A matrix is a 2D array. => we have rows & columns

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

row 1

column 2

A has 2 rows and 3 columns.

A is 2 by 3 / 2×3 / $A \in \mathbb{R}^{2\times 3}$

A vector is one column (column vector) or a 1D array.

a 2D array that is $K \times 1$

$v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, v is a 2×1 matrix

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Basic operations

① A scalar (a number, a constant) $c = 3$. What is $c \cdot A$?

$$B = cA = 3 \cdot \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 & 6 \\ 0 & 9 & 3 \end{bmatrix}$$

② what does it mean to multiply 2 matrices : $C = AB$

$$\underset{2\times 2}{C} = \underset{2\times 3}{A}\,\underset{3\times 2}{B} = \underbrace{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 2 & 1 \end{bmatrix}}_{B} = \begin{bmatrix} 1\cdot 1 - 1\cdot 3 + 2\cdot 2 & 1\cdot 2 + \overset{-1\cdot 0}{0} + 2\cdot 1 \\ 0\cdot 1 + 3\cdot 3 + 1\cdot 2 & 0\cdot 2 + 3\cdot 0 + 1\cdot 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 4 \\ 11 & 1 \end{bmatrix}}_{C}$$

$A = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$ $n \times n$

$B = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$ $n \times n$

$C = \begin{array}{c|c} & j \\ \hline i & \begin{bmatrix} \quad & \circ \\ \quad & \quad \end{bmatrix} \end{array}$ $n \times n$

C requires n^3 $\times$'s
$n^2(n+1)$ $+$'s

multiply $\underbrace{\text{row } i}_{n}$ in A w/ $\underbrace{\text{col } j}_{n}$ in B

n multiplication

$n+1$ additions.

Strassen alg. $\longrightarrow$ $n^{2.8\ldots}$

paper on doing it w/ even fewer $*$'s asymptotically (group at Google).

$$W = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \qquad X = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$$

$3 \times 1 \qquad 3 \times 1$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \to 2 \times 3$$

$$A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix} \to 3 \times 2 \qquad A^T_{ij} = A_{ji}$$

$$W^T = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} \to 3 \times 1$$

1×3

$$\underbrace{W^T X}_{1 \times 1} = \underbrace{1}_{w_1} \cdot \overbrace{2}^{x_1} + \underbrace{-1}_{w_2} \cdot \overbrace{4}^{x_2} + \underbrace{2}_{w_3} \cdot \overbrace{3}^{x_3} = 4$$

$\boxed{W^T X \text{ is the dot product of } W \,\&\, X}$