ITCS 4101: Introduction to Natural Language Processing

N-gram Language Models

Neural Language Models

Recurrent Neural Networks (RNNs)

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Language Modeling (LM)

- **Causal** Language Modeling:
  - **Predict the next word in a sequence**:
    - AI systems use machine _____

      eat?

      learning?

      frogs?

      …
    - **The LM estimates P(word | word_{-1}, word_{-2}, ...)**
      - we want P(learning | machine, use) >> P(about | machine, eat).
  - **Decoder** neural architectures are widely used to train LMs:
    - GPT, Gemini, Llama, Mixtral, Claude, …

Language Modeling (LM)

- **Masked** Language Modeling:
  - **Predict the most likely word in a context**:
    - AI systems use machine _____ models for language understanding .

      eat?

      learning?

      frogs?

      …
    - The LM estimates P(word | word_{-1}, word_{-2}, ...; word_{1}, word_{2}, ...)
      - we want P(learning | machine, use; models, for)

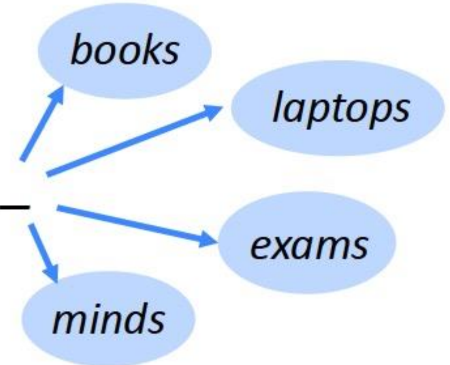
        >> P(frogs | machine, use; models, for).
  - **Encoder** neural architectures are used to train masked LMs.
    - BERT, RoBERTa, …

white slides selected from cs224n @ Stanford

Language Modeling

- **Language Modeling** is the task of predicting what word comes next.

the students opened their ______



- More formally: given a sequence of words $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(t)}$, compute the probability distribution of the next word $\boldsymbol{x}^{(t+1)}$:

$$P(\boldsymbol{x}^{(t+1)} \mid \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$$

where $\boldsymbol{x}^{(t+1)}$ can be any word in the vocabulary $V = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_{|V|}\}$

- A system that does this is called a **Language Model**.

Language Modeling

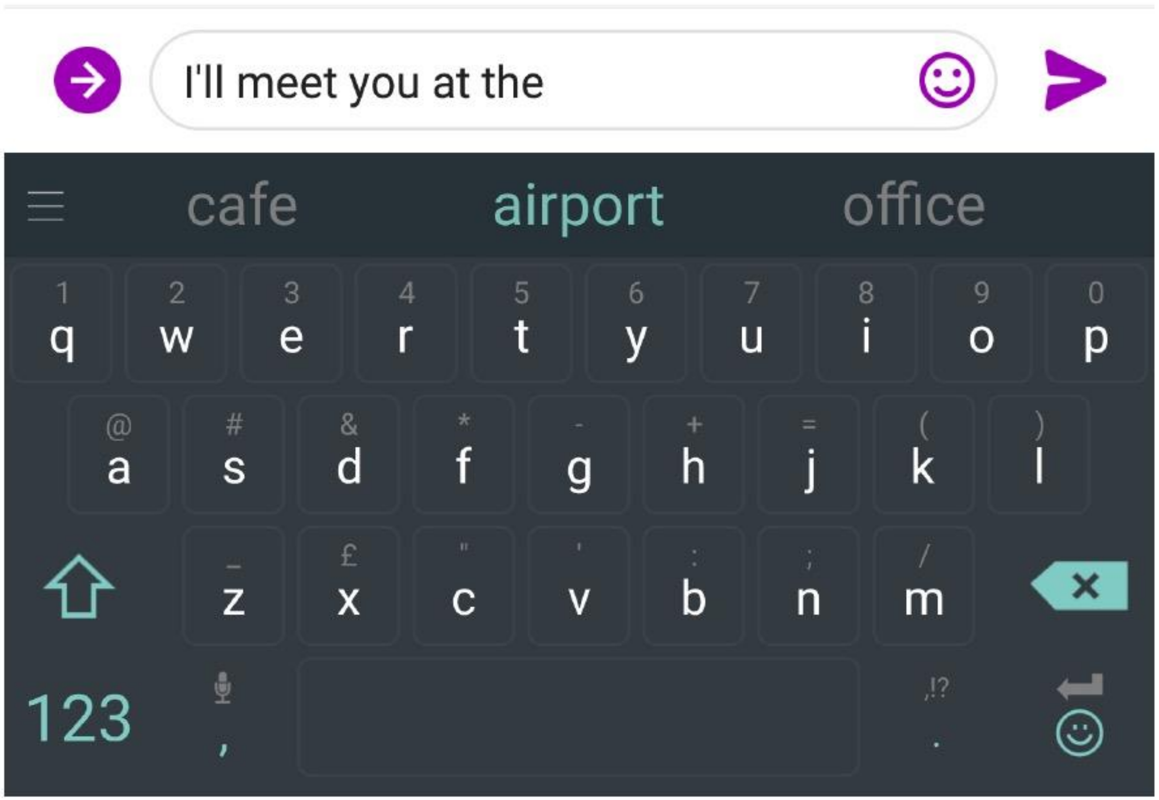
- You can also think of a Language Model as a system that assigns probability to a piece of text.

- For example, if we have some text $x^{(1)}, \ldots, x^{(T)}$, then the probability of this text (according to the Language Model) is:

$$P(\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}) = P(\boldsymbol{x}^{(1)}) \times P(\boldsymbol{x}^{(2)} | \, \boldsymbol{x}^{(1)}) \times \cdots \times P(\boldsymbol{x}^{(T)} | \, \boldsymbol{x}^{(T-1)}, \ldots, \boldsymbol{x}^{(1)})$$

$$= \prod_{t=1}^{T} \underbrace{P(\boldsymbol{x}^{(t)} | \, \boldsymbol{x}^{(t-1)}, \ldots, \boldsymbol{x}^{(1)})}_{\text{This is what our LM provides}}$$

You use Language Models every day!



You use Language Models every day!

Google

what is the |

what is the **weather**
what is the **meaning of life**
what is the **dark web**
what is the **xfl**
what is the **doomsday clock**
what is the **weather today**
what is the **keto diet**
what is the **american dream**
what is the **speed of light**
what is the **bill of rights**

Google Search | I'm Feeling Lucky

n-gram Language Models

the students opened their ______

- **Question**: How to learn a Language Model?
- **Answer** (pre- Deep Learning): learn a *n*-gram Language Model!

- Definition: A *n*-gram is a chunk of *n* consecutive words.
  - unigrams: “the”, “students”, “opened”, ”their”
  - bigrams: “the students”, “students opened”, “opened their”
  - trigrams: “the students opened”, “students opened their”
  - 4-grams: “the students opened their”

- Idea: Collect statistics about how frequent different n-grams are, and use these to predict next word.

n-gram Language Models

- First we make a simplifying assumption: $\boldsymbol{x}^{(t+1)}$ depends only on the preceding *n-1* words.

$$P(\boldsymbol{x}^{(t+1)}|\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(1)}) = P(\boldsymbol{x}^{(t+1)}|\overbrace{\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)}}^{n\text{-1 words}}) \qquad \text{(assumption)}$$

prob of a n-gram

prob of a (n-1)-gram

$$= \frac{\boxed{P(\boldsymbol{x}^{(t+1)},\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}}{\boxed{P(\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}} \qquad \text{(definition of conditional prob)}$$

- **<u>Question:</u>** How do we get these *n*-gram and (*n*-1)-gram probabilities?
- **<u>Answer:</u>** By counting them in some large corpus of text!

$$\approx \frac{\text{count}(\boldsymbol{x}^{(t+1)},\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}{\text{count}(\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})} \qquad \text{(statistical approximation)}$$

n-gram Language Models: Example

Suppose we are learning a 4-gram Language Model.

~~*as the proctor started the clock, the*~~ *students opened their* ______

discard

condition on this

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\text{count}(\text{students opened their } \boldsymbol{w})}{\text{count}(\text{students opened their})}$$

For example, suppose that in the corpus:

- "students opened their" occurred 1000 times
- "students opened their books" occurred 400 times
  - → P(books | students opened their) = 0.4
- "students opened their exams" occurred 100 times
  - → P(exams | students opened their) = 0.1

Should we have discarded the "proctor" context?

Sparsity Problems with n-gram Language Models

Sparsity Problem 1

Problem: What if *"students opened their $\boldsymbol{w}$"* never occurred in data? Then $\boldsymbol{w}$ has probability 0!

(Partial) Solution: Add small δ to the count for every $\boldsymbol{w} \in V$. This is called *smoothing*.

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\boxed{\text{count}(\text{students opened their } \boldsymbol{w})}}{\boxed{\text{count}(\text{students opened their})}}$$

Sparsity Problem 2

Problem: What if *"students opened their"* never occurred in data? Then we can't calculate probability for *any* $\boldsymbol{w}$!

(Partial) Solution: Just condition on *"opened their"* instead. This is called *backoff*.

Note: Increasing *n* makes sparsity problems *worse.* Typically we can't have *n* bigger than 5.

Storage Problems with n-gram Language Models

Storage: Need to store count for all *n*-grams you saw in the corpus.

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\boxed{\text{count}(\text{students opened their } \boldsymbol{w})}}{\text{count}(\text{students opened their})}$$

Increasing *n* or increasing corpus increases model size!

n-gram Language Models in practice

- You can build a simple trigram Language Model over a 1.7 million word corpus (Reuters) in a few seconds on your laptop*

Business and financial news

today the _______

get probability distribution

company	$\boxed{0.153}$
bank	$\boxed{0.153}$
price	0.077
italian	0.039
emirate	0.039
…	

Sparsity problem: not much granularity in the probability distribution

Otherwise, seems reasonable!

*** Try for yourself:** https://nlpforhackers.io/language-models/

Generating text with a n-gram Language Model

- You can also use a Language Model to generate text.

today the ________

condition on this

get probability distribution

company	0.153
bank	0.153
price	0.077
italian	0.039
emirate	0.039
…	

sample

Generating text with a n-gram Language Model

- You can also use a Language Model to generate text.

today the price ________

condition on this

get probability distribution

of	0.308
for	0.050
it	0.046
to	0.046
is	0.031
…	

sample

Generating text with a n-gram Language Model

- You can also use a Language Model to generate text.

today the price of _______

condition on this

get probability distribution

the	0.072
18	0.043
oil	0.043
its	0.036
gold	0.018
…	

sample

Generating text with a n-gram Language Model

- You can also use a Language Model to generate text.

today the price of gold _______

Generating text with a n-gram Language Model

- You can also use a Language Model to generate text.

today the price of gold per ton , while production of shoe lasts and shoe industry , the bank intervened just after it considered and rejected an imf demand to rebuild depleted european stocks , sept 30 end primary 76 cts a share .

Surprisingly grammatical!

…but **incoherent.** We need to consider more than three words at a time if we want to model language well.

But increasing *n* worsens sparsity problem, and increases model size…

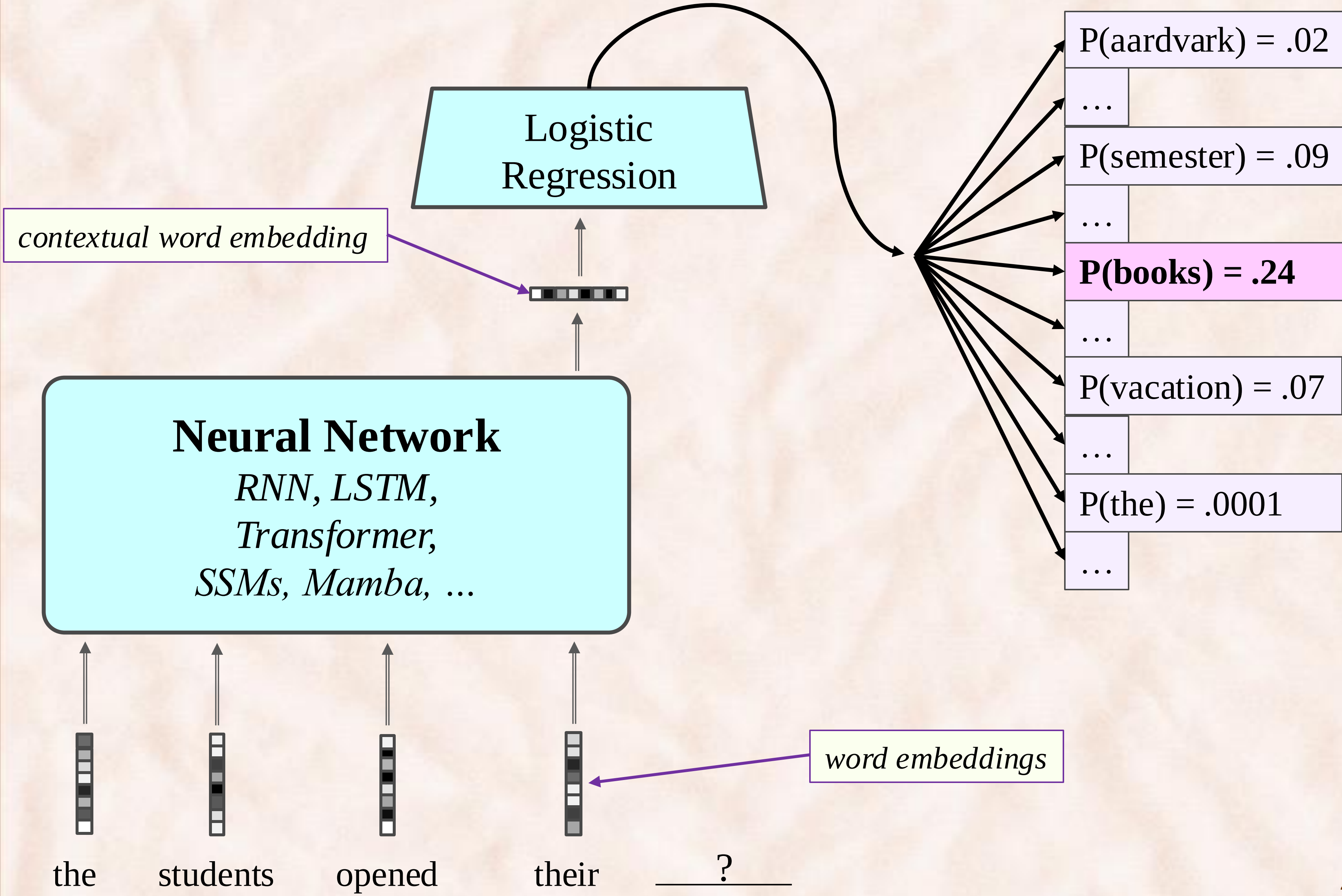
N-gram Language Models are Shallow Learners

- N-gram LMs exhibit extremely little "understanding" of language due to their **weak generalization**.

- N-gram LMs do not generalize to:
  - Unseen words:
    - *spinach* and *kale* are considered completely different words.
    - sim(*spinach*, *kale*) = sim(*spinach*, *laptop*) = 0.
  - Unseen combinations of known words.
  - Longer context.

Neural LMs

https://www.sciencefocus.com/planet-earth/asteroid-impact-change-earths-orbit


P(aardvark) = .02
…
P(semester) = .09
…
P(books) = .24
…
P(vacation) = .07
…
P(the) = .0001
…
Logistic Regression
contextual word embedding
Neural Network
RNN, LSTM, Transformer, SSMs, Mamba, …
word embeddings
the
students
opened
their
?

What can be learned from predicting missing words?

Asheville is a city located in the state of ______.

What can be learned from predicting missing words?

I took ___ dog out for a walk .

What can be learned from predicting missing words?

Upon exiting the restaurant, the man realized ___ left ___ phone at the table .

What can be learned from predicting missing words?

I stopped by the grocery store to buy bread, blueberry pie, milk, and ____ .

What can be learned from predicting missing words?

Overall , the value I got from the two hours watching it was the sum total of the popcorn and the drink .

The movie was _____ .

What can be learned from predicting missing words?

Andrei was eating in the kitchen .

Roxby joined him for breakfast .

After a while , Andrei went to the living room to watch TV .

Once she was done with breakfast , Roxby left the _____ .

What can be learned from predicting missing words?

Dan and Tom go to a restaurant for dinner . Dan leaves his coat on the chair , then goes to the bathroom . While Dan was gone , Tom hangs Dan ’s coat on the coat rack . When Dan comes back , he thinks his coat is on the ___ .

What can be learned from predicting missing words?

I have been thinking of the sequence that goes 1, 2, 4, 7, 11, 16, 22, ______

What can be learned from predicting missing words?

Theorem: $\sqrt{2}$ is an irrational number .

Proof: _____

Theorem: $\sqrt{2}$ is an irrational number .

Proof: Suppose that $\sqrt{2}$ were a _____

Theorem: $\sqrt{2}$ is an irrational number .

Proof: Suppose that $\sqrt{2}$ were a rational number , so by definition

$\sqrt{2}$ = a _____

Word Embeddings

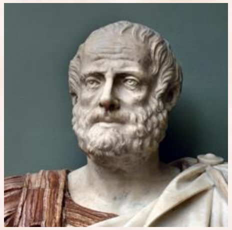
- Neural LMs use as input **word embeddings**:
  - **Vectors** that are **short**, e.g. 256, 512, 768, or 1024, and **dense**, e.g. most if not all entries are non-zero.
    - Very unlike *one-hot encodings*, which are long and sparse.

- We want word embeddings to represent **word meanings**:
  - What do we mean by "word meaning"?
  - How do we train word embeddings to represent word meaning?
    - How do we determine their quality?

What do words mean?

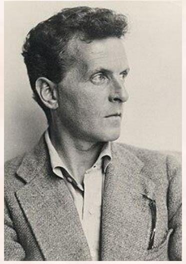
- Classical approach uses a **dictionary** and the **context**.

- Depending on the context, a word is used to refer to a particular *concept*, i.e. *sense*, i.e. *word meaning*:
  - The word "pepper" has multiple meanings, as listed in the dictionary:
    - sense 1: spice from pepper plant
    - sense 2: the pepper plant itself
    - sense 3: another similar plant (Jamaican pepper)
    - sense 4: another plant with peppercorns (California pepper)
    - sense 5: capsicum (i.e. chili, paprika, bell pepper, etc)

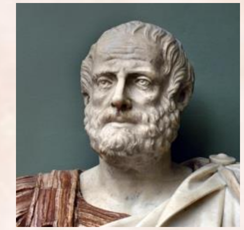


Aristotle: What is a concept?

- Classical theory of concepts:
  - The meaning of a word is a concept that is defined by **necessary** and **sufficient** conditions.
    - The following necessary conditions, jointly, are sufficient for an object *x* to be a **square**:
      - *x* has (exactly) four sides
      - each of *x*'s sides is straight
      - *x* is a closed figure
      - *x* lies in a plane
      - each of *x*'s sides is equal in length to each of the others
      - each of *x*'s interior angles is equal to the others (right angles)
      - the sides of *x* are joined at their ends



Wittgenstein to Aristotle: *But what is a **game**?*



- Philosophical Investigations (1945, # 66):

Don't say "there must be something common, or they would not be called \`games'"—but *look and see* whether there is anything common to all"

  - Is it amusing? Is there competition? Is there long-term strategy?
  - Is skill required? Must luck play a role?
  - Are there cards? Is there a ball?
  - …
- Family Resemblance theory (Rosch and Mervis):
  - Each item has at least one, and probably several, elements in common with one or more items, but no, or few, elements are common to all items.

Game 1	Game 2	Game 3	Game 4
ABC	BCD	ACD	ABD

Distributional Semantics Idea

- Wittgenstein (1945):

  The meaning of a word is its use in the language.

- Harris (1954):

  Words that occur in the same contexts tend to have similar meanings.

- Firth (1935, 1957):

  The complete meaning of a word is always contextual, and no study of meaning apart from a complete context can be taken seriously.

  You shall know a word by the company it keeps.

Distributional Semantics

- The **meaning** of a word is **determined by** the words that appear nearby, i.e. its **context**.
  - **Words that appear in the same contexts tend to have similar meanings.**
  - <u>One of the most successful ideas of modern statistical NLP!</u>

What does recent English borrowing *ongchoi* mean?

- Suppose you see these sentences:
  - Ong choi is delicious **sautéed with garlic**
  - Ong choi is superb **over rice**
  - Ong choi **leaves** with **salty** sauces
- And you've also seen these:
  - …spinach **sautéed with garlic over rice**
  - Chard stems and **leaves** are **delicious**
  - Collard greens and other **salty** leafy greens
- Conclusion:
  - Ongchoi is a leafy green like spinach, chard, or collard greens

Ongchoi: *Ipomoea aquatica "Water Spinach"*

空心菜
kangkong
rau muống
…

Yamaguchi, Wikimedia Commons, public domain

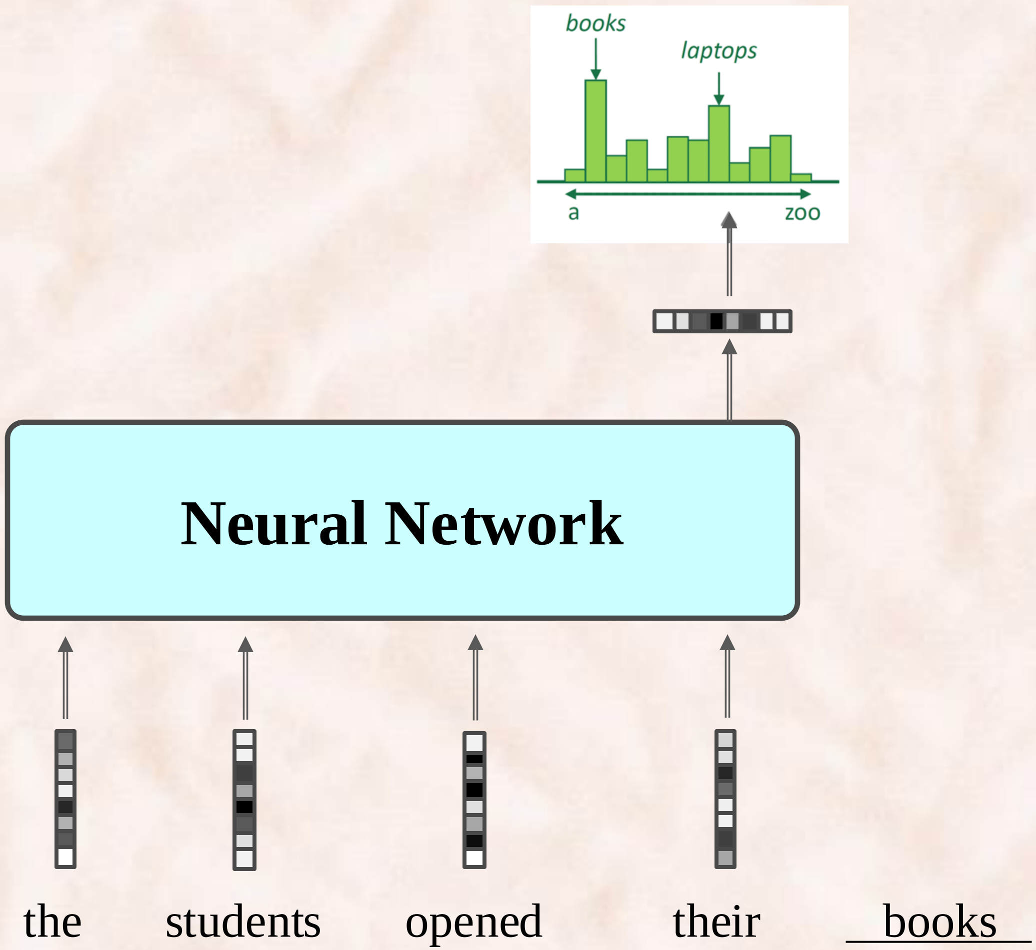
Distributional Semantics and Neural LMs

- Learn word embeddings by training a neural network to **predict a missing word** given words in the **context**.
  - This is a special type of *reconstructing the input* idea used in other modalities, such as computer vision (see *autoencoders*).

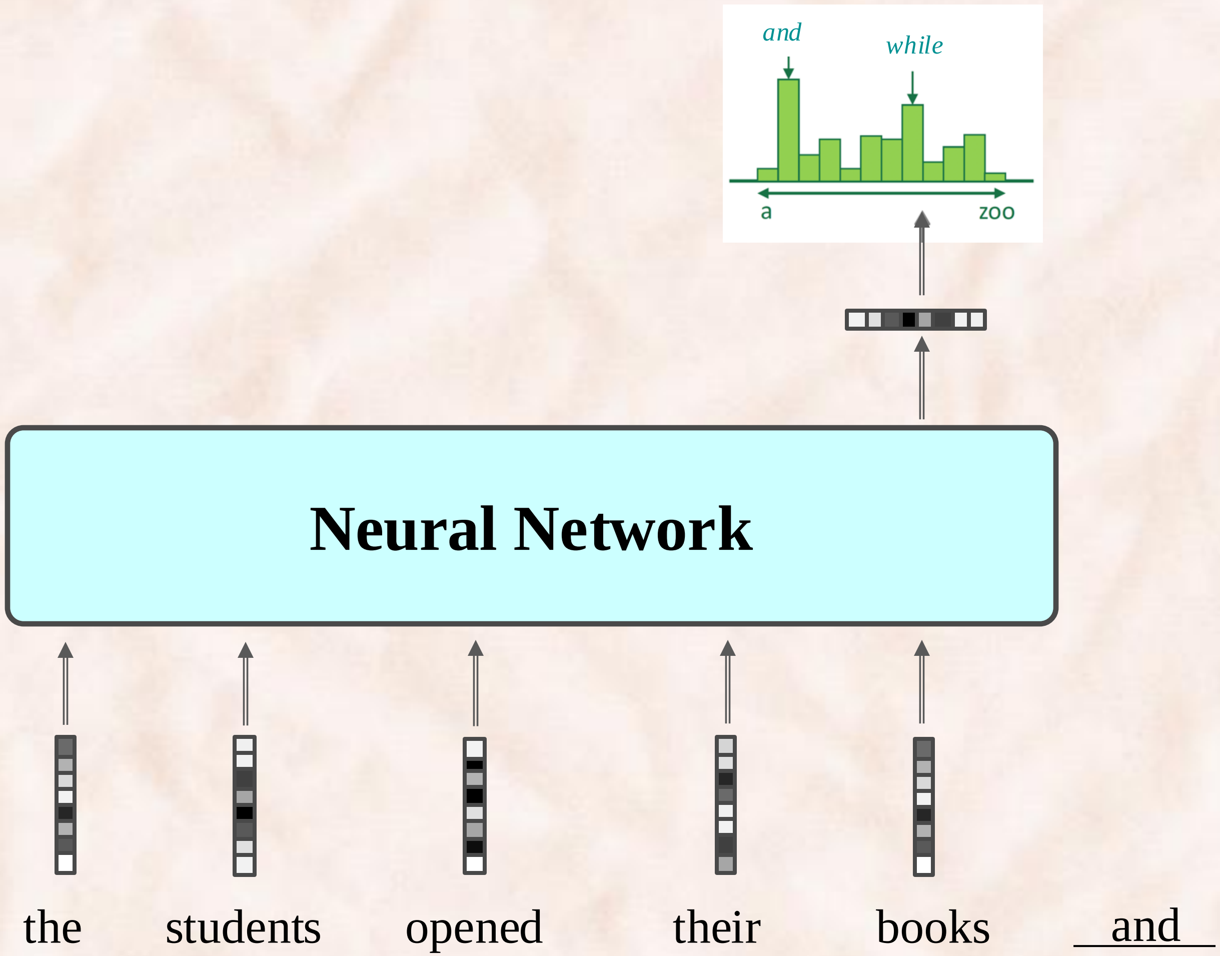
- **Causal LMs** trained using the distributional hypothesis:
  - Context: words so far.
  - Missing word: next word.

- **Masked LMs** trained using the distributional hypothesis:
  - Context: words to the left and to the right of the center word.
  - Missign word: word in the center.

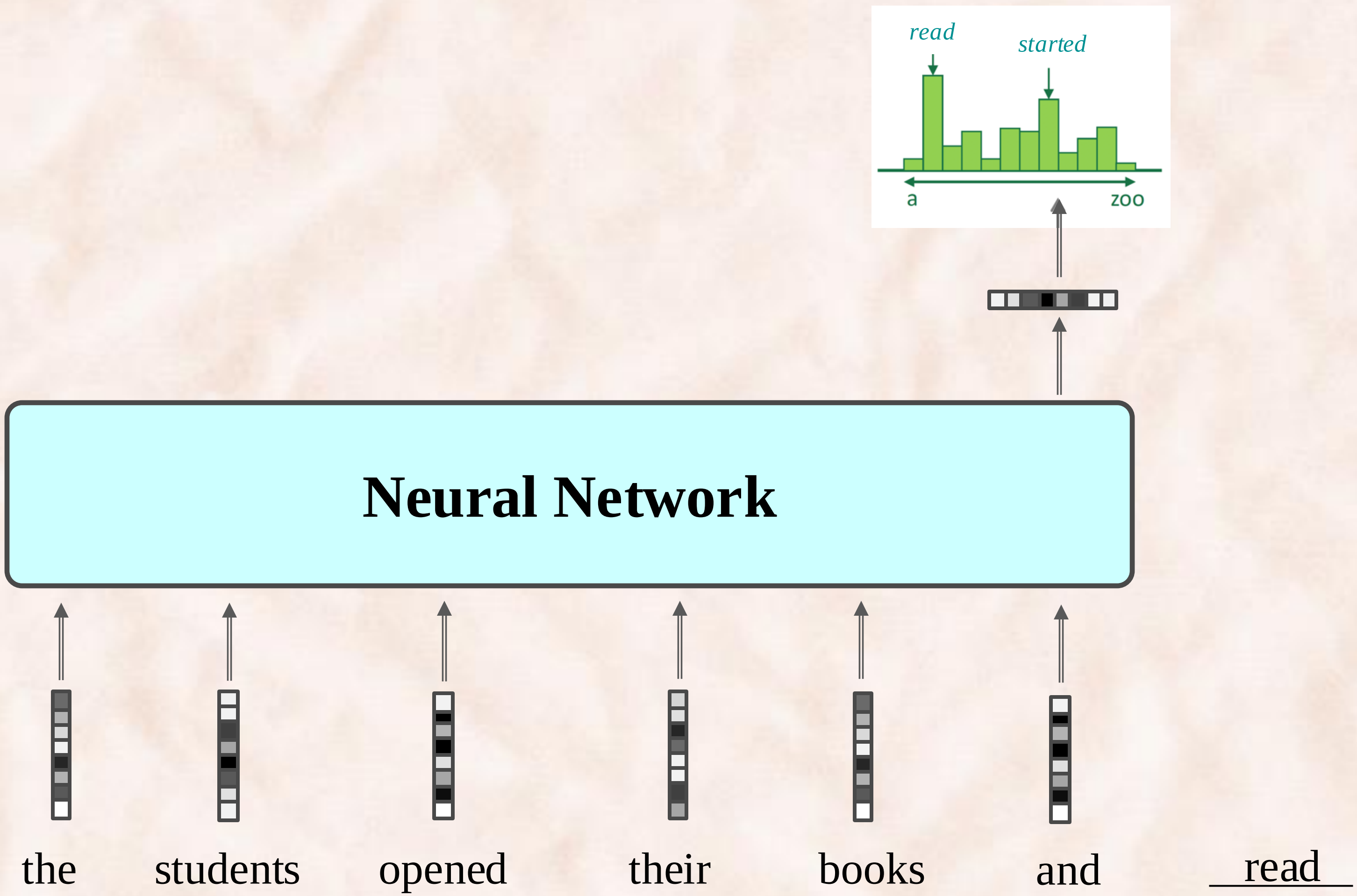
Neural Language Modeling: **Decoders**



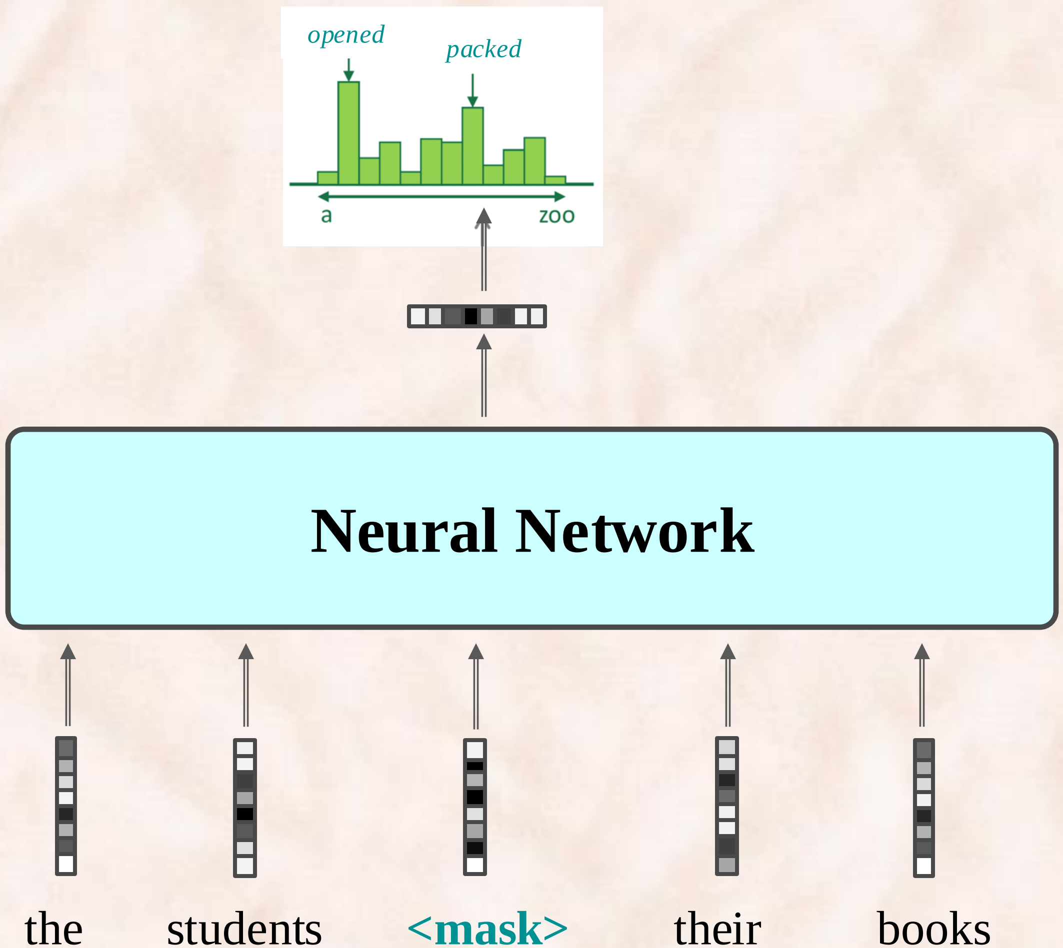
Neural Language Modeling: **Decoders**



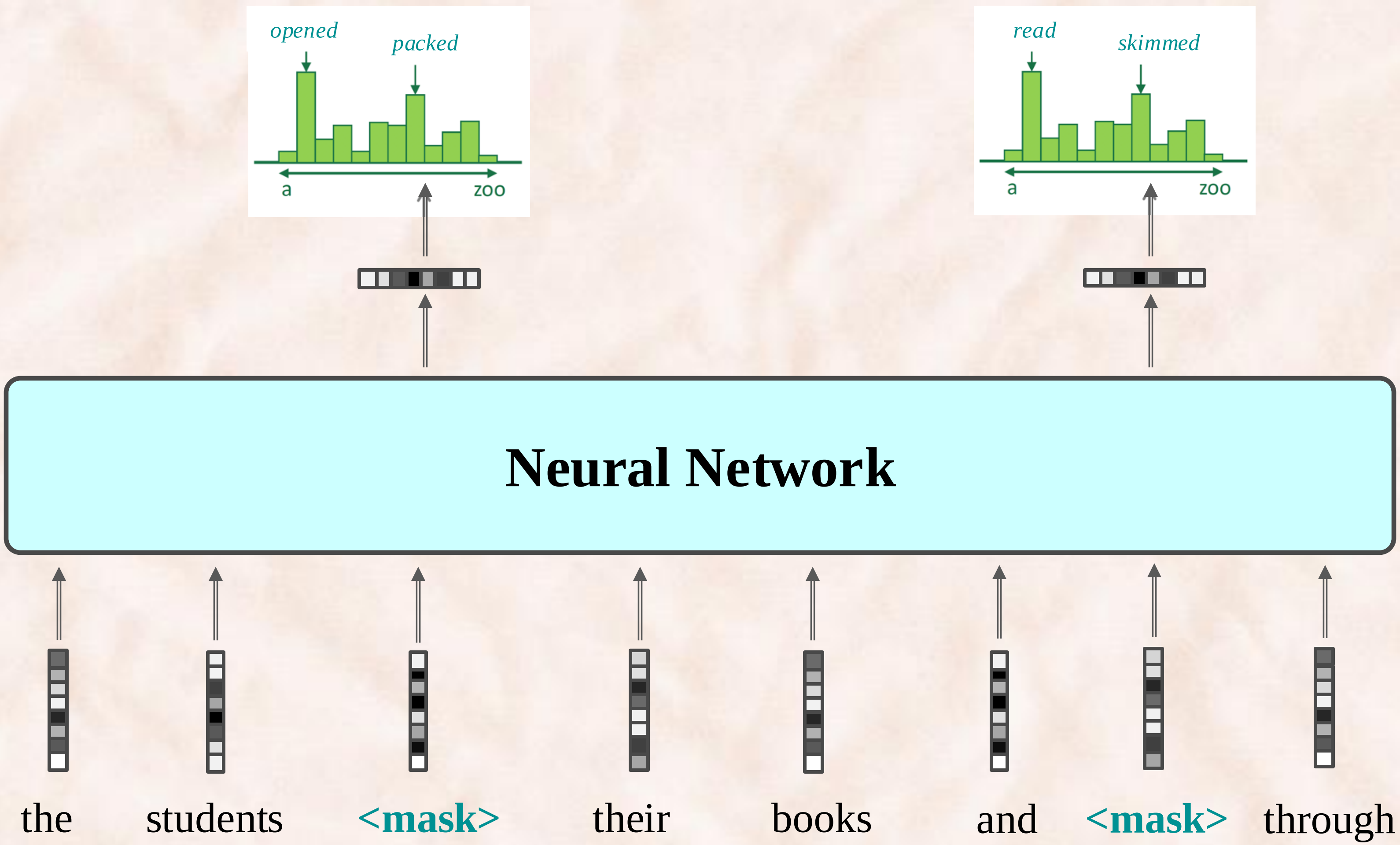
Neural Language Modeling: **Decoders**



Neural Language Modeling: **Encoders**



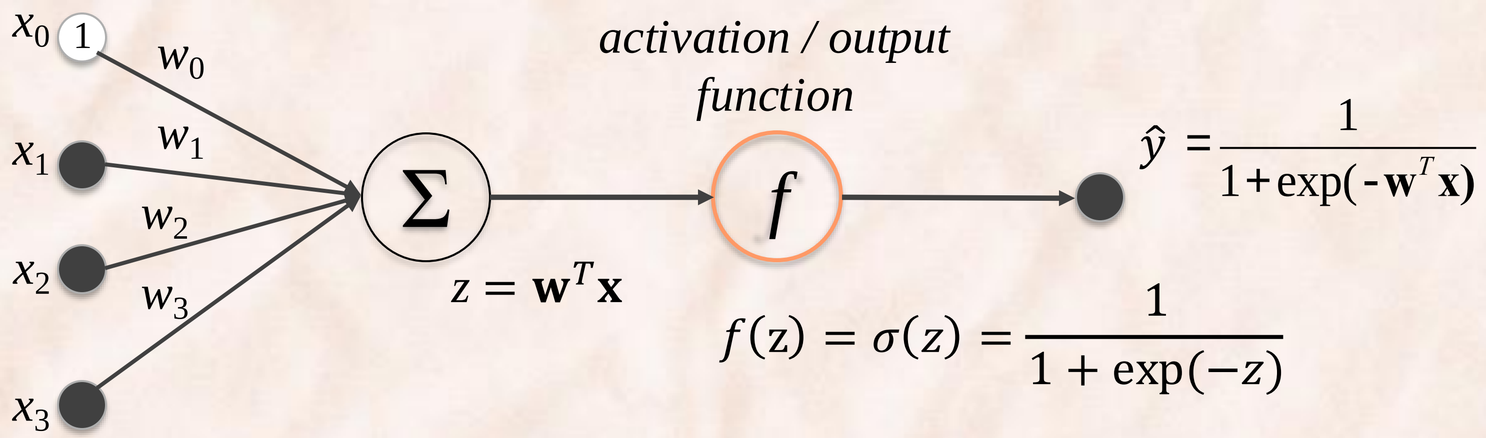
Neural Language Modeling: **Encoders**



Start: Interlude on Neural Networks

- Required reading:
  - Chapter 7 on Neural Networks from the J&M textbook.
    - Sections on Training are optional.

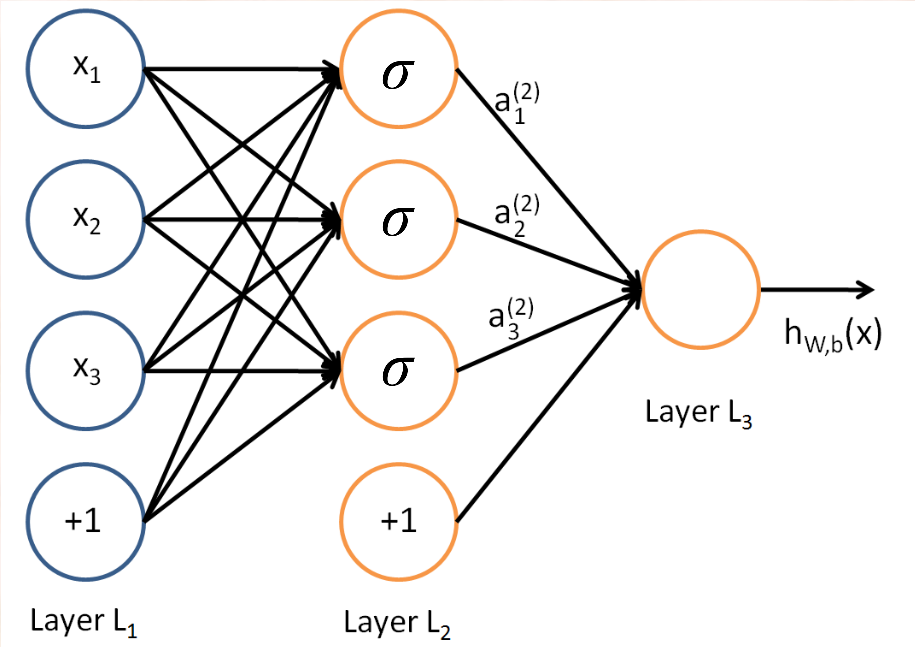
Logistic Neuron = Logistic Regression



- Algebraic interpretation:
  - The output of the neuron is a **linear combination** of inputs from other neurons, **rescaled by** the synaptic **weights**.
    - weights w_i correspond to the synaptic weights (activating or inhibiting).
    - summation corresponds to combination of signals in the soma.
  - It is often transformed through a monotonic **activation function**.

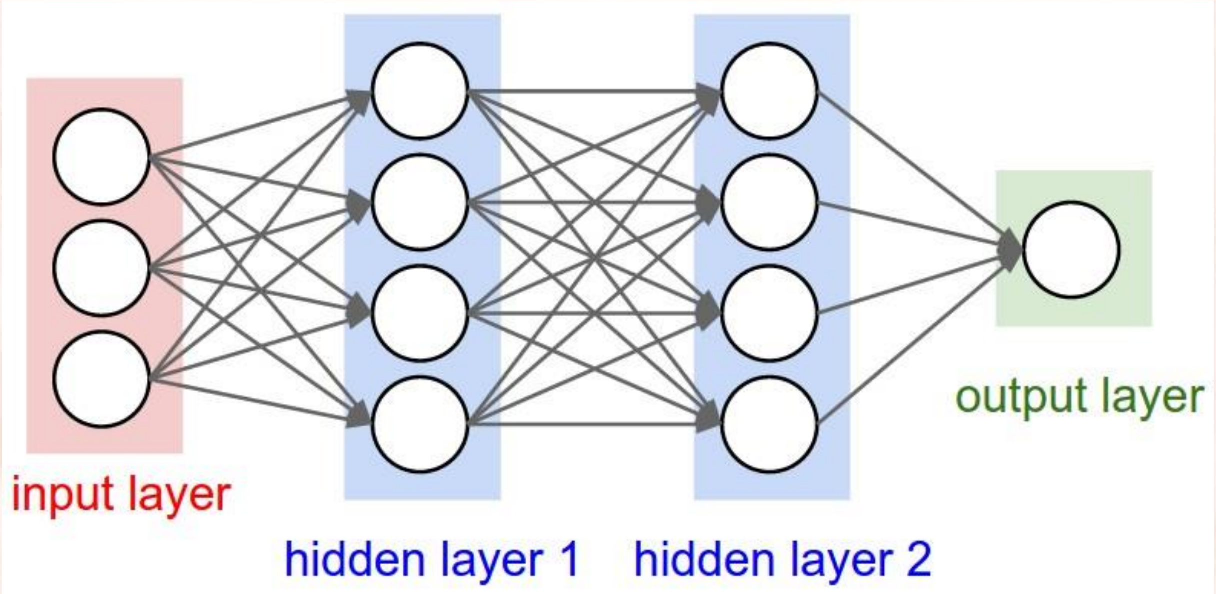
Neural Network Model

- Put together many neurons in layers, such that the output of a neuron on layer *l* can be the input of another neuron on layer *l* + 1:



input layer ***hidden layer*** ***output layer***

Feed-Forward Neural Networks

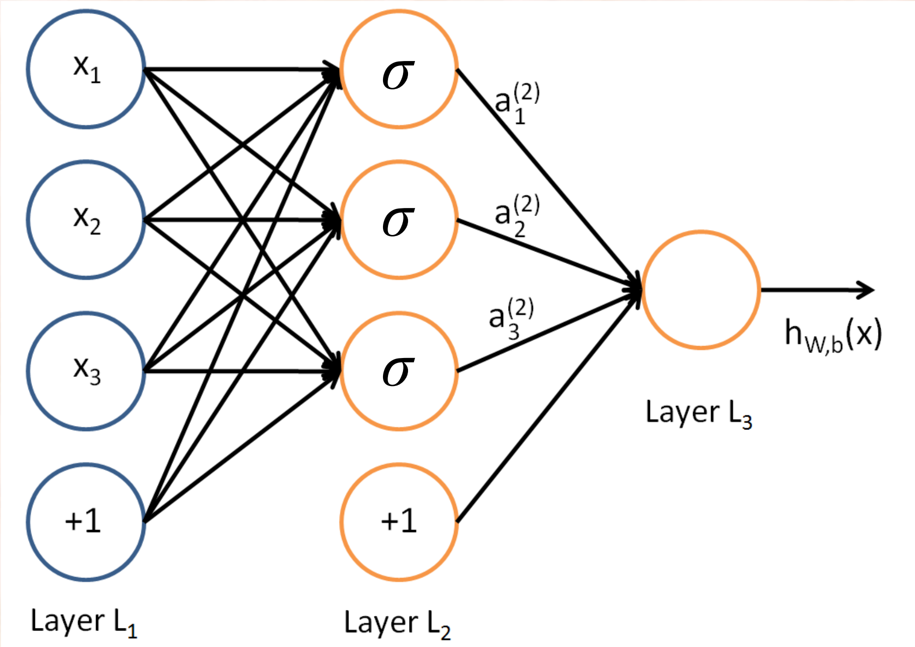


10 10 5 1

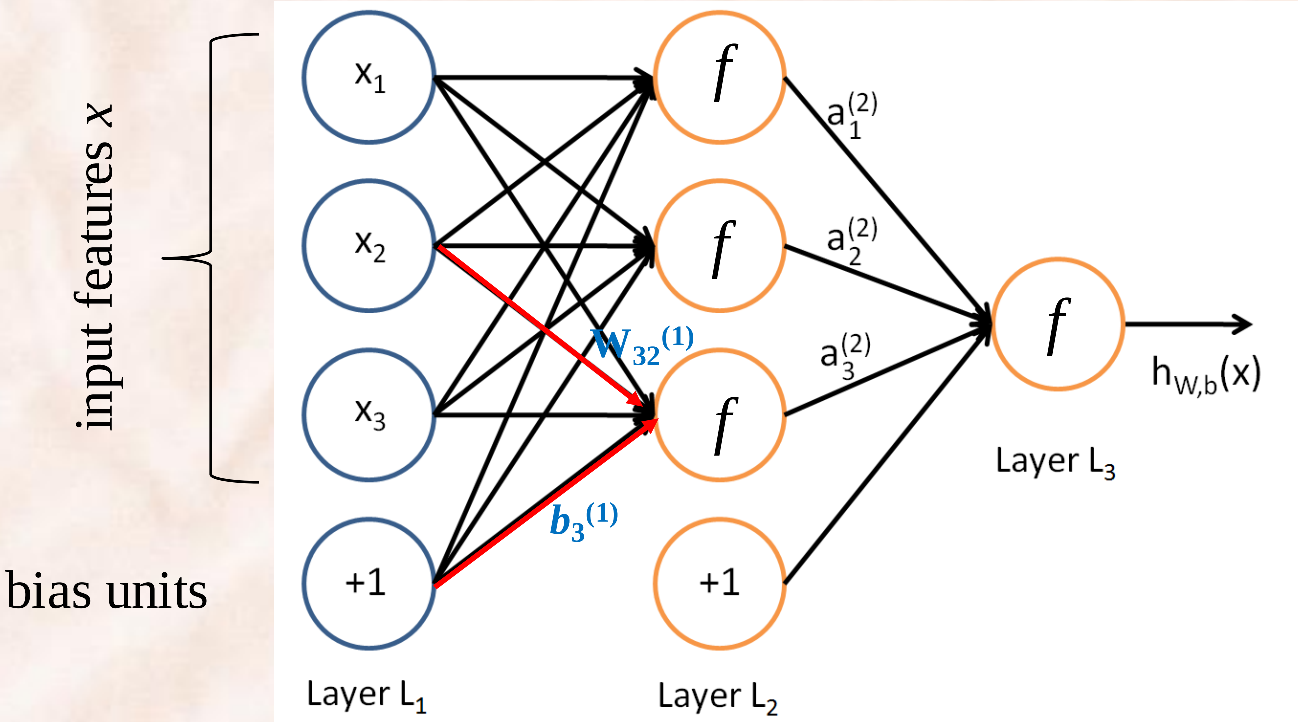
1. For each neuron in hidden layer 1, we need 10 + 1 = 11 params. For the 10 neurons on hidden layer 1, we need in total 10 * 11 = **110 params**.
2. For the 5 neurons on hidden layer 2, we need 5 * 11 = **55 params**.
3. For the output neurons, we need 5 + 1 = **6 params**.

Neural Network Model

- Put together many neurons in layers, such that the output of a neuron can be the input of another:



input layer ***hidden layer*** ***output layer***



- n_l =3 is the number of **layers**.
  - L_1 is the input layer, L_3 is the output layer
- (**W**, **b**) = ($W^{(1)}$, $b^{(1)}$, $W^{(2)}$, $b^{(2)}$) are the parameters:
  - $W^{(l)}_{ij}$ is the **weight** of the connection between unit j in layer l and unit i in layer l + 1.
  - $b^{(l)}_i$ is the **bias** associated unit unit i in layer l + 1.
- $a^{(l)}_i$ is the **activation** of unit i in layer l, e.g. $a^{(1)}_{\mathrm{i}} = x_{\mathrm{i}}$ and $a^{(3)}_1 = h_{W,b}(x)$.

Inference: Forward Propagation

- The activations in the hidden layer are:

$$a_1^{(2)} = f(W_{11}^{(1)} x_1 + W_{12}^{(1)} x_2 + W_{13}^{(1)} x_3 + b_1^{(1)})$$
$$a_2^{(2)} = f(W_{21}^{(1)} x_1 + W_{22}^{(1)} x_2 + W_{23}^{(1)} x_3 + b_2^{(1)})$$
$$a_3^{(2)} = f(W_{31}^{(1)} x_1 + W_{32}^{(1)} x_2 + W_{33}^{(1)} x_3 + b_3^{(1)})$$

- The activations in the output layer are:

$$h_{W,b}(x) = a_1^{(3)} = f(W_{11}^{(2)} a_1^{(2)} + W_{12}^{(2)} a_2^{(2)} + W_{13}^{(2)} a_3^{(2)} + b_1^{(2)})$$

- Compressed notation:

$a_i^{(l)} = f(z_i^{(l)})$ where $z_i^{(2)} = \sum_{j=1}^{n} W_{ij}^{(1)} x_j + b_i^{(1)}$

Forward Propagation

- Forward propagation (unrolled):

$$a_1^{(2)} = f(W_{11}^{(1)}x_1 + W_{12}^{(1)}x_2 + W_{13}^{(1)}x_3 + b_1^{(1)})$$
$$a_2^{(2)} = f(W_{21}^{(1)}x_1 + W_{22}^{(1)}x_2 + W_{23}^{(1)}x_3 + b_2^{(1)})$$
$$a_3^{(2)} = f(W_{31}^{(1)}x_1 + W_{32}^{(1)}x_2 + W_{33}^{(1)}x_3 + b_3^{(1)})$$
$$h_{W,b}(x) = a_1^{(3)} = f(W_{11}^{(2)}a_1^{(2)} + W_{12}^{(2)}a_2^{(2)} + W_{13}^{(2)}a_3^{(2)} + b_1^{(2)})$$

- Forward propagation (compressed):

$$z^{(2)} = W^{(1)}x + b^{(1)}$$
$$a^{(2)} = f(z^{(2)})$$
$$z^{(3)} = W^{(2)}a^{(2)} + b^{(2)}$$
$$h_{W,b}(x) = a^{(3)} = f(z^{(3)})$$

- Element-wise application:

$f(\mathbf{z}) = [f(z_1), f(z_2), f(z_3)]$

Forward Propagation

- Forward propagation (compressed):

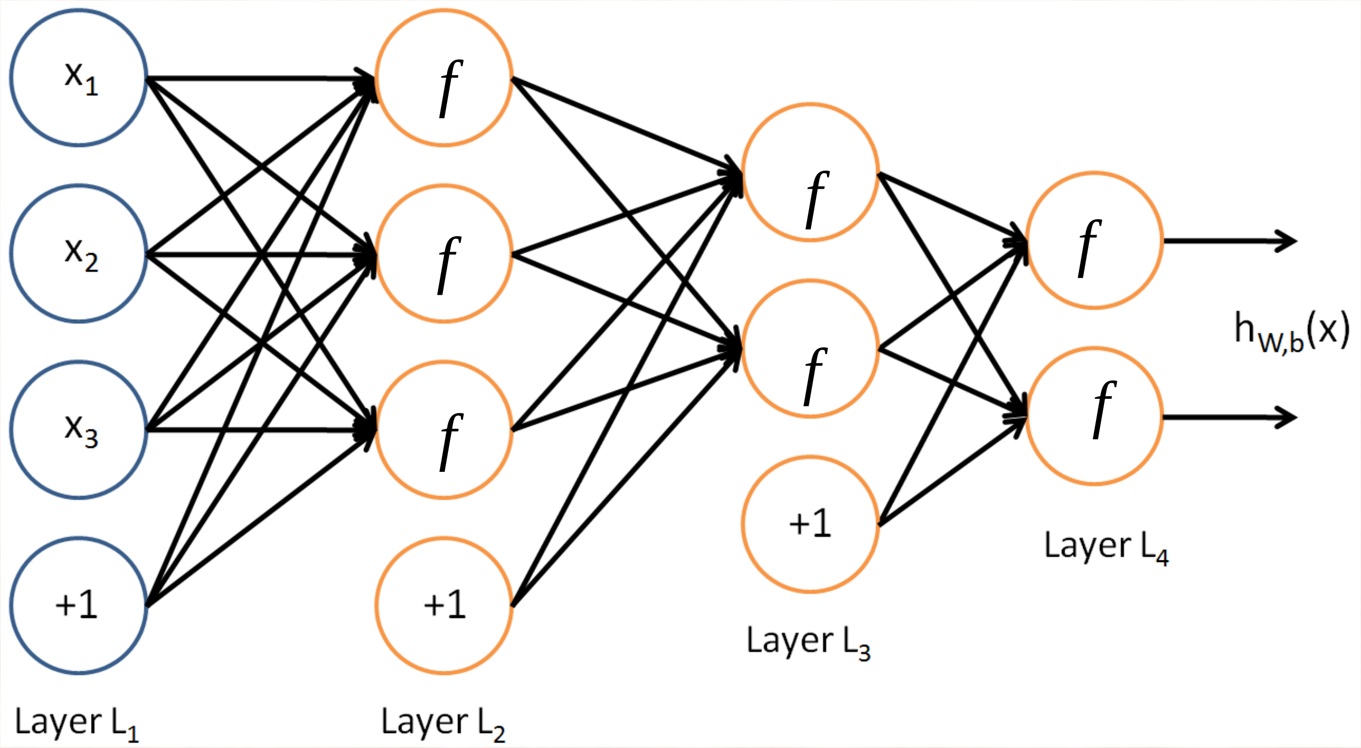
$$
\begin{aligned}
z^{(2)} &= W^{(1)}x + b^{(1)} \\
a^{(2)} &= f(z^{(2)}) \\
z^{(3)} &= W^{(2)}a^{(2)} + b^{(2)} \\
h_{W,b}(x) &= a^{(3)} = f(z^{(3)})
\end{aligned}
$$

- Composed of two ***forward propagation steps***:

$$
\begin{aligned}
z^{(l+1)} &= W^{(l)}a^{(l)} + b^{(l)} \\
a^{(l+1)} &= f(z^{(l+1)})
\end{aligned}
$$

Multiple Hidden Units, Multiple Outputs

- Write down the forward propagation steps for:



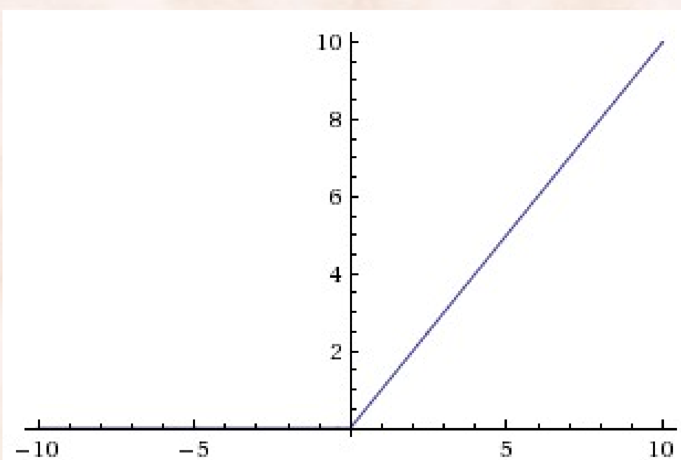
ReLU and Generalizations

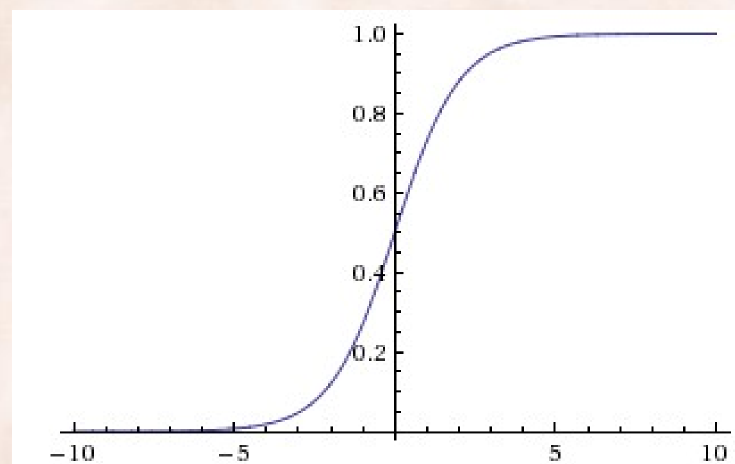
- It has become more common to use piecewise linear activation functions for hidden units instead of σ:
  - **ReLU**: the rectifier activation $g(z) = \max\{0, z\}$.
  - **Absolute value ReLU**: $g(z) = |z|$.
  - **Maxout**: $g(a_1, \ldots, a_k) = \max\{a_1, \ldots, a_k\}$.
    - needs k weight vectors instead of 1.
  - **Leaky ReLU**: $g(a) = \max\{0, a\} + \alpha \min(0, a)$.

⇒ the network computes a *piecewise linear function* (up to the output activation function).

ReLU vs. Sigmoid and Tanh

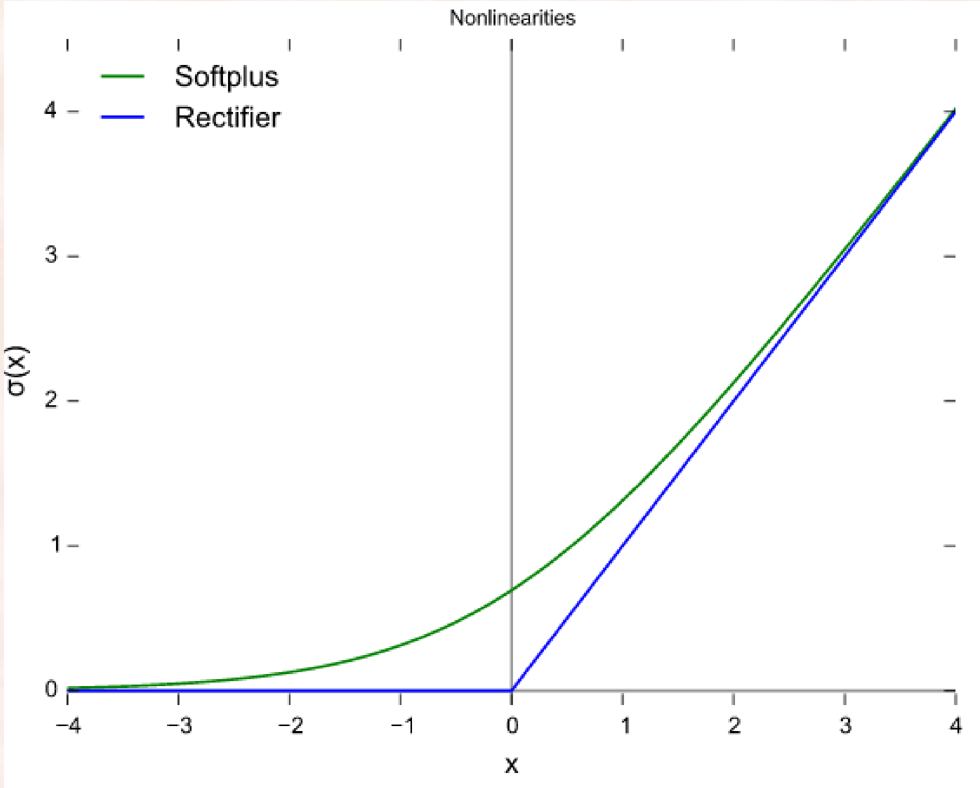
- Sigmoid and Tanh saturate for values not close to 0:
  - "kill" gradients, bad behavior for gradient-based learning.
- ReLU does not saturate for values > 0:
  - greatly accelerates learning, fast implementation.
  - fragile during training and can "die", due to 0 gradient:
    - initialize all *b*'s to a small, positive value, e.g. 0.1.





ReLU vs. Softplus

- Softplus $g(z) = \ln(1+e^z)$ is a smooth version of the rectifier.
  - Saturates less than ReLU, yet ReLU still does better [Glorot, 2011].



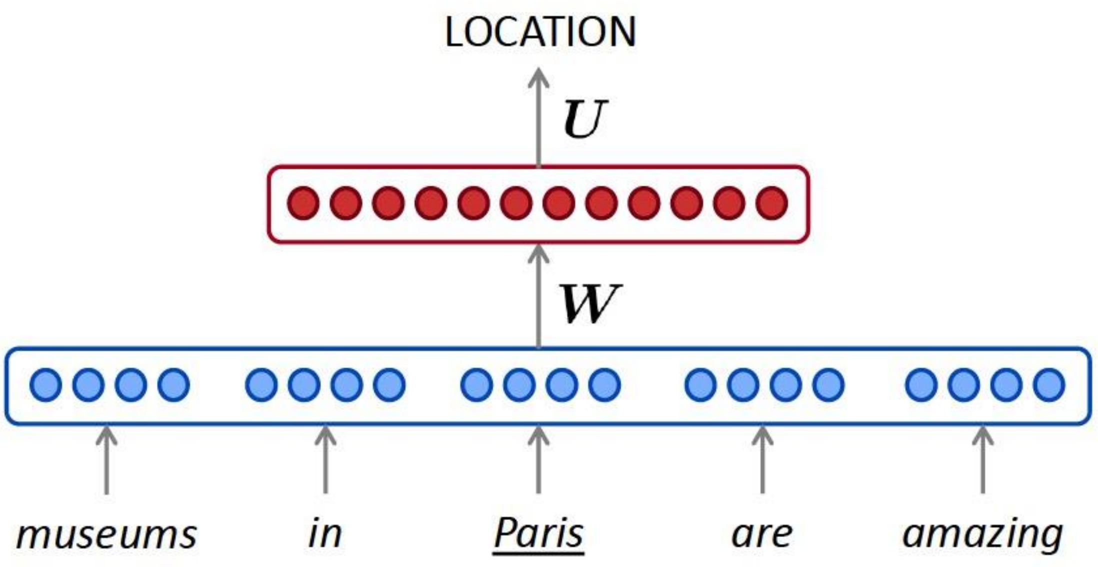
End: Interlude on Neural Networks

- Required reading:
  - Chapter 7 on Neural Networks from the J&M textbook.
    - Sections on Training are optional.

How to build a *neural* Language Model?

- Recall the Language Modeling task:
  - Input: sequence of words $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(t)}$
  - Output: prob dist of the next word $P(\boldsymbol{x}^{(t+1)} | \; \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$

- How about a window-based neural model?
  - We can apply this to Named Entity Recognition:



A fixed-window neural Language Model

~~*as the proctor started the clock*~~ *the students opened their* ______

discard

fixed window

A fixed-window neural Language Model

output distribution

$$\hat{\boldsymbol{y}} = \mathrm{softmax}(\boldsymbol{U}\boldsymbol{h} + \boldsymbol{b}_2) \in \mathbb{R}^{|V|}$$

hidden layer

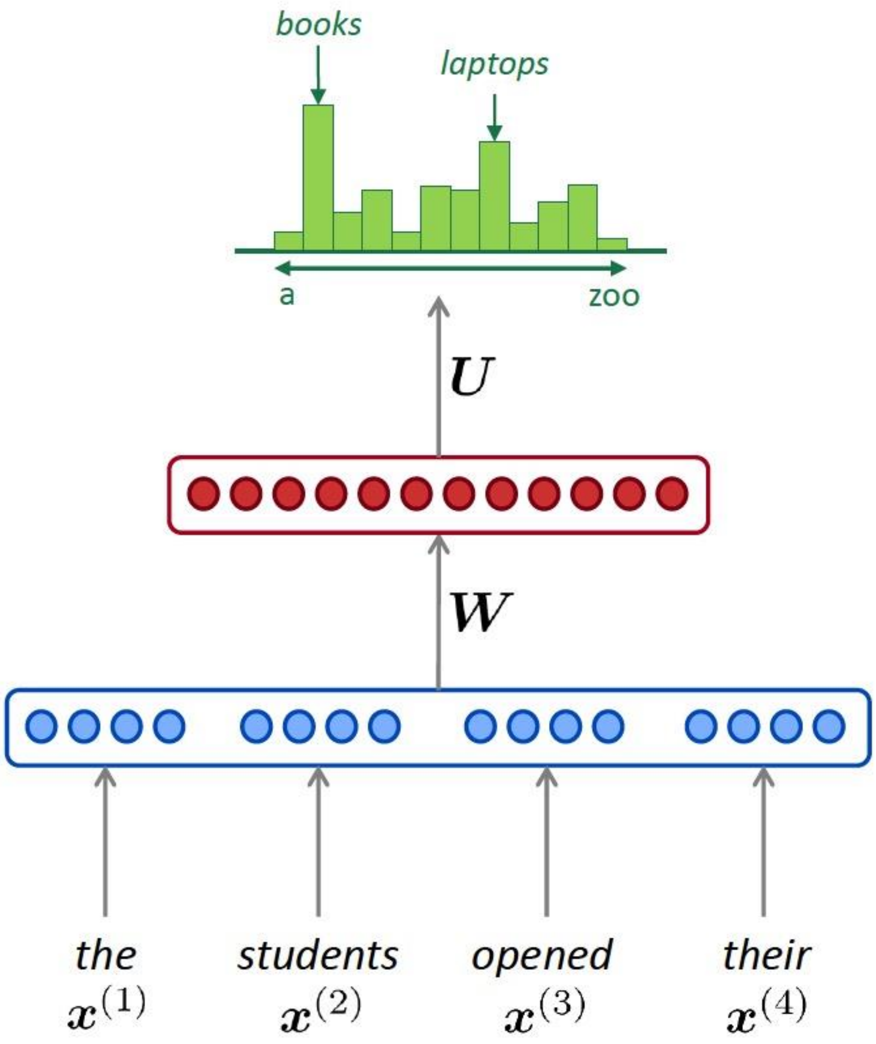
$$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{e} + \boldsymbol{b}_1)$$

concatenated word embeddings

$$\boldsymbol{e} = [\boldsymbol{e}^{(1)}; \boldsymbol{e}^{(2)}; \boldsymbol{e}^{(3)}; \boldsymbol{e}^{(4)}]$$

words / one-hot vectors

$$\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \boldsymbol{x}^{(3)}, \boldsymbol{x}^{(4)}$$



A fixed-window neural Language Model

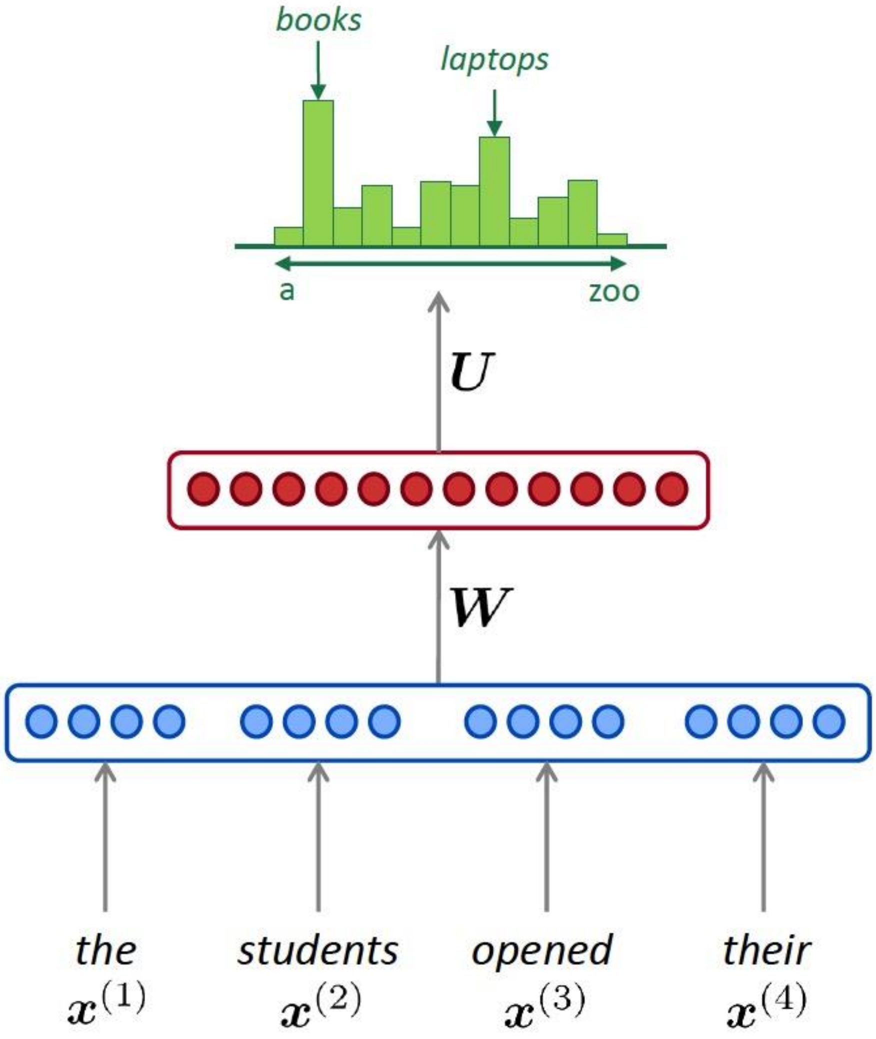
Improvements over *n*-gram LM:

- No sparsity problem
- Don't need to store all observed *n*-grams

Remaining **problems**:

- Fixed window is too small
- Enlarging window enlarges $\boldsymbol{W}$
- Window can never be large enough!
- $\boldsymbol{x}^{(1)}$ and $\boldsymbol{x}^{(2)}$ are multiplied by completely different weights in $\boldsymbol{W}$. No symmetry in how the inputs are processed.

We need a neural architecture that can process *any length input*

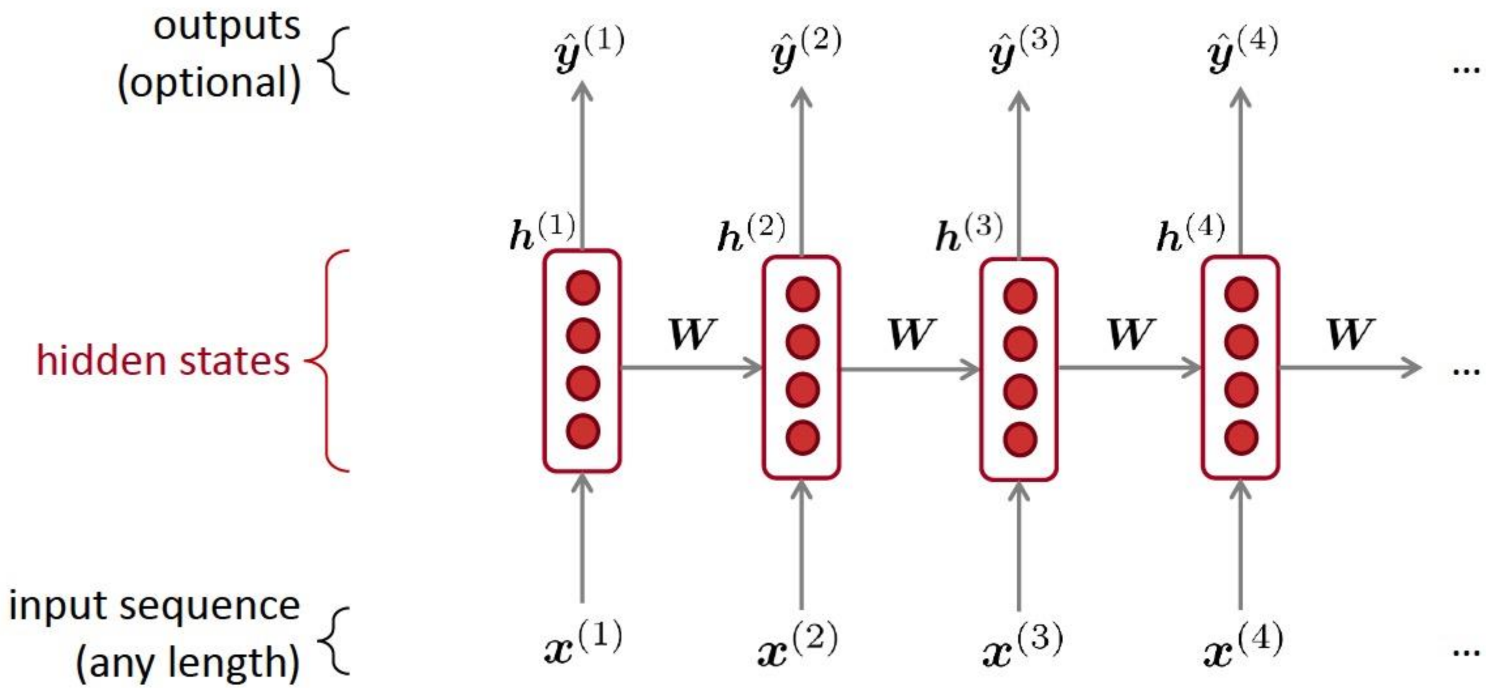


Recurrent Neural Networks (RNN)

A family of neural architectures

Core idea: Apply the same weights $\boldsymbol{W}$ *repeatedly*

Slides from the CS224N at Stanford



A RNN Language Model

$$\hat{\boldsymbol{y}}^{(4)} = P(\boldsymbol{x}^{(5)}|\text{the students opened their})$$

output distribution

$$\hat{\boldsymbol{y}}^{(t)} = \text{softmax}\left(\boldsymbol{U}\boldsymbol{h}^{(t)} + \boldsymbol{b}_2\right) \in \mathbb{R}^{|V|}$$

hidden states

$$\boldsymbol{h}^{(t)} = \sigma\left(\boldsymbol{W}_h \boldsymbol{h}^{(t-1)} + \boldsymbol{W}_e \boldsymbol{e}^{(t)} + \boldsymbol{b}_1\right)$$

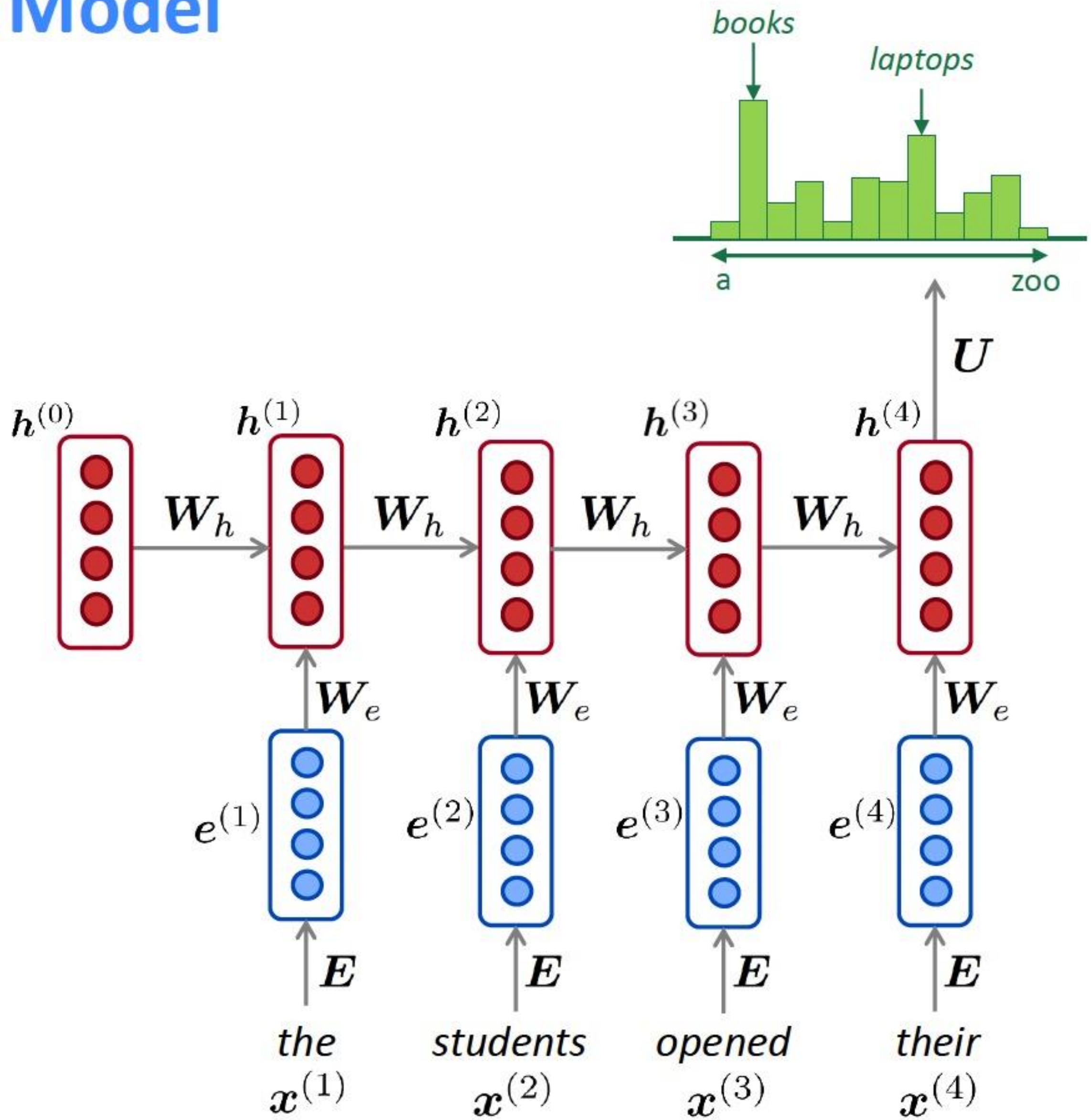
$\boldsymbol{h}^{(0)}$ is the initial hidden state

word embeddings

$$\boldsymbol{e}^{(t)} = \boldsymbol{E}\boldsymbol{x}^{(t)}$$

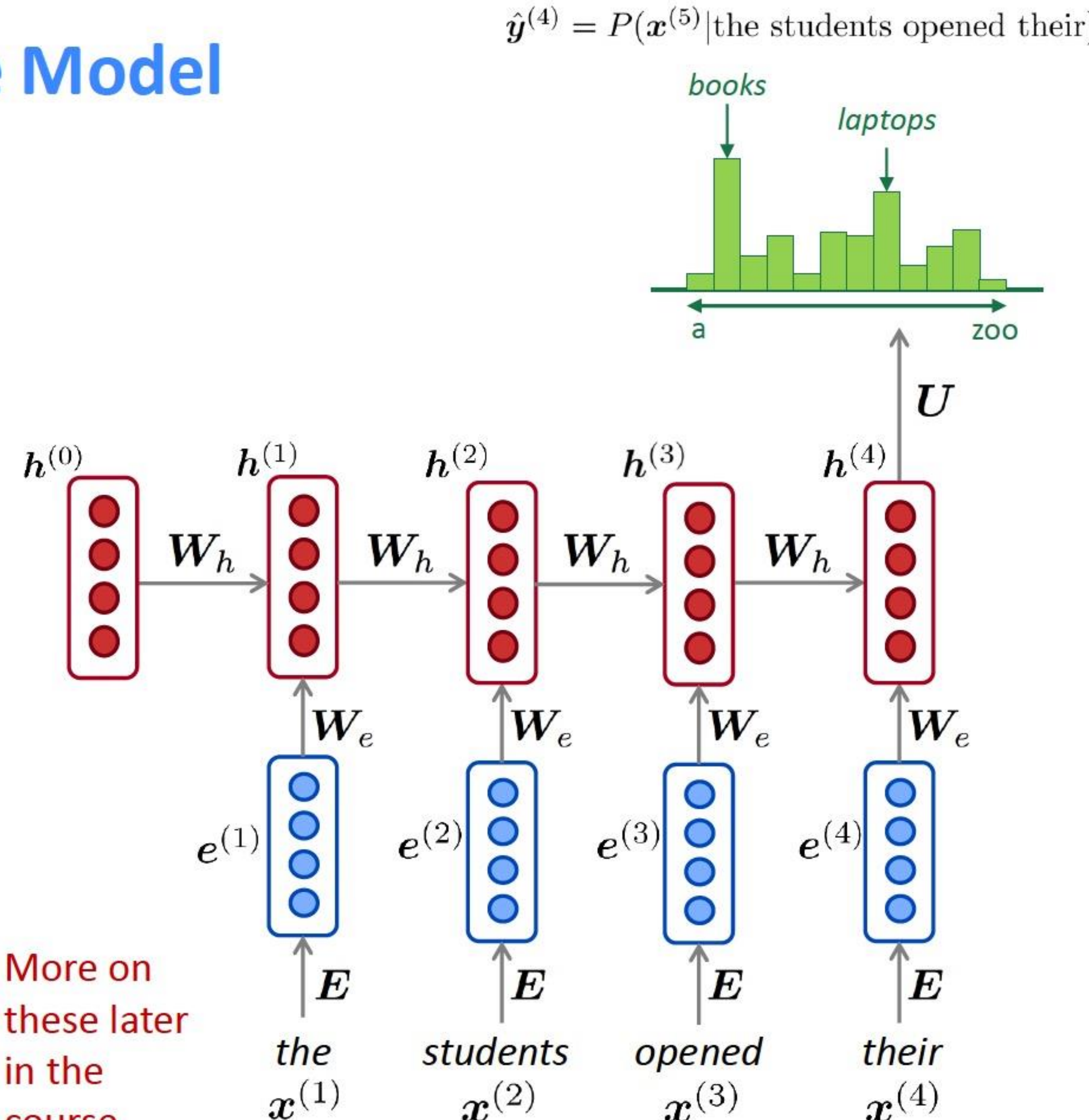
words / one-hot vectors

$$\boldsymbol{x}^{(t)} \in \mathbb{R}^{|V|}$$



***Note**: this input sequence could be much longer, but this slide doesn't have space!*

A RNN Language Model



RNN **Advantages**:

- Can process any length input
- Computation for step *t* can (in theory) use information from many steps back
- Model size doesn't increase for longer input
- Same weights applied on every timestep, so there is symmetry in how inputs are processed.

RNN **Disadvantages**:

- Recurrent computation is slow
- In practice, difficult to access information from many steps back

More on these later in the course

Start Supplemental Material: Training RNNs

Training a RNN Language Model

- Get a big corpus of text which is a sequence of words $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$
- Feed into RNN-LM; compute output distribution $\hat{\boldsymbol{y}}^{(t)}$ for *every step t.*
  - i.e. predict probability dist of *every word*, given words so far

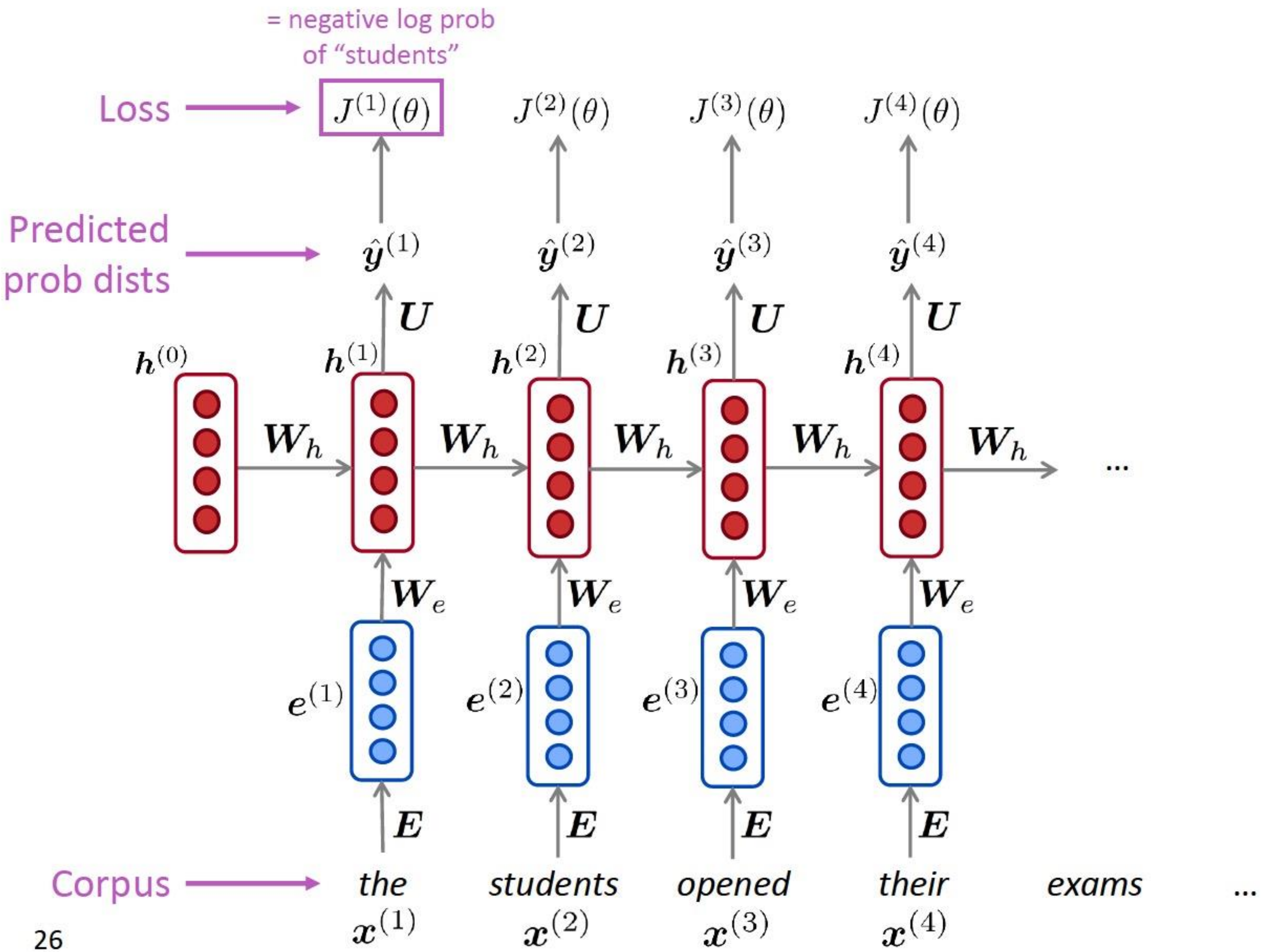
- Loss function on step *t* is cross-entropy between predicted probability distribution $\hat{\boldsymbol{y}}^{(t)}$, and the true next word $\boldsymbol{y}^{(t)}$ (one-hot for $\boldsymbol{x}^{(t+1)}$):

$$J^{(t)}(\theta) = CE(\boldsymbol{y}^{(t)}, \hat{\boldsymbol{y}}^{(t)}) = -\sum_{w \in V} \boldsymbol{y}_w^{(t)} \log \hat{\boldsymbol{y}}_w^{(t)} = -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}_{t+1}}^{(t)}$$

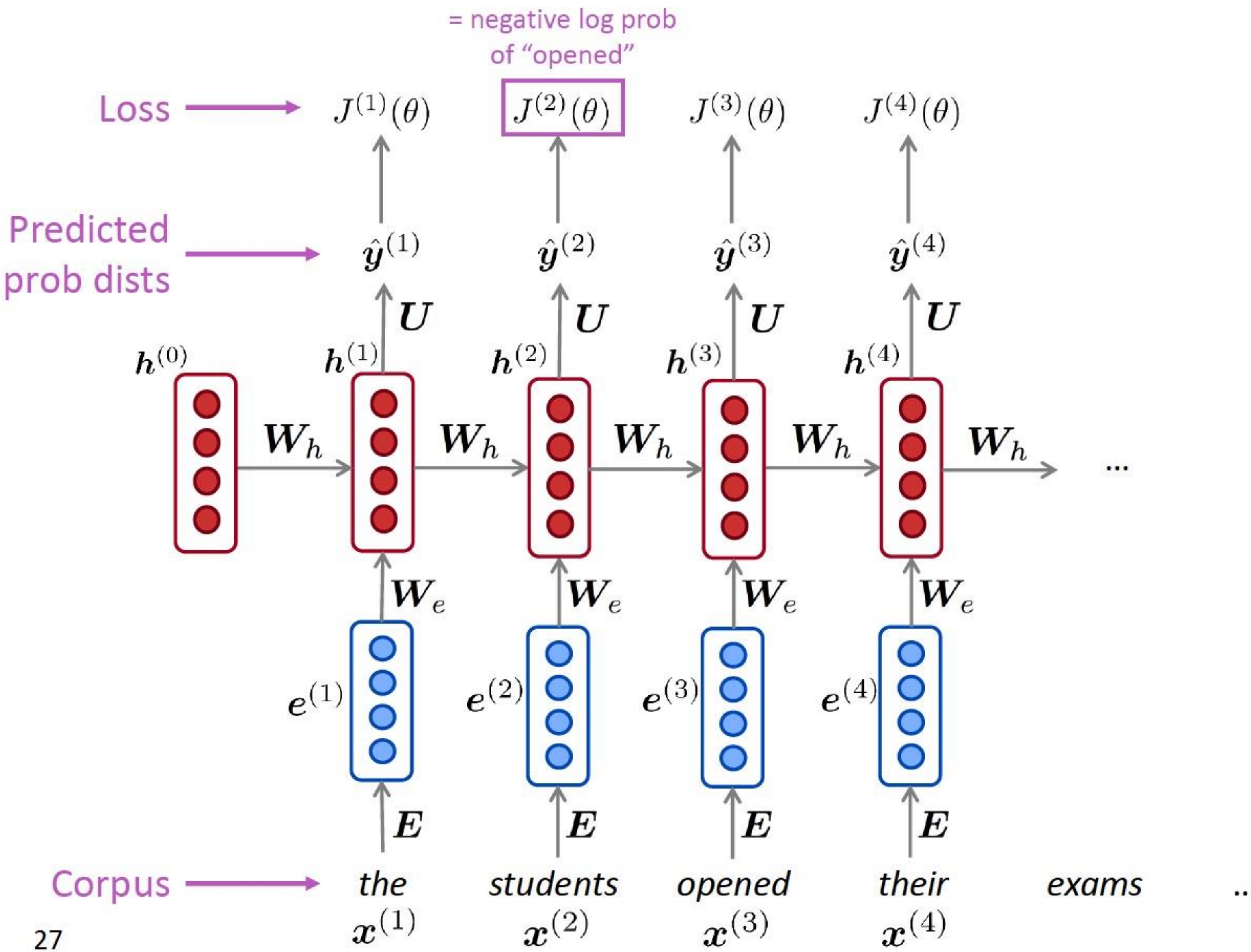
- Average this to get overall loss for entire training set:

$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J^{(t)}(\theta) = \frac{1}{T} \sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}_{t+1}}^{(t)}$$

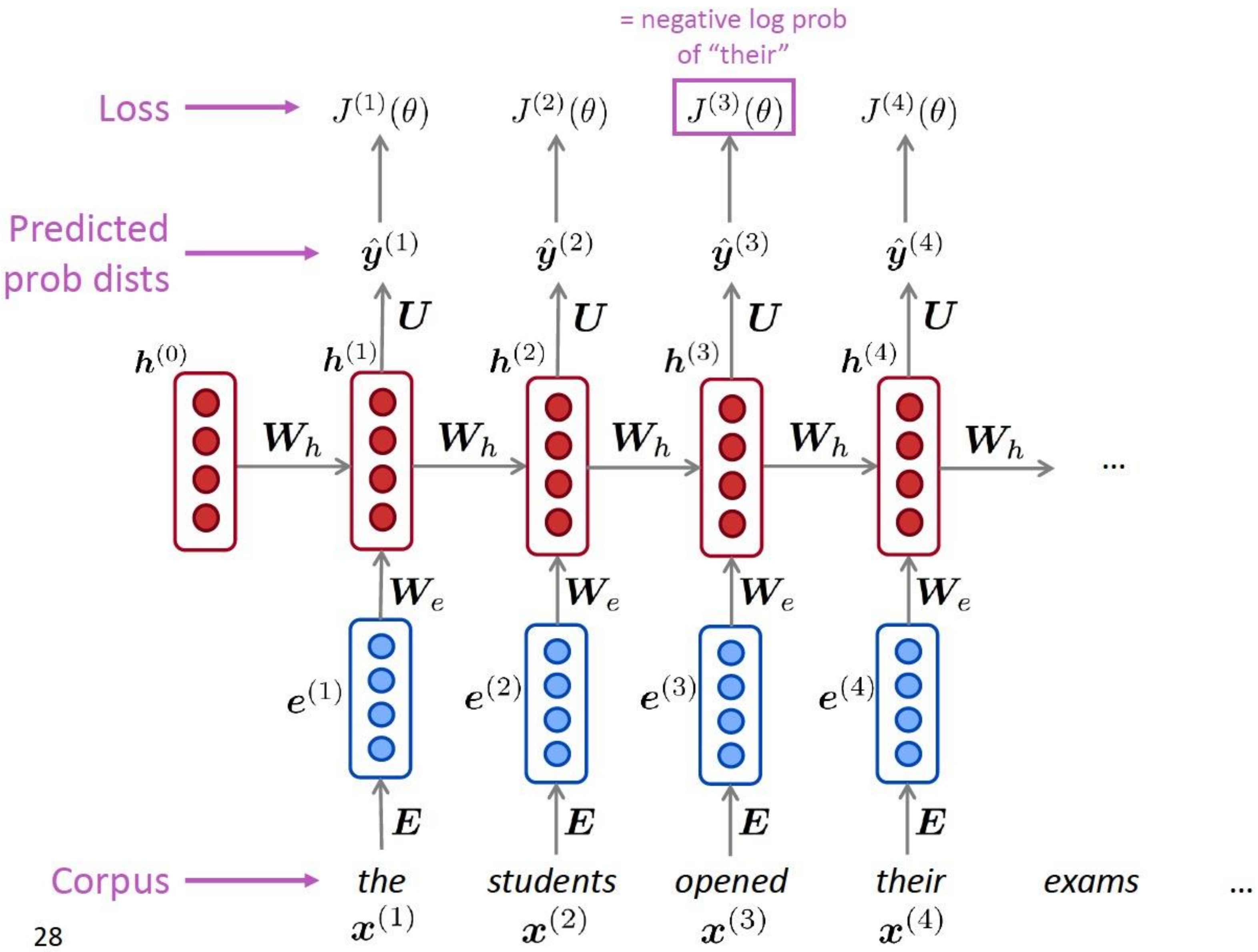
Training a RNN Language Model



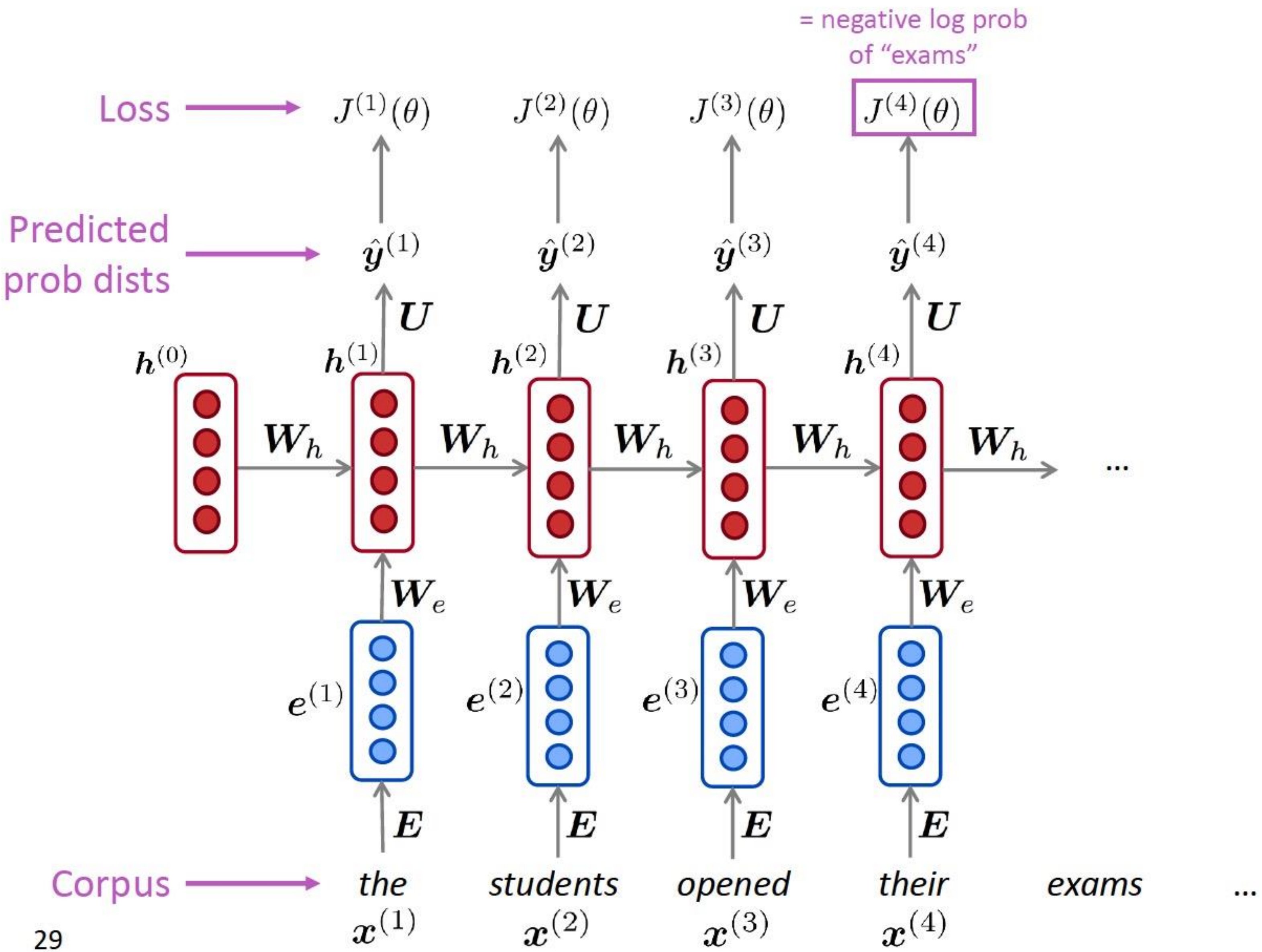
Training a RNN Language Model



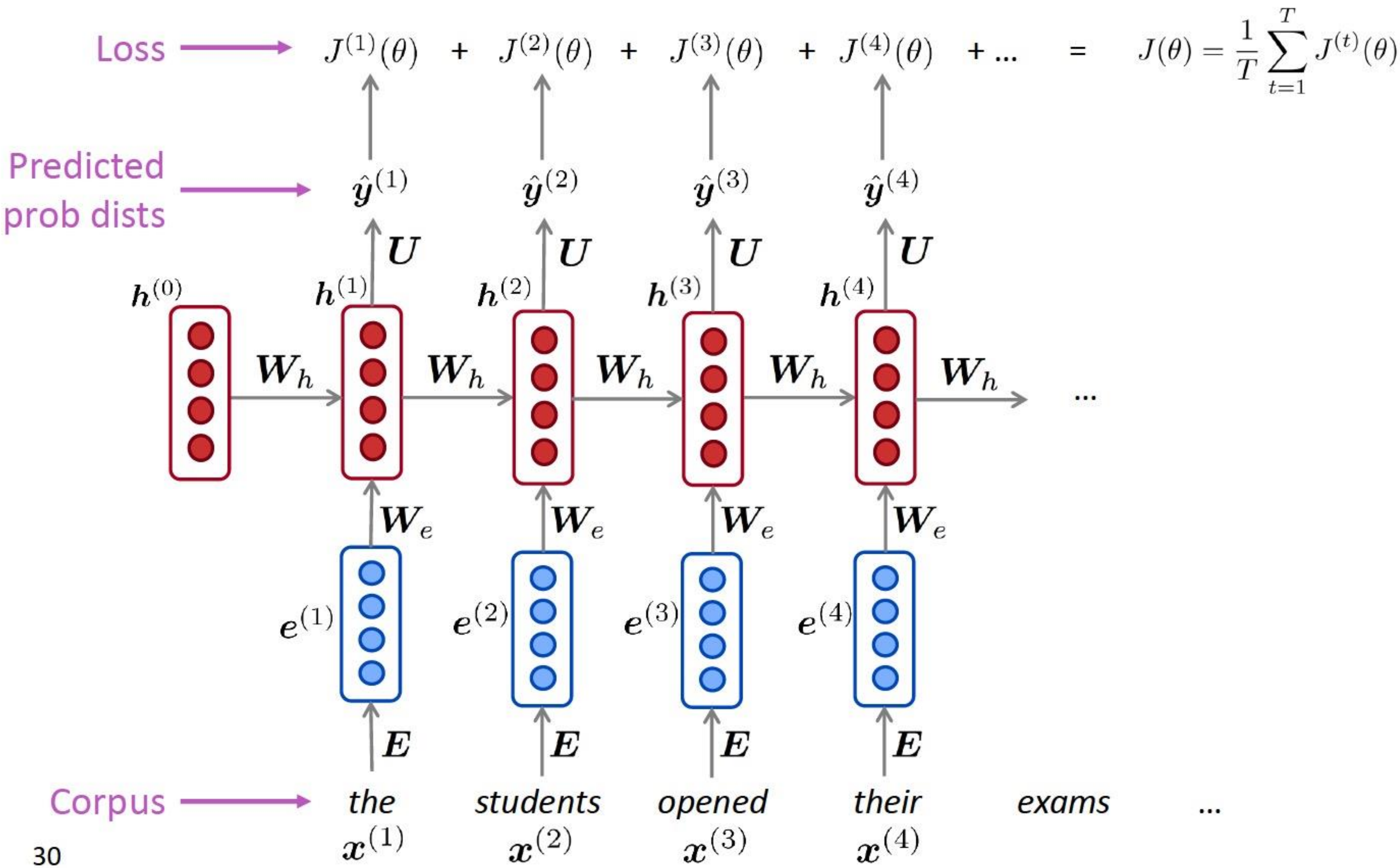
Training a RNN Language Model



Training a RNN Language Model



Training a RNN Language Model



Training a RNN Language Model

- However: Computing loss and gradients across entire corpus $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$ is too expensive!

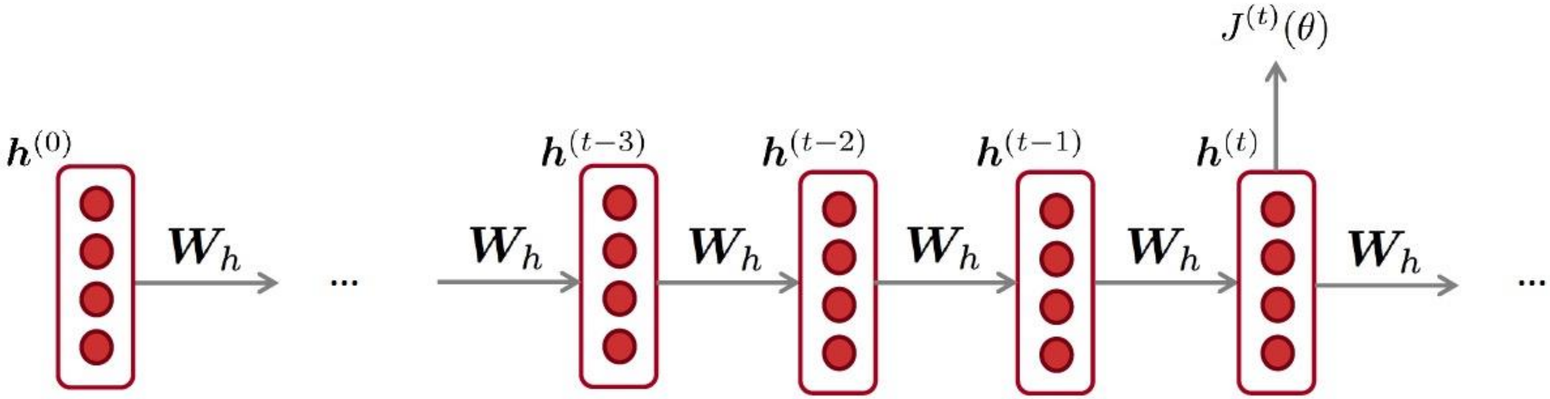
$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J^{(t)}(\theta)$$

- In practice, consider $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$ as a sentence (or a document)

- Recall: Stochastic Gradient Descent allows us to compute loss and gradients for small chunk of data, and update.

- Compute loss $J(\theta)$ for a sentence (actually a batch of sentences), compute gradients and update weights. Repeat.

Backpropagation for RNNs



Question: What's the derivative of $J^{(t)}(\theta)$ w.r.t. the repeated weight matrix $\boldsymbol{W}_h$?

Answer: $$\frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} = \sum_{i=1}^{t} \left. \frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} \right|_{(i)}$$

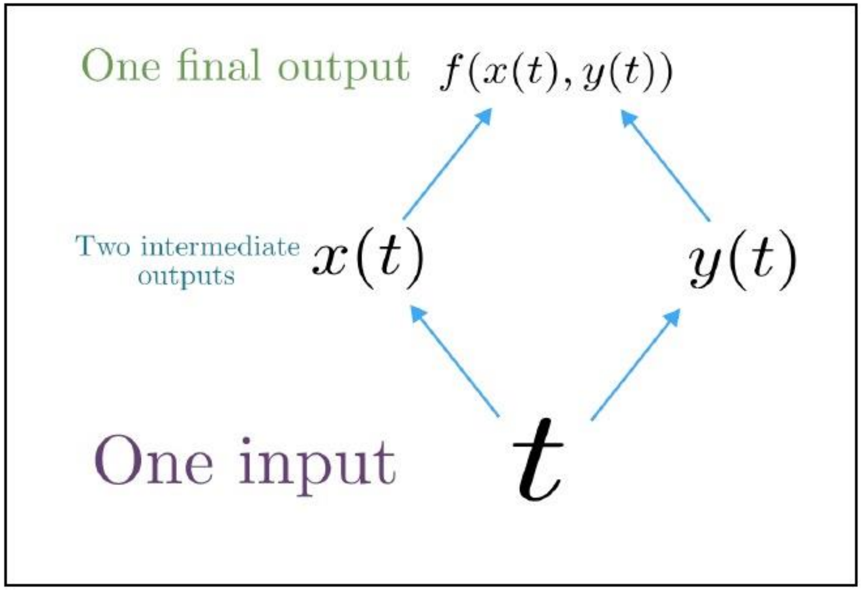
"The gradient w.r.t. a repeated weight is the sum of the gradient w.r.t. each time it appears"

Why?

Multivariable Chain Rule

- Given a multivariable function $f(x, y)$, and two single variable functions $x(t)$ and $y(t)$, here's what the multivariable chain rule says:

$$\underbrace{\frac{d}{dt} f(x(t), y(t))}_{\text{Derivative of composition function}} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$



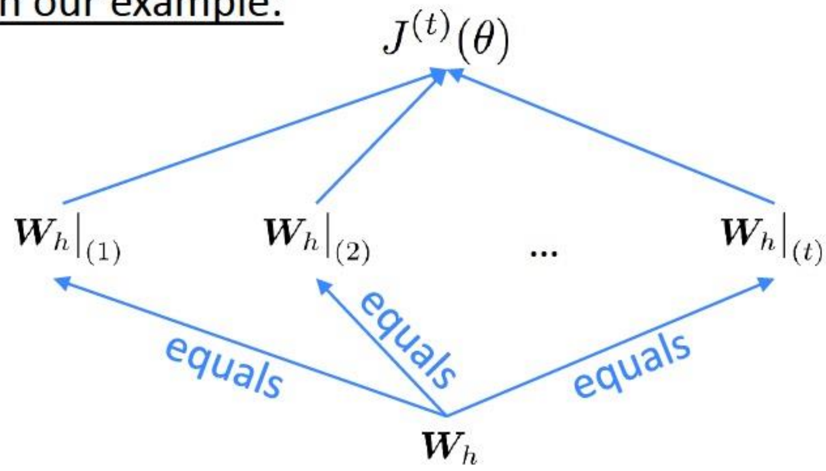
Source:
https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/differentiating-vector-valued-functions/a/multivariable-chain-rule-simple-version

Backpropagation for RNNs: Proof sketch

- Given a multivariable function $f(x, y)$, and two single variable functions $x(t)$ and $y(t)$, here's what the multivariable chain rule says:

$$\underbrace{\frac{d}{dt} f(x(t), y(t))}_{\text{Derivative of composition function}} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

In our example:



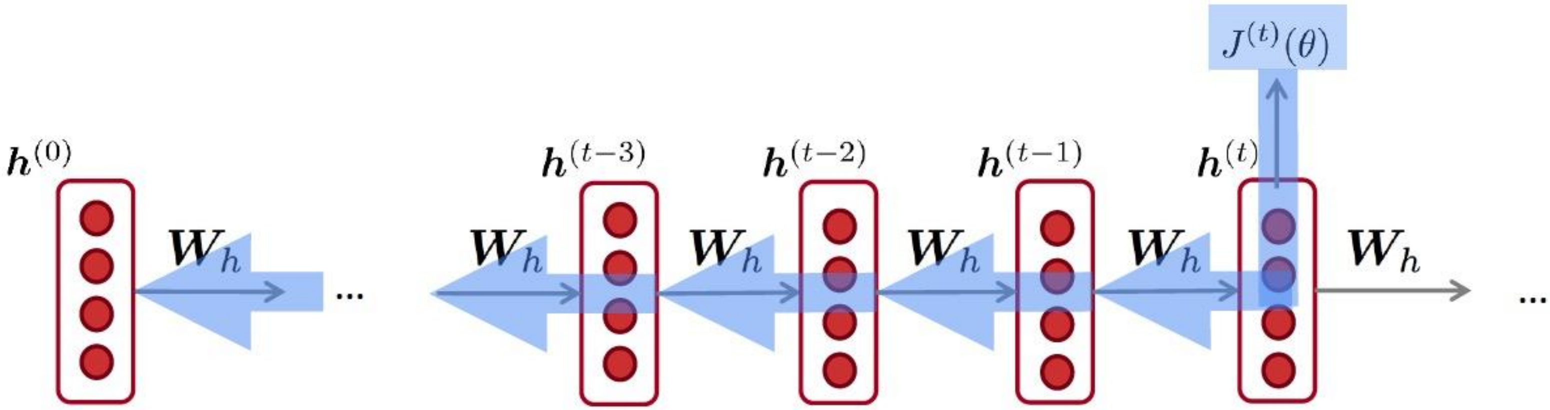
Apply the multivariable chain rule:

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{W}_h} = \sum_{i=1}^{t} \left.\frac{\partial J^{(t)}}{\partial \boldsymbol{W}_h}\right|_{(i)} \underbrace{\boxed{\frac{\partial \boldsymbol{W}_h|_{(i)}}{\partial \boldsymbol{W}_h}}}_{=1}$$

$$= \sum_{i=1}^{t} \left.\frac{\partial J^{(t)}}{\partial \boldsymbol{W}_h}\right|_{(i)}$$

Source:
https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/differentiating-vector-valued-functions/a/multivariable-chain-rule-simple-version

Backpropagation for RNNs



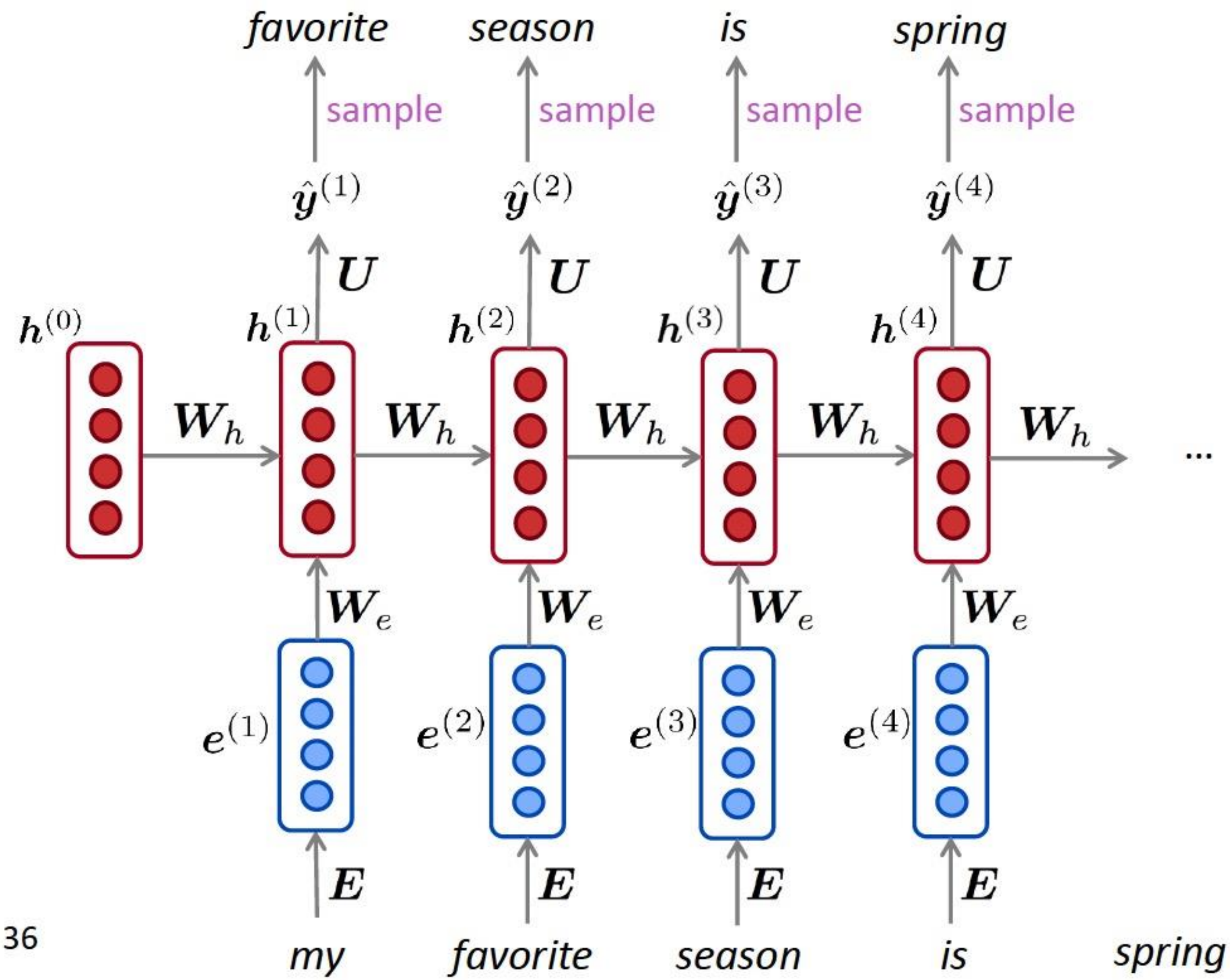
$$\frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} = \boxed{\sum_{i=1}^{t} \left. \frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} \right|_{(i)}}$$

Question: How do we calculate this?

Answer: Backpropagate over timesteps *i*=*t*,…,0, summing gradients as you go.
This algorithm is called **"backpropagation through time"**

Generating text with a RNN Language Model

Just like a n-gram Language Model, you can use a RNN Language Model to generate text by repeated sampling. Sampled output is next step's input.



End Supplemental Material: Training RNNs

Generating text with a RNN Language Model

- Let's have some fun!
- You can train a RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on Obama speeches:

The United States will step up to the cost of a new challenges of the American people that will share the fact that we created the problem. They were attacked and so that they have to say that all the task of the final days of war that I will not be able to get this done.

Source: https://medium.com/@samim/obama-rnn-machine-generated-political-speeches-c8abd18a2ea0

Generating text with a RNN Language Model

- Let's have some fun!
- You can train a RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on *Harry Potter*:

"Sorry," Harry shouted, panicking—"I'll leave those brooms in London, are they?"

"No idea," said Nearly Headless Nick, casting low close by Cedric, carrying the last bit of treacle Charms, from Harry's shoulder, and to answer him the common room perched upon it, four arms held a shining knob from when the spider hadn't felt it seemed. He reached the teams too.

Source: https://medium.com/deep-writing/harry-potter-written-by-artificial-intelligence-8a9431803da6

Generating text with a RNN Language Model

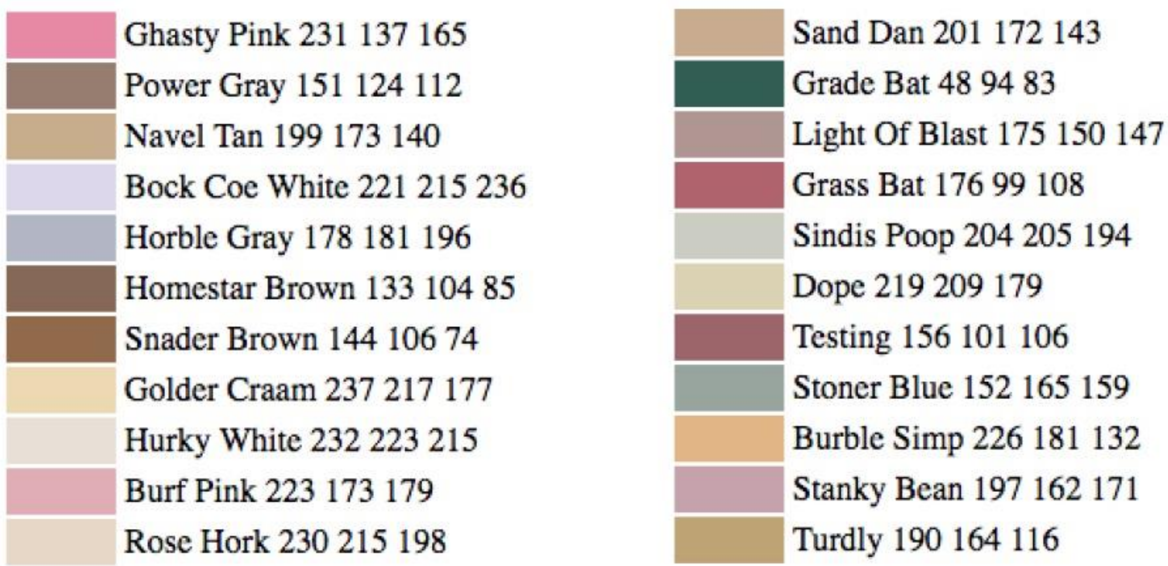
- Let's have some fun!
- You can train a RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on recipes:

```
     Title: CHOCOLATE RANCH BARBECUE
Categories: Game, Casseroles, Cookies, Cookies
     Yield: 6 Servings

     2 tb Parmesan cheese -- chopped
     1 c  Coconut milk
     3    Eggs, beaten

  Place each pasta over layers of lumps. Shape mixture into the moderate oven and simmer
  until firm. Serve hot in bodied fresh, mustard, orange and cheese.

  Combine the cheese and salt together the dough in a large skillet; add the ingredients
  and stir in the chocolate and pepper.
```

Source: https://gist.github.com/nylki/1efbaa36635956d35bcc

Generating text with a RNN Language Model

- Let's have some fun!
- You can train a RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on paint color names:



This is an example of a character-level RNN-LM (predicts what character comes next)

Source: http://aiweirdness.com/post/160776374467/new-paint-colors-invented-by-neural-network

Evaluating Language Models

- The standard evaluation metric for Language Models is perplexity.

$$\text{perplexity} = \underbrace{\prod_{t=1}^{T} \left(\frac{1}{P_{\text{LM}}(\boldsymbol{x}^{(t+1)} \mid \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})} \right)}_{\text{Inverse probability of corpus, according to Language Model}}{}^{1/T}$$

Normalized by number of words

- This is equal to the exponential of the cross-entropy loss $J(\theta)$:

$$= \prod_{t=1}^{T} \left(\frac{1}{\hat{\boldsymbol{y}}^{(t)}_{\boldsymbol{x}_{t+1}}} \right)^{1/T} = \exp\left(\frac{1}{T} \sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}^{(t)}_{\boldsymbol{x}_{t+1}} \right) = \exp(J(\theta))$$

$\boxed{\textbf{Lower}\text{ perplexity is better!}}$

RNNs have greatly improved perplexity

n-gram model →

Increasingly complex RNNs

Model	Perplexity
Interpolated Kneser-Ney 5-gram (Chelba et al., 2013)	67.6
RNN-1024 + MaxEnt 9-gram (Chelba et al., 2013)	51.3
RNN-2048 + BlackOut sampling (Ji et al., 2015)	68.3
Sparse Non-negative Matrix factorization (Shazeer et al., 2015)	52.9
LSTM-2048 (Jozefowicz et al., 2016)	43.7
2-layer LSTM-8192 (Jozefowicz et al., 2016)	30
Ours small (LSTM-2048)	43.9
Ours large (2-layer LSTM-2048)	39.8

Perplexity improves (lower is better)

Source: https://research.fb.com/building-an-efficient-neural-language-model-over-a-billion-words/

Why should we care about Language Modeling?

- Language Modeling is a benchmark task that helps us measure our progress on understanding language

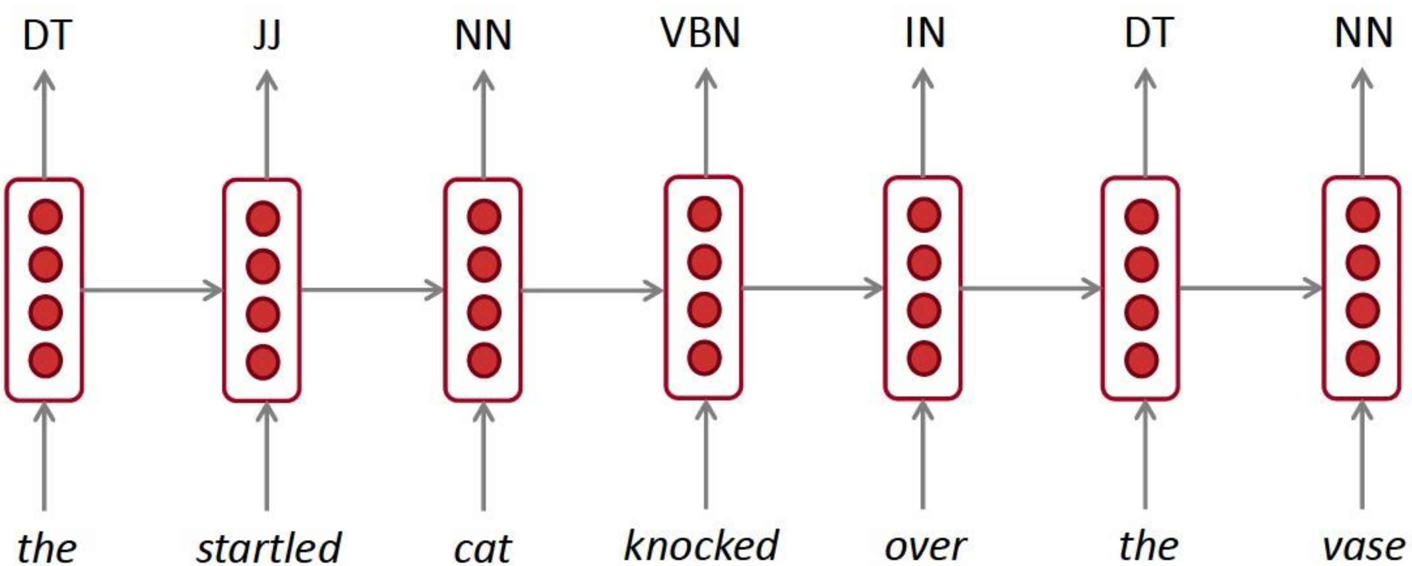
- Language Modeling is a subcomponent of many NLP tasks, especially those involving generating text or estimating the probability of text:
    - Predictive typing
    - Speech recognition
    - Handwriting recognition
    - Spelling/grammar correction
    - Authorship identification
    - Machine translation
    - Summarization
    - Dialogue
    - etc.

Recap

- **Language Model**: A system that predicts the next word
- **Recurrent Neural Network**: A family of neural networks that:
  - Take sequential input of any length
  - Apply the same weights on each step
  - Can optionally produce output on each step
- Recurrent Neural Network ≠ Language Model
- We've shown that RNNs are a great way to build a LM.
- But RNNs are useful for much more!

RNNs can be used for tagging

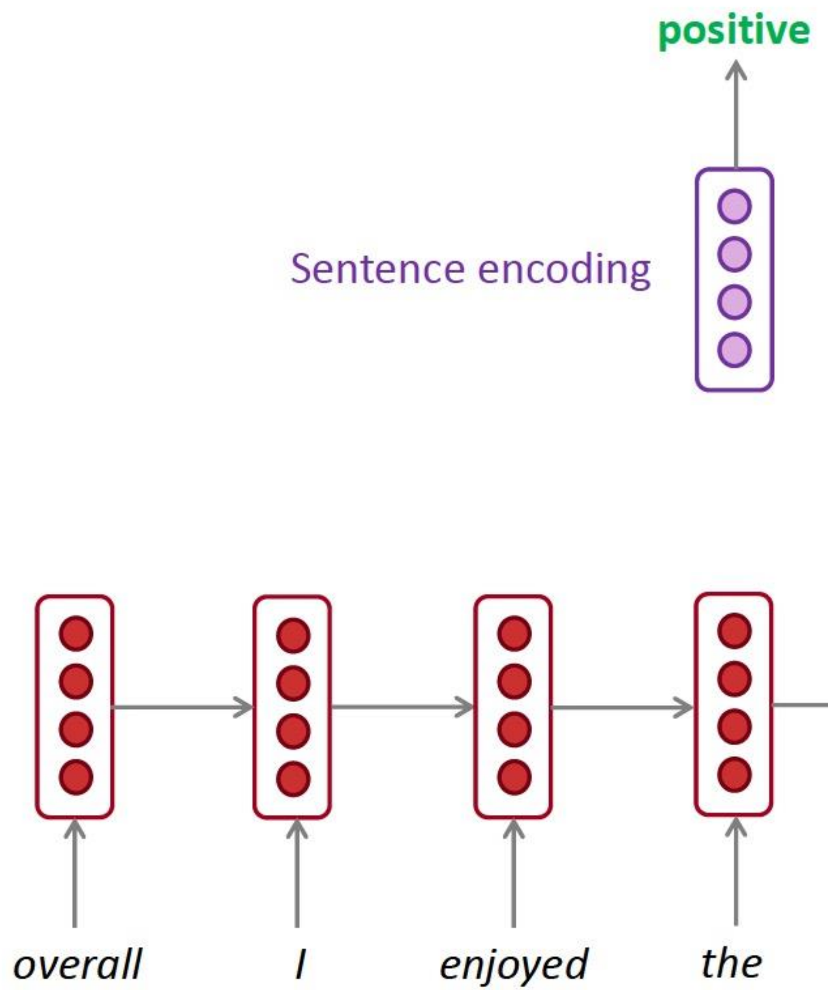
e.g. part-of-speech tagging, named entity recognition



RNNs can be used for sentence classification

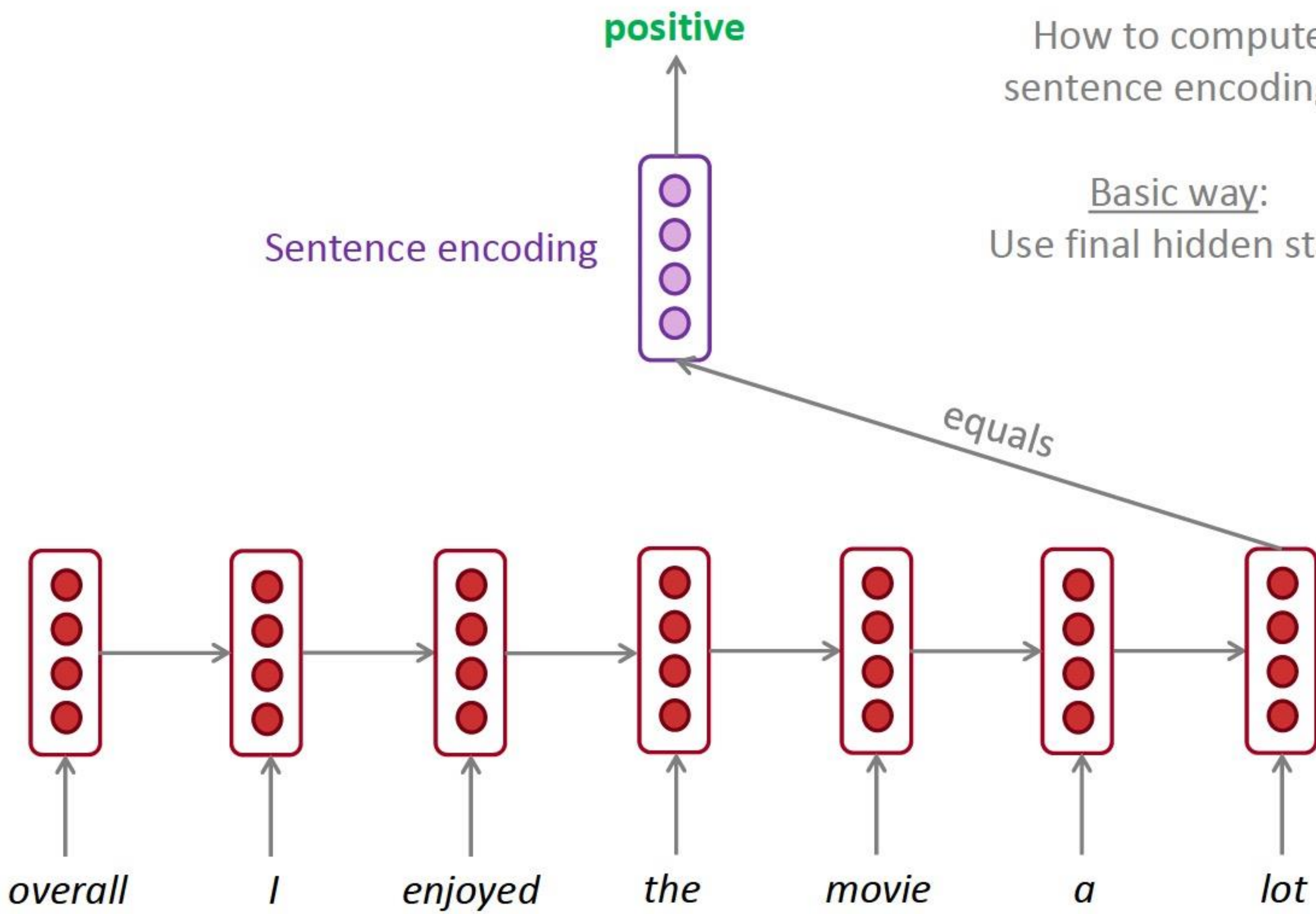
e.g. sentiment classification

How to compute sentence encoding?



RNNs can be used for sentence classification

e.g. sentiment classification

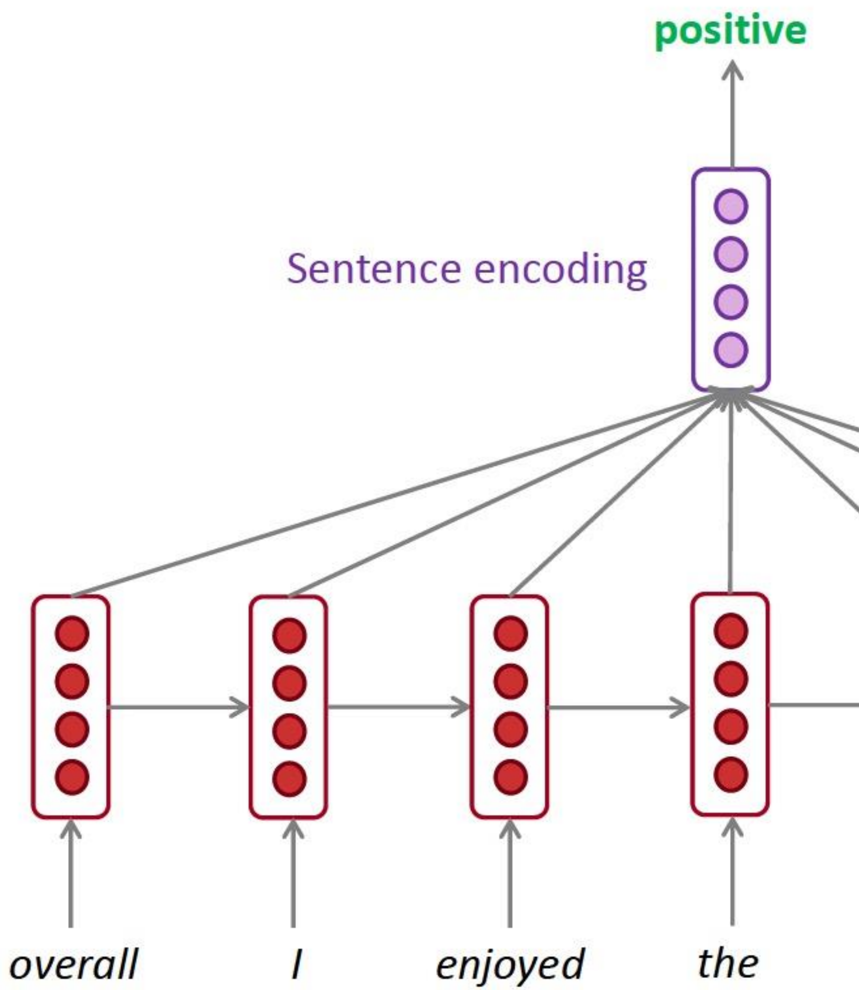


How to compute sentence encoding?

Basic way:
Use final hidden state

RNNs can be used for sentence classification

e.g. sentiment classification



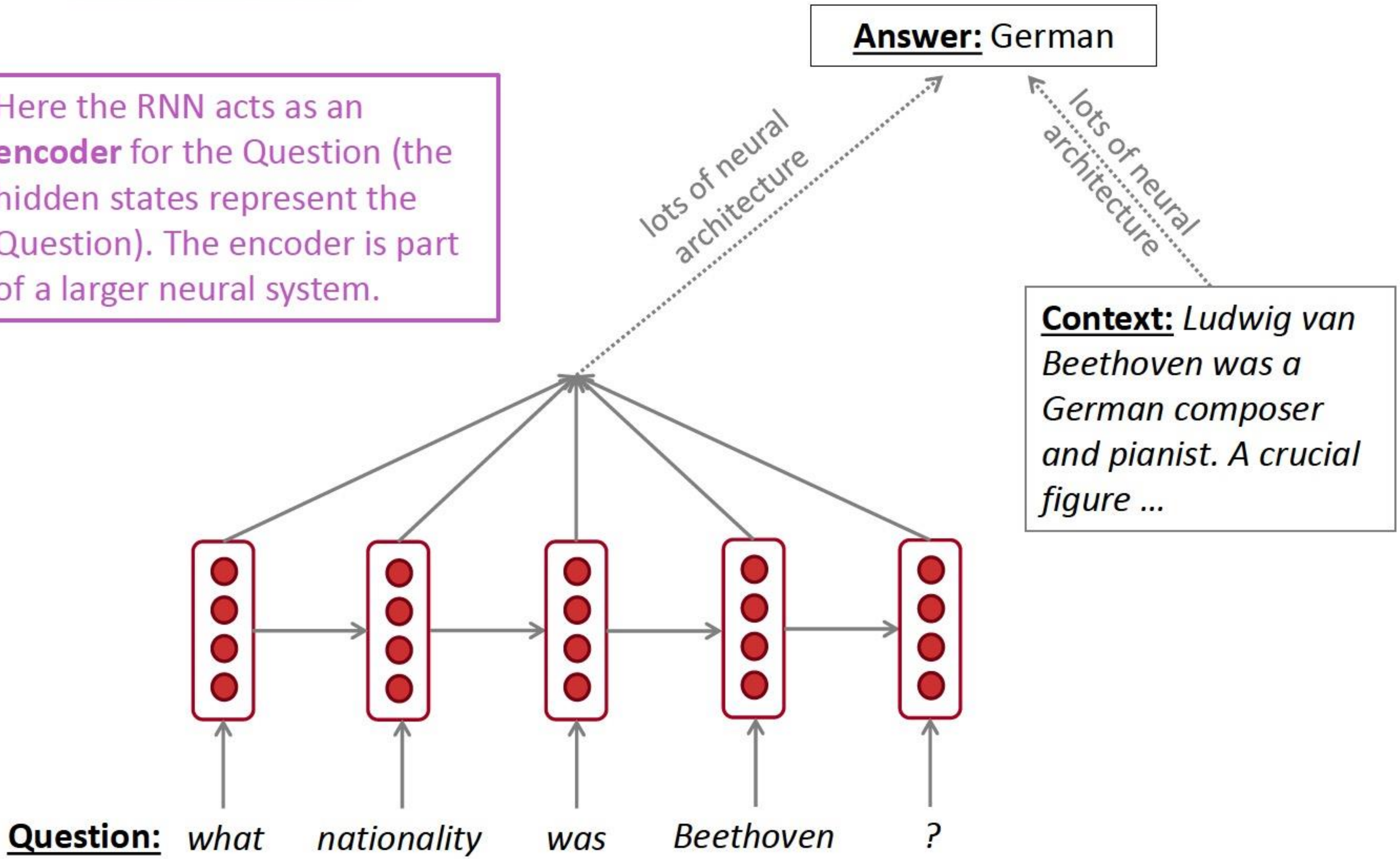
How to compute sentence encoding?

Usually better:
Take element-wise max or mean of all hidden states

RNNs can be used as an encoder module

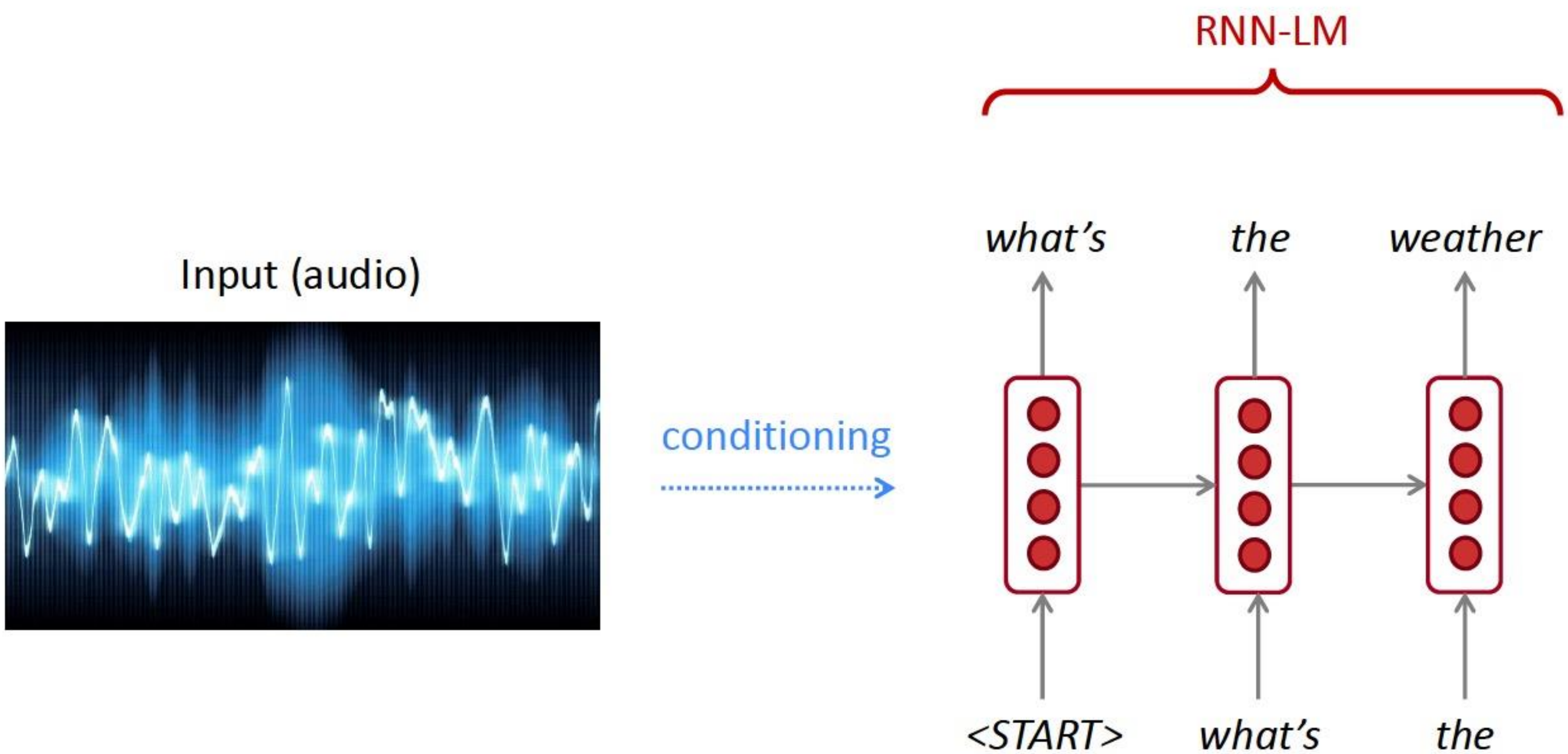
e.g. question answering, machine translation, *many other tasks!*

Here the RNN acts as an **encoder** for the Question (the hidden states represent the Question). The encoder is part of a larger neural system.



RNN-LMs can be used to generate text

e.g. speech recognition, machine translation, summarization



This is an example of a *conditional language model*.
We'll see Machine Translation in much more detail later.

A note on terminology

RNN described in this lecture = "vanilla RNN"

<u>Next lecture:</u> You will learn about other RNN flavors

like GRU  and LSTM  and multi-layer RNNs 

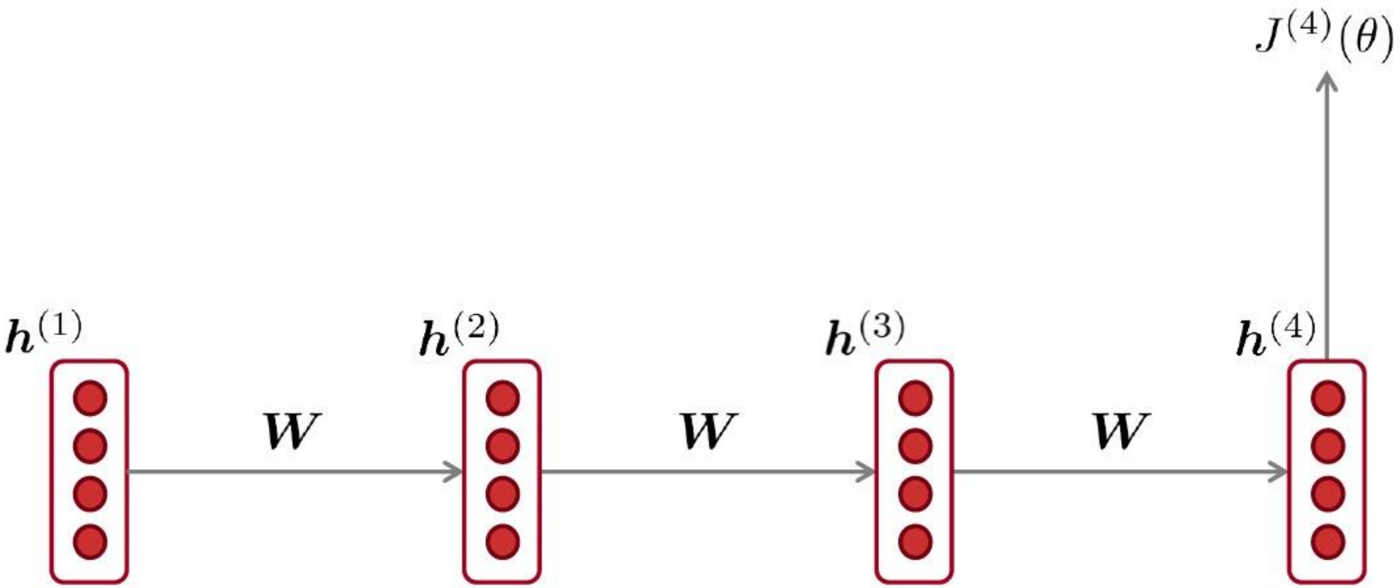
<u>By the end of the course:</u> You will understand phrases like
"stacked bidirectional LSTM with residual connections and self-attention"

- Problems with RNNs!
  - Vanishing gradients

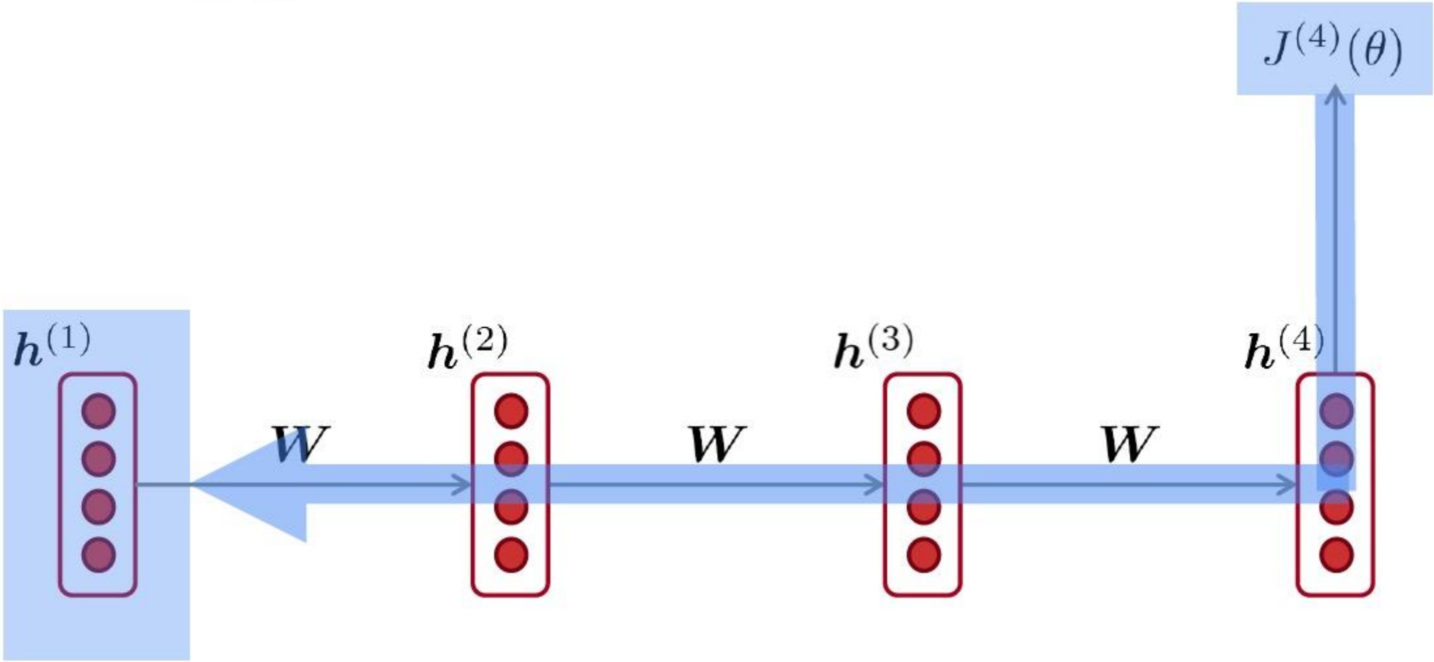
$\Big\downarrow$ motivates

- Fancy RNN variants!
  - LSTM
  - GRU
  - multi-layer
  - bidirectional

Vanishing gradient intuition

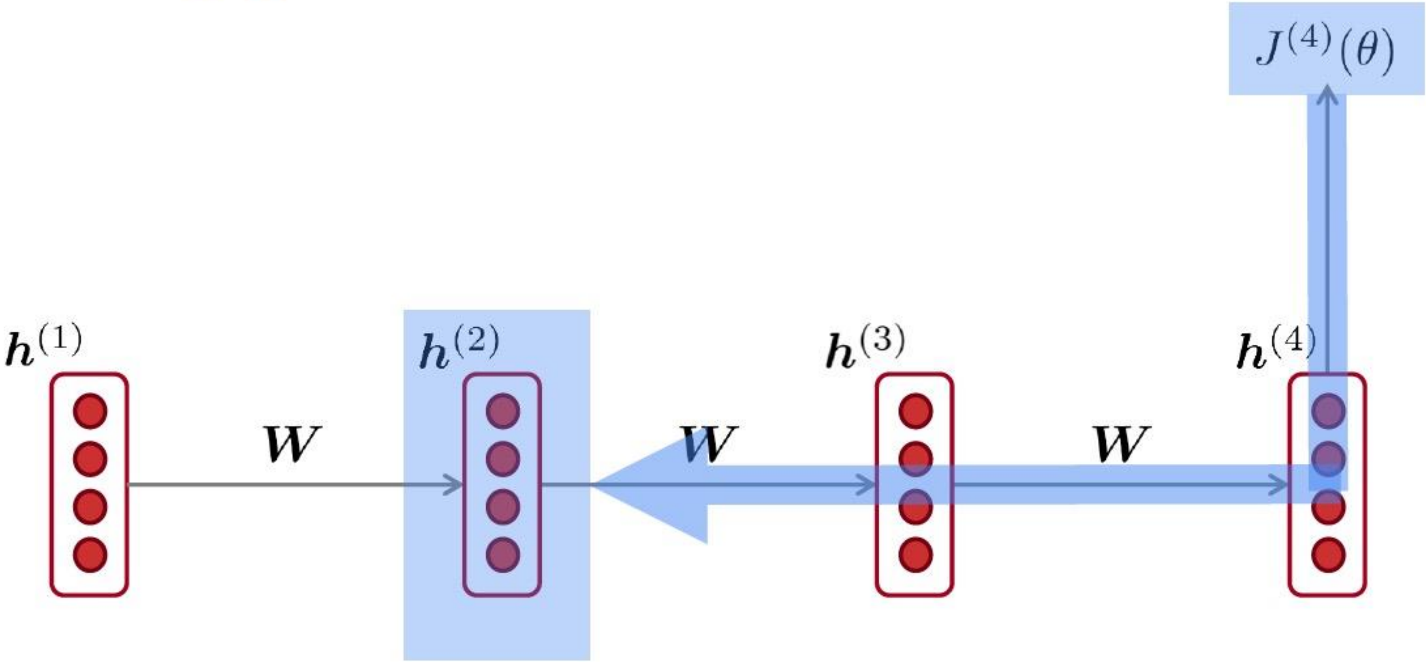


Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \ ?$$

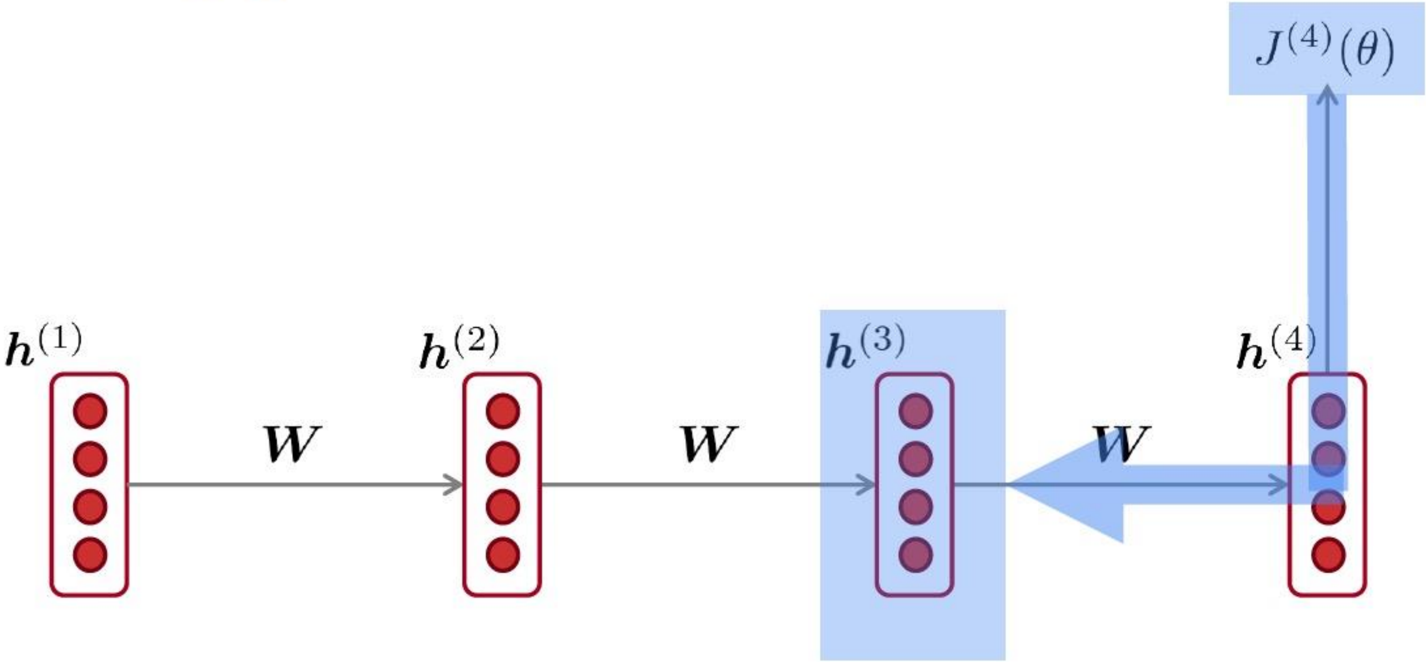
Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(2)}}$$

chain rule!

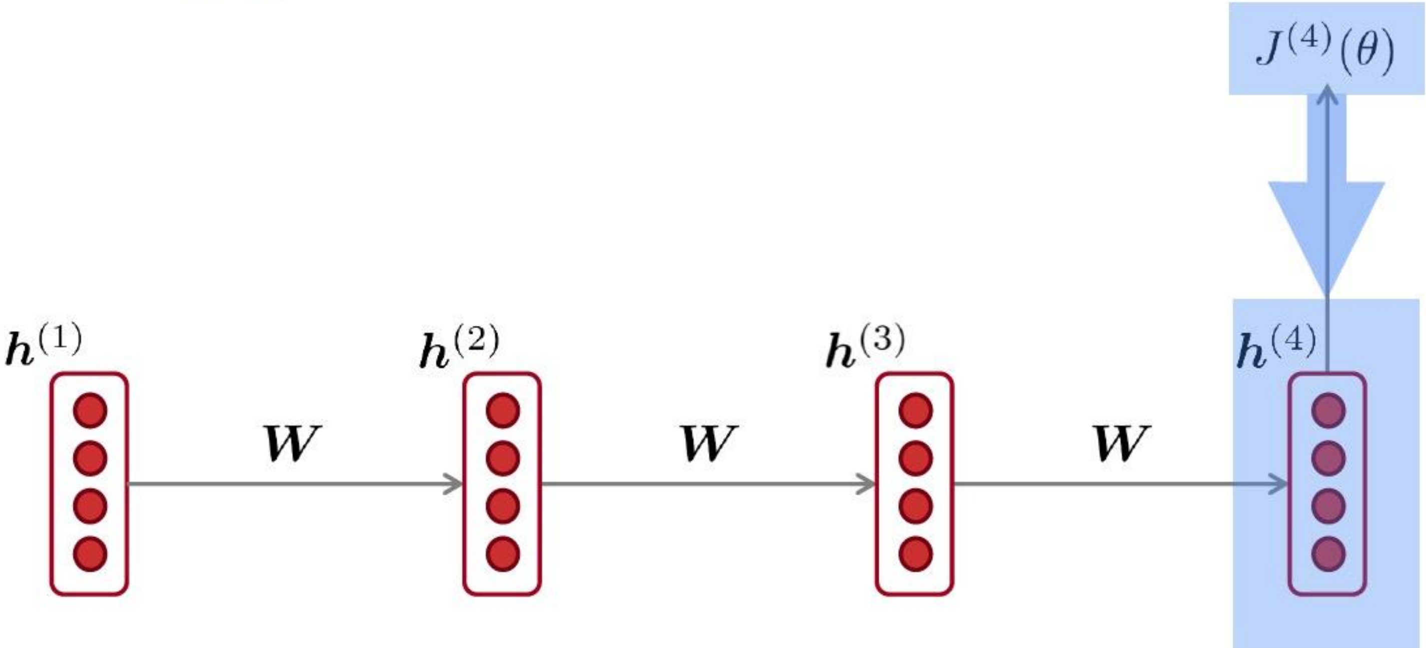
Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}} \times \quad \frac{\partial \boldsymbol{h}^{(3)}}{\partial \boldsymbol{h}^{(2)}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(3)}}$$

chain rule!

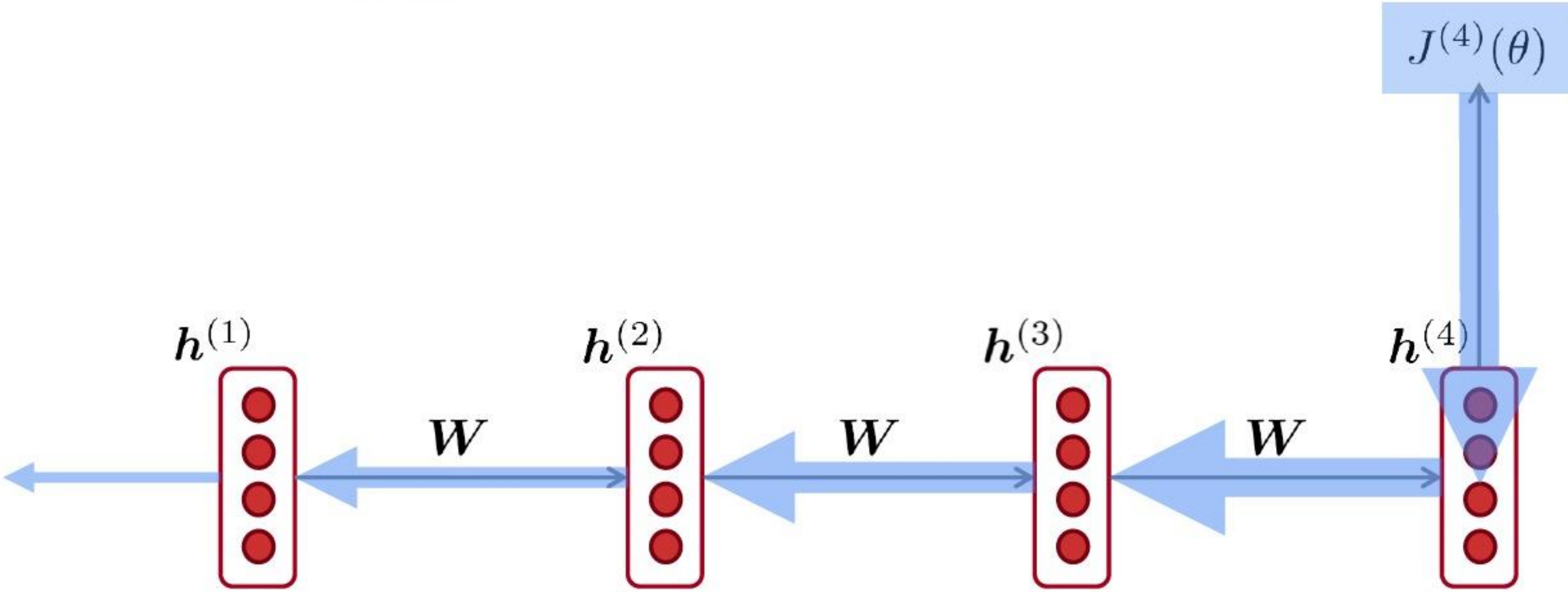
Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}} \times \frac{\partial \boldsymbol{h}^{(3)}}{\partial \boldsymbol{h}^{(2)}} \times \frac{\partial \boldsymbol{h}^{(4)}}{\partial \boldsymbol{h}^{(3)}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(4)}}$$

chain rule!

Vanishing gradient intuition

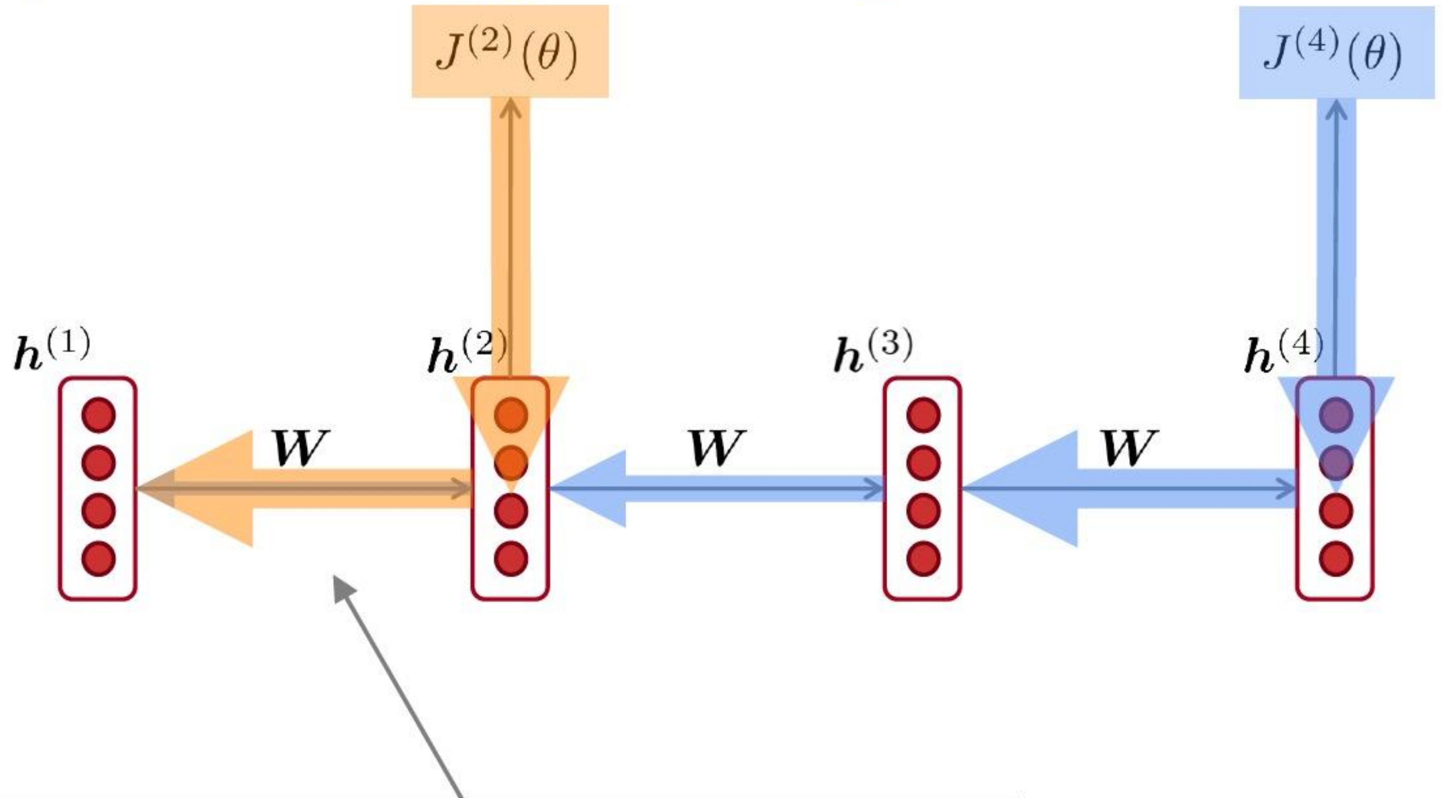


$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \boxed{\frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}}} \times \boxed{\frac{\partial \boldsymbol{h}^{(3)}}{\partial \boldsymbol{h}^{(2)}}} \times \boxed{\frac{\partial \boldsymbol{h}^{(4)}}{\partial \boldsymbol{h}^{(3)}}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(4)}}$$

What happens if these are small?

<u>Vanishing gradient problem:</u>
When these are small, the gradient signal gets smaller and smaller as it backpropagates further

Why is vanishing gradient a problem?



Gradient signal from faraway is lost because it's much smaller than gradient signal from close-by.

So model weights are only updated only with respect to near effects, not long-term effects.

Why is vanishing gradient a problem?

- Another explanation: Gradient can be viewed as a measure of *the effect of the past on the future*

- If the gradient becomes vanishingly small over longer distances (step *t* to step *t+n*), then we can't tell whether:
  1. There's no dependency between step *t* and *t+n* in the data
  2. We have wrong parameters to capture the true dependency between *t* and *t+n*

Effect of vanishing gradient on RNN-LM

- **LM task:** *When she tried to print her tickets, she found that the printer was out of toner. She went to the stationery store to buy more toner. It was very overpriced. After installing the toner into the printer, she finally printed her ________*

- To learn from this training example, the RNN-LM needs to model the dependency between *"tickets"* on the 7th step and the target word *"tickets"* at the end.

- But if gradient is small, the model can't learn this dependency
  - So the model is unable to predict similar long-distance dependencies at test time

Effect of vanishing gradient on RNN-LM

- **LM task:** *The writer of the books* ___ → *is* / *are*

- **Correct answer**: *The writer of the books is planning a sequel*

- **Syntactic recency:** *The writer of the books is* (correct)

- **Sequential recency:** *The writer of the books are* (incorrect)

- Due to vanishing gradient, RNN-LMs are better at learning from sequential recency than syntactic recency, so they make this type of error more often than we'd like [Linzen et al 2016]

"Assessing the Ability of LSTMs to Learn Syntax-Sensitive Dependencies", Linzen et al, 2016. https://arxiv.org/pdf/1611.01368.pdf

How to fix vanishing gradient problem?

- The main problem is that *it's too difficult for the RNN to learn to preserve information over many timesteps*.

- In a vanilla RNN, the hidden state is constantly being rewritten

$$\boldsymbol{h}^{(t)} = \sigma\left(\boldsymbol{W}_h \boldsymbol{h}^{(t-1)} + \boldsymbol{W}_x \boldsymbol{x}^{(t)} + \boldsymbol{b}\right)$$

- How about a RNN with separate memory?

Long Short-Term Memory (LSTM)

- A type of RNN proposed by Hochreiter and Schmidhuber in 1997 as a solution to the vanishing gradients problem.

- On step *t*, there is a hidden state $\boldsymbol{h}^{(t)}$ *and* a cell state $\boldsymbol{c}^{(t)}$
  - Both are vectors length *n*
  - The cell stores long-term information
  - The LSTM can erase, write and read information from the cell

- The selection of which information is erased/written/read is controlled by three corresponding gates
  - The gates are also vectors length *n*
  - On each timestep, each element of the gates can be open (1), closed (0), or somewhere in-between.
  - The gates are dynamic: their value is computed based on the current context

"Long short-term memory", Hochreiter and Schmidhuber, 1997. https://www.bioinf.jku.at/publications/older/2604.pdf

Long Short-Term Memory (LSTM)

We have a sequence of inputs $\boldsymbol{x}^{(t)}$, and we will compute a sequence of hidden states $\boldsymbol{h}^{(t)}$ and cell states $\boldsymbol{c}^{(t)}$. On timestep *t*:

Sigmoid function: all gate values are between 0 and 1

Forget gate: controls what is kept vs forgotten, from previous cell state

Input gate: controls what parts of the new cell content are written to cell

Output gate: controls what parts of cell are output to hidden state

$$\boldsymbol{f}^{(t)} = \sigma\left(\boldsymbol{W}_f \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_f \boldsymbol{x}^{(t)} + \boldsymbol{b}_f\right)$$

$$\boldsymbol{i}^{(t)} = \sigma\left(\boldsymbol{W}_i \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_i \boldsymbol{x}^{(t)} + \boldsymbol{b}_i\right)$$

$$\boldsymbol{o}^{(t)} = \sigma\left(\boldsymbol{W}_o \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_o \boldsymbol{x}^{(t)} + \boldsymbol{b}_o\right)$$

New cell content: this is the new content to be written to the cell

Cell state: erase ("forget") some content from last cell state, and write ("input") some new cell content

Hidden state: read ("output") some content from the cell

$$\tilde{\boldsymbol{c}}^{(t)} = \tanh\left(\boldsymbol{W}_c \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_c \boldsymbol{x}^{(t)} + \boldsymbol{b}_c\right)$$

$$\boldsymbol{c}^{(t)} = \boldsymbol{f}^{(t)} \circ \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \circ \tilde{\boldsymbol{c}}^{(t)}$$

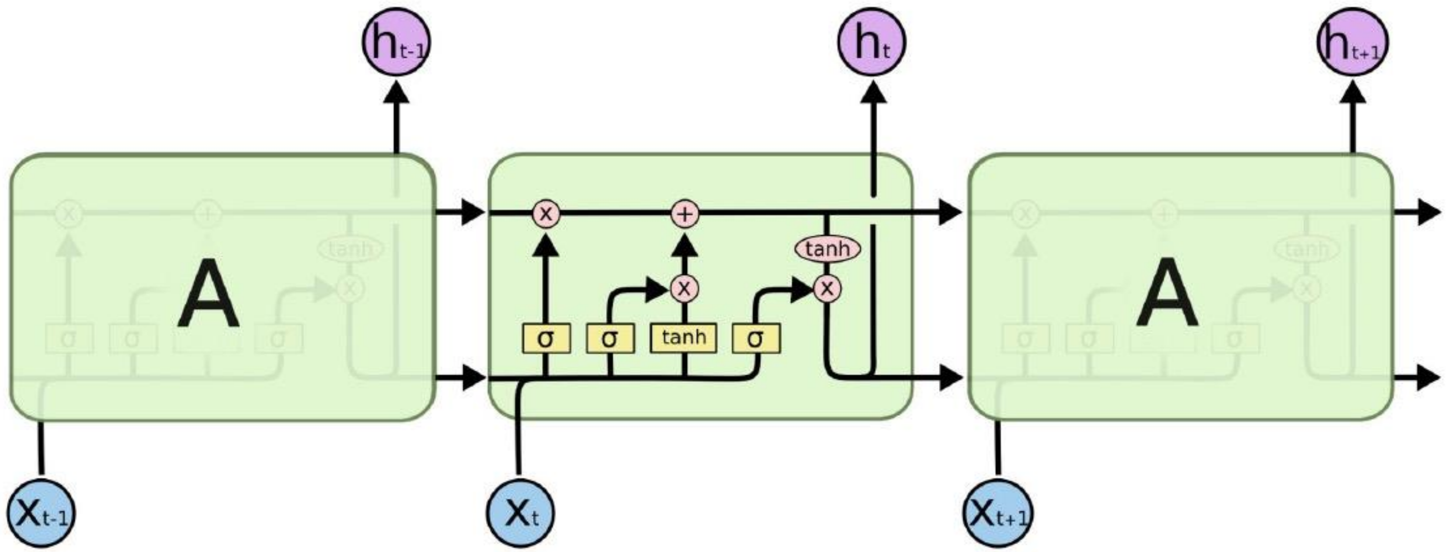
$$\boldsymbol{h}^{(t)} = \boldsymbol{o}^{(t)} \circ \tanh \boldsymbol{c}^{(t)}$$

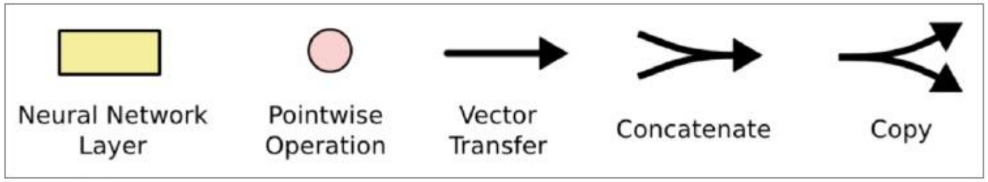
All these are vectors of same length *n*

Gates are applied using element-wise product

Long Short-Term Memory (LSTM)

You can think of the LSTM equations visually like this:





Source: http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Long Short-Term Memory (LSTM)

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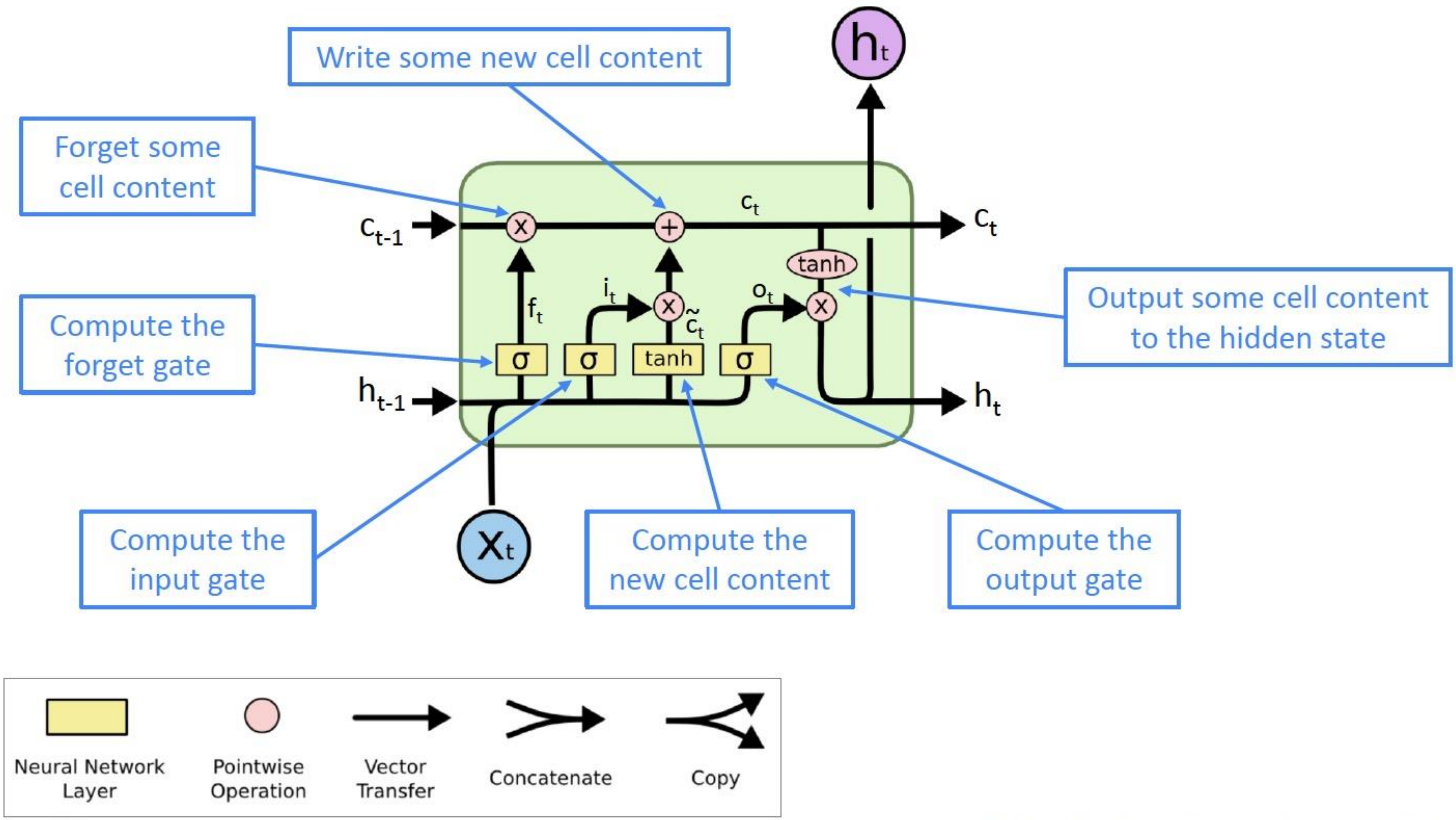


Source: http://colah.github.io/posts/2015-08-Understanding-LSTMs/

How does LSTM solve vanishing gradients?

- The LSTM architecture makes it easier for the RNN to preserve information over many timesteps
  - e.g. if the forget gate is set to remember everything on every timestep, then the info in the cell is preserved indefinitely
  - By contrast, it's harder for vanilla RNN to learn a recurrent weight matrix W_h that preserves info in hidden state

- LSTM doesn't *guarantee* that there is no vanishing/exploding gradient, but it does provide an easier way for the model to learn long-distance dependencies

LSTMs: real-world success

- In 2013-2015, LSTMs started achieving state-of-the-art results
  - Successful tasks include: handwriting recognition, speech recognition, machine translation, parsing, image captioning
  - LSTM became the dominant approach

- Now (2019), other approaches (e.g. Transformers) have become more dominant for certain tasks.
  - For example in **WMT** (a MT conference + competition):
  - In WMT 2016, the summary report contains ”RNN” 44 times
  - In WMT 2018, the report contains “RNN” 9 times and “Transformer” 63 times

Source: "Findings of the 2016 Conference on Machine Translation (WMT16)", Bojar et al. 2016, http://www.statmt.org/wmt16/pdf/W16-2301.pdf
Source: "Findings of the 2018 Conference on Machine Translation (WMT18)", Bojar et al. 2018, http://www.statmt.org/wmt18/pdf/WMT028.pdf

Gated Recurrent Units (GRU)

- Proposed by Cho et al. in 2014 as a simpler alternative to the LSTM.
- On each timestep *t* we have input $\boldsymbol{x}^{(t)}$ and hidden state $\boldsymbol{h}^{(t)}$ (no cell state).

Update gate: controls what parts of hidden state are updated vs preserved

Reset gate: controls what parts of previous hidden state are used to compute new content

$$\boldsymbol{u}^{(t)} = \sigma\left(\boldsymbol{W}_u \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_u \boldsymbol{x}^{(t)} + \boldsymbol{b}_u\right)$$

$$\boldsymbol{r}^{(t)} = \sigma\left(\boldsymbol{W}_r \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_r \boldsymbol{x}^{(t)} + \boldsymbol{b}_r\right)$$

New hidden state content: reset gate selects useful parts of prev hidden state. Use this and current input to compute new hidden content.

Hidden state: update gate simultaneously controls what is kept from previous hidden state, and what is updated to new hidden state content

$$\tilde{\boldsymbol{h}}^{(t)} = \tanh\left(\boldsymbol{W}_h(\boldsymbol{r}^{(t)} \circ \boldsymbol{h}^{(t-1)}) + \boldsymbol{U}_h \boldsymbol{x}^{(t)} + \boldsymbol{b}_h\right)$$

$$\boldsymbol{h}^{(t)} = (1 - \boldsymbol{u}^{(t)}) \circ \boldsymbol{h}^{(t-1)} + \boldsymbol{u}^{(t)} \circ \tilde{\boldsymbol{h}}^{(t)}$$

How does this solve vanishing gradient?
Like LSTM, GRU makes it easier to retain info long-term (e.g. by setting update gate to 0)

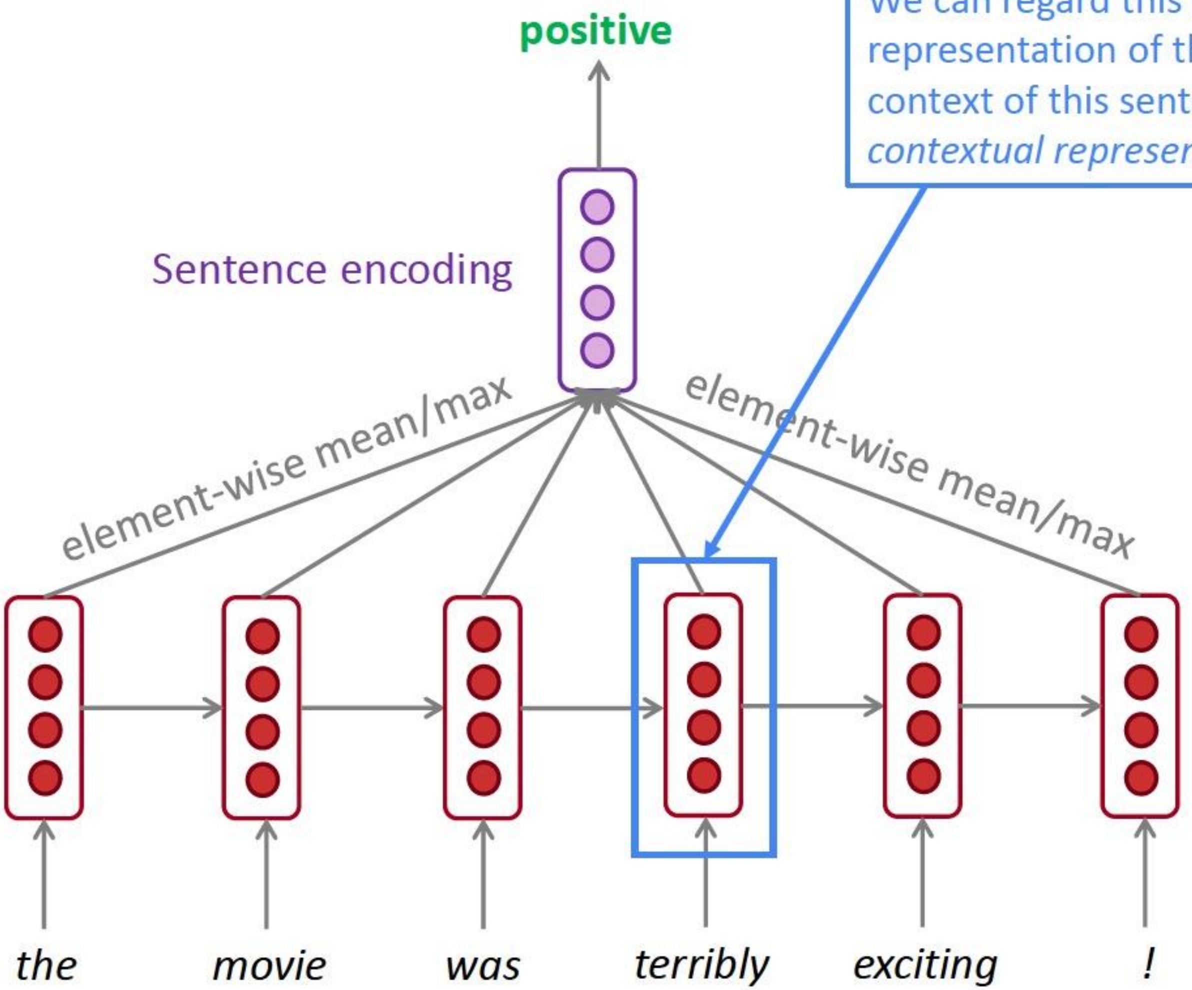
"Learning Phrase Representations using RNN Encoder–Decoder for Statistical Machine Translation", Cho et al. 2014, https://arxiv.org/pdf/1406.1078v3.pdf

LSTM vs GRU

- Researchers have proposed many gated RNN variants, but LSTM and GRU are the most widely-used
- The biggest difference is that GRU is quicker to compute and has fewer parameters
- There is no conclusive evidence that one consistently performs better than the other
- LSTM is a good default choice (especially if your data has particularly long dependencies, or you have lots of training data)
- Rule of thumb: start with LSTM, but switch to GRU if you want something more efficient

Bidirectional RNNs: motivation

Task: Sentiment Classification



We can regard this hidden state as a representation of the word *"terribly"* in the context of this sentence. We call this a *contextual representation.*

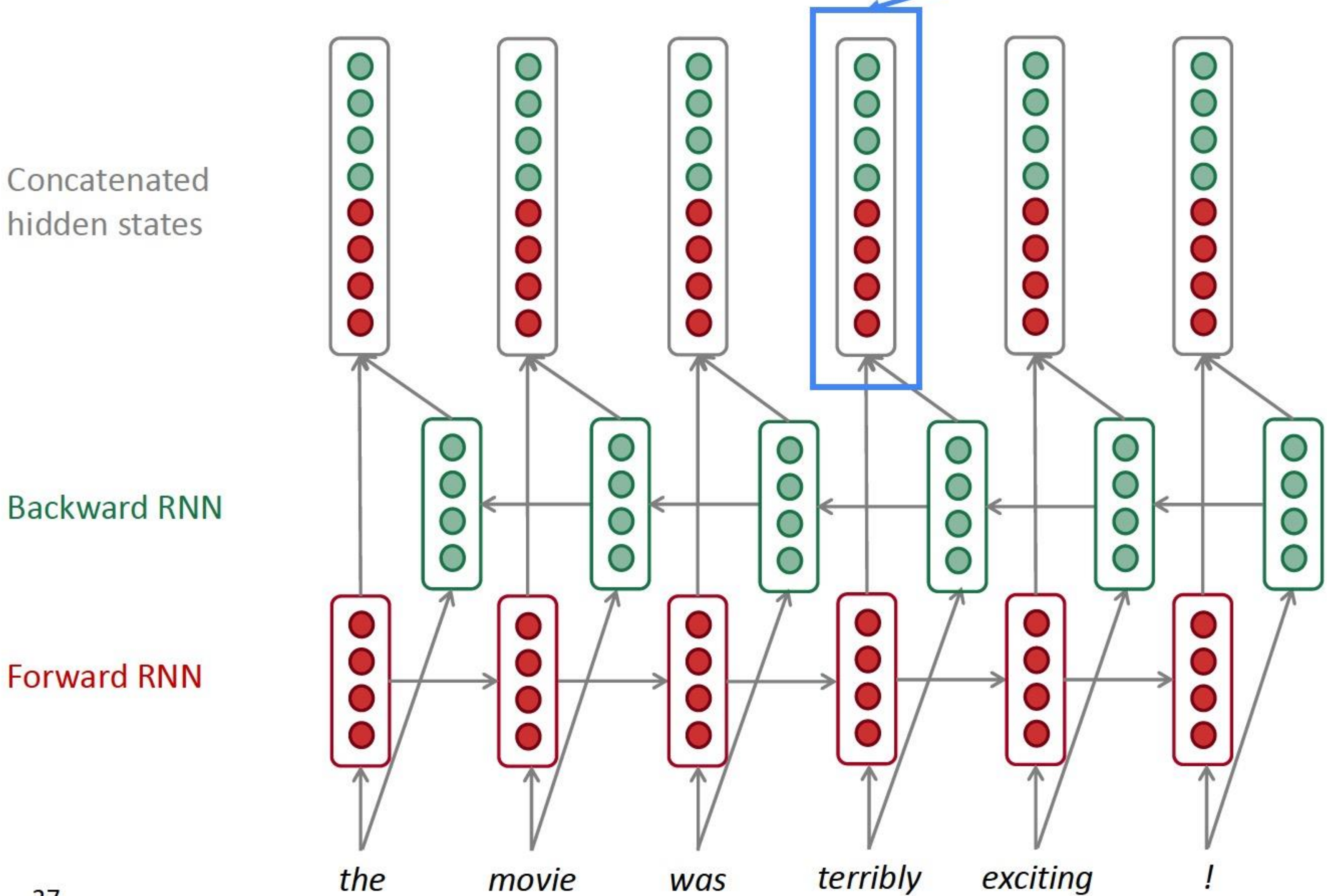
These contextual representations only contain information about the *left* context (e.g. *"the movie was"*).

What about *right* context?

In this example, *"exciting"* is in the right context and this modifies the meaning of *"terribly"* (from negative to positive)

Bidirectional RNNs

This contextual representation of "terribly" has both left and right context!



Bidirectional RNNs

On timestep *t*:

This is a general notation to mean "compute one forward step of the RNN" – it could be a vanilla, LSTM or GRU computation.

Forward RNN $\overrightarrow{\boldsymbol{h}}^{(t)} = \boxed{\mathrm{RNN}_{\mathrm{FW}}}(\overrightarrow{\boldsymbol{h}}^{(t-1)}, \boldsymbol{x}^{(t)})$

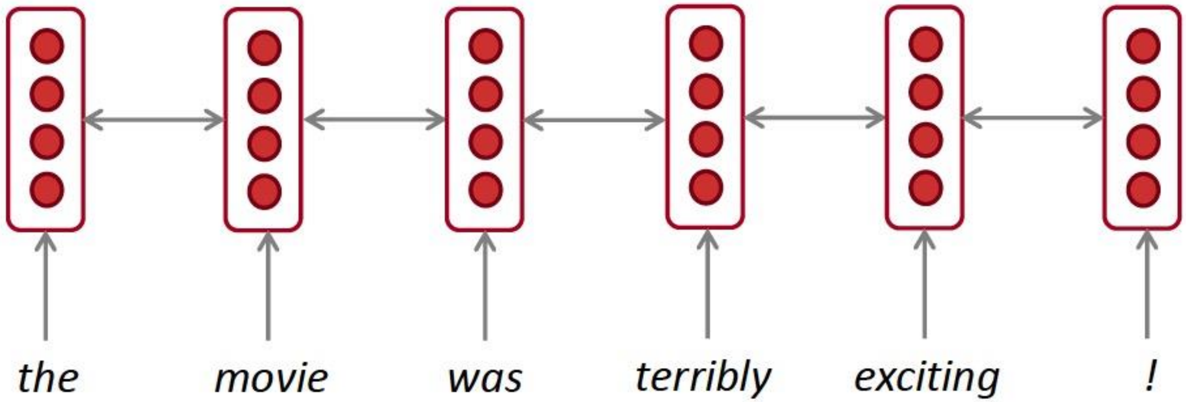
Backward RNN $\overleftarrow{\boldsymbol{h}}^{(t)} = \mathrm{RNN}_{\mathrm{BW}}(\overleftarrow{\boldsymbol{h}}^{(t+1)}, \boldsymbol{x}^{(t)})$

Generally, these two RNNs have separate weights

Concatenated hidden states $\boxed{\boldsymbol{h}^{(t)}} = [\overrightarrow{\boldsymbol{h}}^{(t)}; \overleftarrow{\boldsymbol{h}}^{(t)}]$

We regard this as "the hidden state" of a bidirectional RNN. This is what we pass on to the next parts of the network.

Bidirectional RNNs: simplified diagram



The two-way arrows indicate bidirectionality and the depicted hidden states are assumed to be the concatenated forwards+backwards states.

Bidirectional RNNs

- Note: bidirectional RNNs are only applicable if you have access to the entire input sequence.
  - They are **not** applicable to Language Modeling, because in LM you *only* have left context available.

- If you do have entire input sequence (e.g. any kind of encoding), bidirectionality is powerful (you should use it by default).

- For example, BERT (**Bidirectional** Encoder Representations from Transformers) is a powerful pretrained contextual representation system built on bidirectionality.
  - You will learn more about BERT later in the course!

Multi-layer RNNs

- RNNs are already “deep” on one dimension (they unroll over many timesteps)

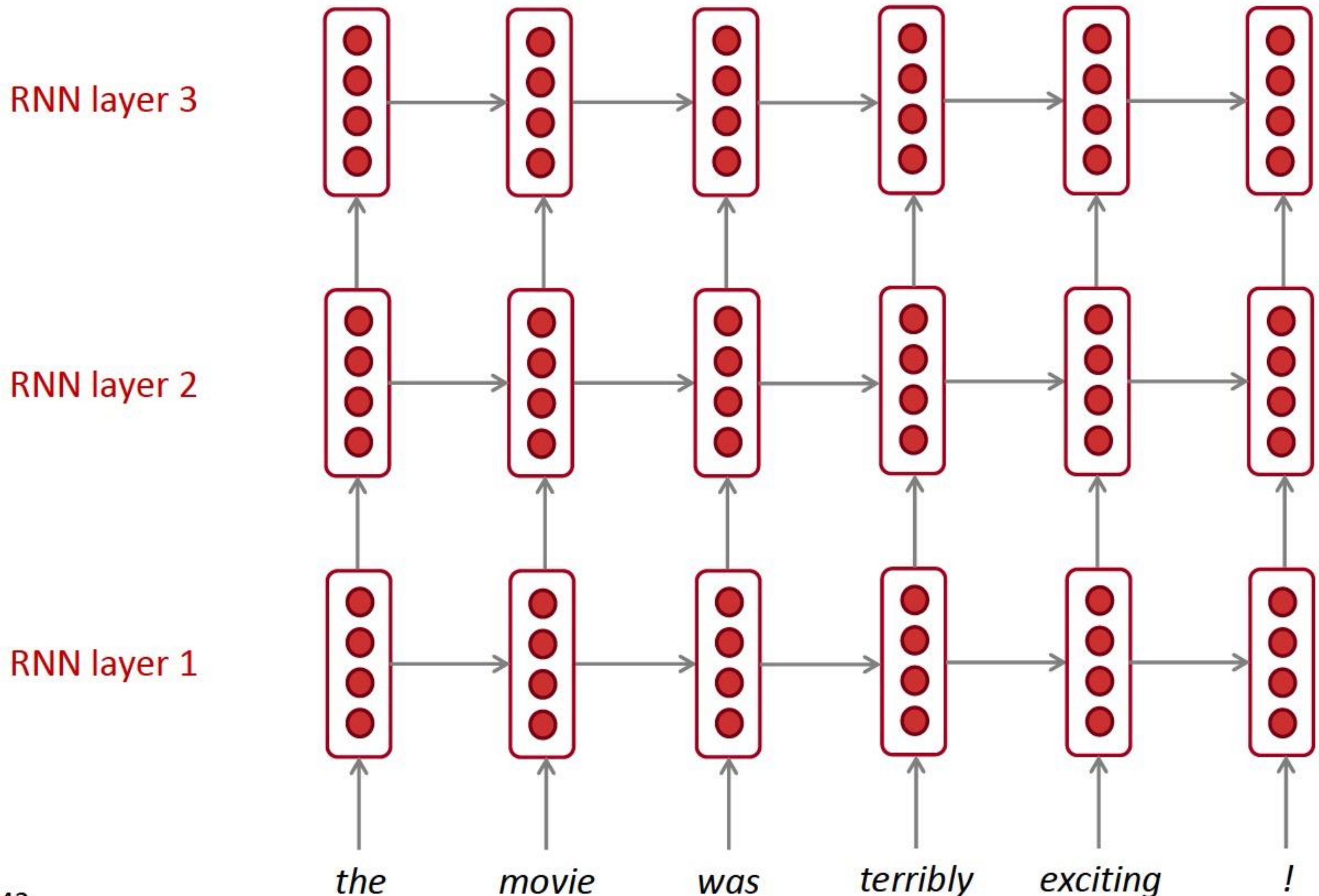
- We can also make them “deep” in another dimension by applying multiple RNNs – this is a multi-layer RNN.

- This allows the network to compute more complex representations
  - The lower RNNs should compute lower-level features and the higher RNNs should compute higher-level features.

- Multi-layer RNNs are also called *stacked RNNs*.

Multi-layer RNNs

The hidden states from RNN layer *i* are the inputs to RNN layer *i+1*



Multi-layer RNNs in practice

- High-performing RNNs are often multi-layer (but aren't as deep as convolutional or feed-forward networks)

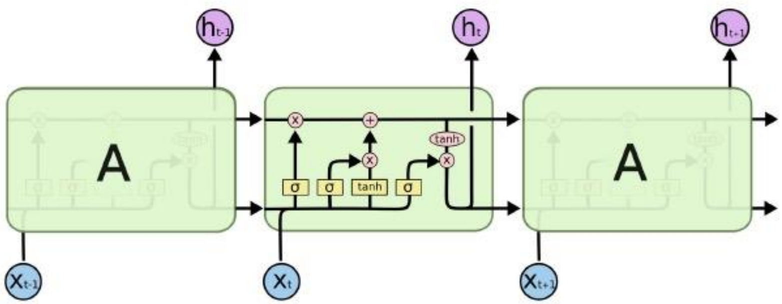
- For example: In a 2017 paper, Britz et al find that for Neural Machine Translation, 2 to 4 layers is best for the encoder RNN, and 4 layers is best for the decoder RNN
  - However, skip-connections/dense-connections are needed to train deeper RNNs (e.g. 8 layers)

- Transformer-based networks (e.g. BERT) can be up to 24 layers
  - You will learn about Transformers later; they have a lot of skipping-like connections

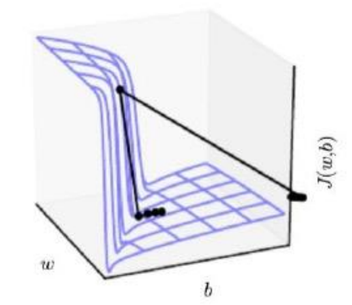
"Massive Exploration of Neural Machine Translation Architecutres", Britz et al, 2017. https://arxiv.org/pdf/1703.03906.pdf

In summary

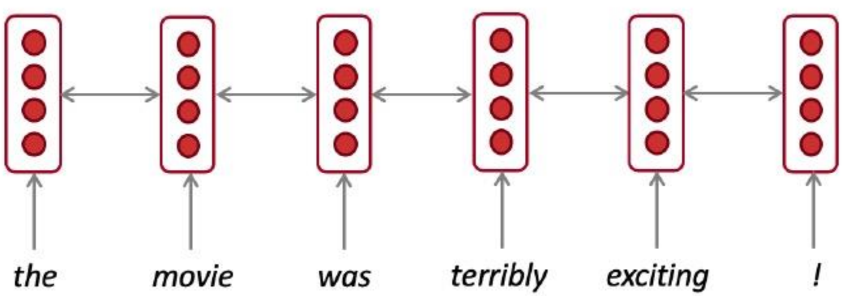
Lots of new information today! What are the practical takeaways?



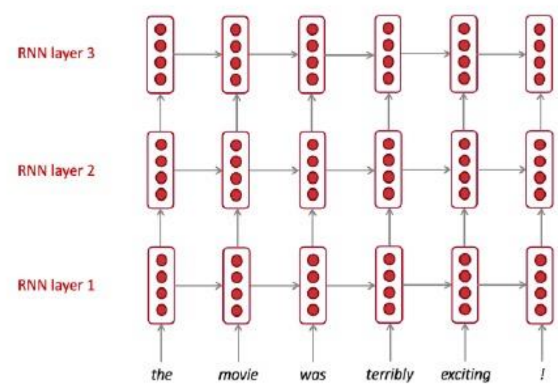
1. LSTMs are powerful but GRUs are faster



2. Clip your gradients



3. Use bidirectionality when possible



4. Multi-layer RNNs are powerful, but you might need skip/dense-connections if it's deep

Required Readings

- Chapter 7 on Neural Networks from the J&M textbook.
  - Sections on Training are optional.
- Chapter 8 on RNNs and LSTMs from the J&M textbook.
  - Sections on Training are optional.