
      begin
      $mark(J^{[i,i+m]}(k))$ and store
      $[J^{[i,i+m]}(k), d(x)]$, where $x \in J^{[i,i+m]}(k)$,
      as new rows in Table 4;
      end
    $k := k + 1$
    end
  $i := i + 1$
  end
$m := m + 1$
end

Rule	Scale	Region	Genre	Emotion	sma
$(J^{IV}_4)^* = \{X_{10}\}$	Iwato	Iwato	ethnic	neutral	s
$(J^I_1 \cdot J^{II}_3)^* = \{X_{22}\}$	Altered Pentatonic	Western	neutral	neutral	m
$(J^I_3 \cdot J^{II}_3)^* = \{X_{35}\}$	Diminished	neutral	neutral	not happy	a
$(J^I_4 \cdot J^{II}_4)^* = \{X_{34}\}$	Augmented	neutral	neutral	happy	a
$(J^I_1 \cdot J^{III}_1)^* = \{X_{10}\}$	Iwato	Iwato	ethnic	neutral	s
$(J^I_1 \cdot J^{III}_3)^* = \{X_{22}\}$	Altered Pentatonic	Western	neutral	neutral	m
$(J^I_2 \cdot J^{III}_1)^* = \{X_{32}\}$	Major 11th	neutral	neutral	happy	a
$(J^I_3 \cdot J^{III}_4)^* = \{X_{28}\}$	Minor 7th Major	neutral	neutral	happy	a
$(J^I_4 \cdot J^{III}_3)^* = \{X_{27}\}$	Major 7th Minor	neutral	neutral	not happy	a
$(J^I_4 \cdot J^{III}_4)^* = \{X_{26}\}$	Major 7th Major	neutral	neutral	happy	a
$(J^I_1 \cdot J^{IV}_2)^* = \{X_{22}\}$	Altered Pentatonic	Western	neutral	neutral	m
$(J^I_1 \cdot J^{IV}_3)^* = \{X_{15}\}$	Pelog	Western	neutral	neutral	s
$(J^I_3 \cdot J^{IV}_2)^* = \{X_{18}\}$	Pentatonic 3	Western	neutral	neutral	m
$(J^{II}_1 \cdot J^{III}_2)^* = \{X_{33}\}$	Minor 11th	neutral	neutral	not happy	a
$(J^{II}_1 \cdot J^{III}_3)^* = \{X_5\}$	Augmented	Western	Jazz	feel-good	s
$(J^{II}_3 \cdot J^{III}_4)^* = \{X_{26}\}$	Major 7th Major	neutral	neutral	happy	a
$(J^{II}_4 \cdot J^{III}_1)^* = \{X_{10}\}$	Iwato	Iwato	ethnic	neutral	s
$(J^{II}_4 \cdot J^{III}_3)^* = \{X_{29}\}$	Minor 7th Minor	neutral	neutral	not happy	a
$(J^{II}_4 \cdot J^{III}_4)^* = \{X_{28}\}$	Minor 7th Major	neutral	neutral	happy	a
$(J^{II}_1 \cdot J^{IV}_2)^* = \{X_{33}\}$	Minor 11th	neutral	neutral	not happy	a
$(J^{II}_1 \cdot J^{IV}_3)^* = \{X_{31}\}$	Minor 9th	neutral	neutral	not happy	a
$(J^I_2 * (J^{II}_3 \cdot J^{IV}_2)^* = \{X_{19}\}$	Pentatonic 4	Western	neutral	neutral	m
$(J^I_2 * (J^{II}_2 \cdot J^{III}_2)^* = \{X_6\}$	Whole Tone	Western	Jazz	push-pull	s
$(J^I_2 * (J^{II}_1 \cdot J^{IV}_1)^* = \{X_{12}\}$	Hirajoshi	Hirajoshi	ethnic	neutral	s
$(J^I_3 * (J^{III}_3 \cdot J^{IV}_3)^* = \{X_{18}\}$	Pentatonic 3	Western	neutral	neutral	m
$(J^I_3 * (J^{II}_2 \cdot J^{IV}_2)^* = \{X_{18}\}$	Pentatonic 3	Western	neutral	neutral	m
$(J^I_3 * (J^{II}_2 \cdot J^{III}_3)^* = \{X_{18}\}$	Pentatonic 3	Western	neutral	neutral	m
$(J^{II}_1 * (J^I_2 \cdot J^{IV}_3)^* = \{X_{31}\}$	Minor 9th	neutral	neutral	not happy	a
$(J^{II}_1 * (J^I_1 \cdot J^{IV}_1)^* = \{X_{12}\}$	Hirajoshi	Hirajoshi	ethnic	neutral	s
$(J^{II}_2 * (J^I_3 \cdot J^{III}_3)^* = \{X_{18}\}$	Pentatonic 3	Western	neutral	neutral	m
$(J^{II}_2 * (J^I_1 \cdot J^{IV}_1)^* = \{X_7\}$	Balinese	Balinese	ethnic	neutral	s
$(J^{II}_3 * (J^{III}_2 \cdot J^{IV}_2)^* = \{X_{19}\}$	Pentatonic 4	Western	neutral	neutral	m
$(J^{II}_3 * (J^I_2 \cdot J^{IV}_2)^* = \{X_{19}\}$	Pentatonic 4	Western	neutral	neutral	m
$(J^{III}_1 * (J^{II}_2 \cdot J^{IV}_2)^* = \{X_{32}\}$	Major 11th	neutral	neutral	happy	a
$(J^{III}_2 * (J^{II}_2 \cdot J^{IV}_2)^* = \{X_6\}$	Whole Tone	Western	Jazz	push-pull	s
$(J^{III}_3 * (J^{II}_3 \cdot J^{IV}_2)^* = \{X_{22}\}$	Altered Pentatonic	Western	neutral	neutral	m
$(J^{III}_3 * (J^I_3 \cdot J^{IV}_1)^* = \{X_{22}\}$	Altered Pentatonic	Western	neutral	neutral	m
$(J^{III}_4 * (J^{II}_2 \cdot J^{IV}_3)^* = \{X_{15}\}$	Pelog	Western	neutral	neutral	s
$(J^{III}_4 * (J^{II}_2 \cdot J^{IV}_1)^* = \{X_7\}$	Balinese	Balinese	ethnic	neutral	s
$(J^{III}_4 * (J^{II}_1 \cdot J^{IV}_1)^* = \{X_{12}\}$	Hirajoshi	Hirajoshi	ethnic	neutral	s
$(J^{III}_4 * (J^I_2 \cdot J^{IV}_1)^* = \{X_{12}\}$	Hirajoshi	Hirajoshi	ethnic	neutral	s
$(J^{III}_4 * (J^I_1 \cdot J^{IV}_1)^* = \{X_7\}$	Balinese	Balinese	ethnic	neutral	s
$(J^{IV}_1 * (J^{II}_2 \cdot J^{III}_4)^* = \{X_7\}$	Balinese	Balinese	ethnic	neutral	s
$(J^{IV}_1 * (J^{II}_1 \cdot J^{III}_4)^* = \{X_{12}\}$	Hirajoshi	Hirajoshi	ethnic	neutral	s
$(J^{IV}_1 * (J^I_3 \cdot J^{III}_1)^* = \{X_7\}$	Balinese	Balinese	ethnic	neutral	s
$(J^{IV}_1 * (J^I_2 \cdot J^{II}_1)^* = \{X_{12}\}$	Hirajoshi	Hirajoshi	ethnic	neutral	s
$(J^{IV}_1 * (J^I_1 \cdot J^{II}_2)^* = \{X_7\}$	Balinese	Balinese	ethnic	neutral	s
$(J^{IV}_2 * (J^{II}_3 \cdot J^{III}_2)^* = \{X_{19}\}$	Pentatonic 4	Western	neutral	neutral	m
$(J^{IV}_2 * (J^{II}_2 \cdot J^{III}_1)^* = \{X_{32}\}$	Major 11th	neutral	neutral	happy	a
$(J^{IV}_3 * (J^I_2 \cdot J^{III}_4)^* = \{X_{31}\}$	Minor 9th	neutral	neutral	not happy	a

Table 4. Knowledge generated by LMC

Figure 5. 4-Bar example of Pentatonic Minor Scale played in the key of C

Figure 6. 4-Bar example of Pentatonic Minor Scale played in the key of C changed to Egyptian in C

Bar ♯	1	2	3	4	5	6	7	8
Bar 1 Notes	$A\sharp$	G	$A\sharp$	C	C	$D\sharp$	∅	∅
Bar 1 Jump	5	4	5	Root	Root	2	∅	∅
Bar 2 Notes	$D\sharp$	C	C	F	C	$A\sharp$	C	∅
Bar 2 Jump	2	Root	Root	3	Root	5	Root	∅
Bar 3 Notes	$A\sharp$	G	$A\sharp$	G	G	$D\sharp$	∅	∅
Bar 3 Jump	5	4	5	4	4	2	∅	∅
Bar 4 Notes	G	C	$D\sharp$	$A\sharp$	C	C	∅	∅
Bar 4 Jump	4	Root	2	5	Root	Root	∅	∅

Table 5. 4-Bar example in letter-notation

Scale	Root	i	ii	iii	iv
C Pentatonic Minor	C	$D^\sharp$	F	G	$A^\sharp$
	∅	3	2	2	3
C Egyptian	C	D	F	G	$A^\sharp$
	∅	2	3	2	3
Changes to make	Root	i	ii	iii	iv
C Egyptian	C	$D^\sharp$	F	G	$A^\sharp$
	Keep	Change to D	Keep	Keep	Keep

Table 6. Comparing note pattern and jump sequences of C Pentatonic Minor and Egytian

5. Action Rules to Manipulate Emotions of the Song

In this section we give an example showing how action rules can be used to manipulate the scalar components of a retrieved segment of music or song to change its emotional content and tone. To explain our approach, we expound upon the first example and continue using the Pentatonic Minor Scale played in the key of C as illustrated in the figures below.

In the example, illustrated in Figure 5, 25 notes are played in the C Pentatonic Minor Scale. They are: $A\sharp$, *G*, $A\sharp$, *F*, $D\sharp$, *G*, *C* and C. As it was shown in the first example, the QAS can find the segment and identify the scale as C Pentatonic Minor Scale. For the current example we create the hypothetical case where a user, upon retrieving the above segment of music says: "*I like jazz and this certainly is western Jazz but today I'd like the computer to keep the song but make it sound more Egyptian!*"

Essentially QAS views the 25 notes as seen in Table 5 which illustrates the relationship between each named note in each bar and the corresponding jump number. Then in Table 6, it compares the original jump sequence to the intended jump sequence of the Egyptian scale. Table 6 illustrates that the change is only needed at the C Pentatonic Minor Scale and the C Egyptian Scale shown by its attribute values (J_2^{II}, $D\sharp$), (J_2^{II}, D), correspondingly. This means that the action rule simply states: $(J_2^{II}, D\sharp \rightarrow D) \rightarrow$ *(Scale: Pentatonic → Egyptian)*. It will result in the song shown in Figure 6.

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References

[1] Chew, E., *Modeling Tonality: Applications to Music Cognition*, in **Processing of the 23rd Annual Meeting of the Cognitive Science Society**, Springer, Vol. 2445, 2002, 18

[2] Cook, N. Fujisawa T, X., *The Psychophysics of Harmony Perception: Harmony is a Three-Tone Phenomenon*, in **Empirical Musicology Review**, Parncutt and Richard Publishing, Vol. 1, No. 4, 2006, 204-209

[3] Coulter, L.O., *Mathematics, Music and the Arts: Making Finite Math Relevant to the Arts Major*, in **Proceedings of the 35th Annual Meeting of Florida Section Mathematical Association of America**, 2002, 11-18

[4] Gaasterland, T., *Cooperative answering through controlled query relaxation*, in **IEEE Expert**, Vol. 12, No. 5, 1997, 48-59

[5] Godfrey, P., *Minimization in cooperative response to failing database queries*, in **International Journal of Cooperative Information Systems**, Vol. 6, No. 2, 1993, 95-149

[6] Grzymala-Busse J., *A new version of the rule induction system LERS*, in **Fundamenta Informaticae**, Vol. 31, No. 1, 1997, 27-39

[7] Kastner, M.P., Crowder, R.G, *Perception of the Major/Minor Distinction*, in Emotional connations in young children, **Music Perception**, Vol. 8, No. 2, 1984, 189-202

[8] Leman, M., *Schema-Based Tone Center Recognition of Musical Signals*, in **Journal of Music Research**, Vol. 23, 1994, 169-204

[9] Pawlak, Z., *Information systems - theoretical foundations*, Information Systems Journal, Vol. 6, 1991, 205-218

[10] Raś, Z.W., Dardzińska, A., *Intelligent query answering*, in **Encyclopedia of Data Warehousing and Mining**, (Ed. J. Wang), Idea Group Inc., 2005, 639-643

[11] Raś, Z.W., Tzacheva, A., Tsay, L.-S., *Action rules*, in **Encyclopedia of Data Warehousing and Mining**, (Ed. J. Wang), Idea Group Inc., 2005, 1-5

[12] Raś, Z., Wieczorkowska, A., *Action Rules: how to increase profit of a company*, in Principles of Data Mining and Knowledge Discovery, (Eds. D.A. Zighed, J. Komorowski, J. Zytkow), Proceedings of PKDD'00, Lyon, France, LNAI, No. 1910, Springer-Verlag, 2000, 587-592

[13] Tramo, M.J., Cariani P.A, Delgutte B., Braida L.D., *Neurobiological Foundations for the Theory of Harmony in Western Tonal Music*, in **Annals of the New York Academy of Sciences**, Vol. 930, 2001, 112

[14] Tsay, L.-S., Raś, Z.W., *Action Rules Discovery System DEAR, Method and Experiments*, Journal of Experimental and Theoretical Artificial Intelligence, Taylor & Francis, Vol. 17, No. 1-2, 2005, 119-128

[15] Wang, M., Zhang, N., Zhu, H., *User-adaptive music emotion recognition*, Signal Processing, Proceedings, ICSP′04, 7th International Conference, Vol. 2, 2004, 1352-1355