Calculus Practice Set: Derivatives & Gradients

Univariate Derivatives and Multivariate Gradients (Up to 3 Variables)

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1 Overview

This problem set consists of 25 practice exercises generated with Gemini 3.6 Thinking, covering single-variable differentiation and multi-variable gradient computations. The exercises progress from fundamental rules (power, exponential, logarithmic, product, quotient, and chain rules) to composite multivariate functions.

Part 1: Practice Questions

Section A: Single-Variable Derivatives

Compute the first derivative $f'(x)$ for each of the following functions:

1. **(Foundational Polynomial)**

$$f(x) = 4x^5 - 3x^3 + 7x - 12$$

2. **(Foundational Basic Functions)**

$$f(x) = 5e^x + 2\ln(x) - \frac{3}{x} \quad (x > 0)$$

3. **(Product Rule Warm-up)**

$$f(x) = (3x^2 + 1)(2x - 5)$$

4. **(Quotient Rule Warm-up)**

$$f(x) = \frac{x^2 + 4}{x - 1} \quad (x \neq 1)$$

5. **(Chain Rule Power Composition)**

$$f(x) = (5x^3 - 2)^4$$

6. **(Chain Rule Exponential)**

$$f(x) = e^{4x-1}$$

7. **(Logarithmic Composition)**

$$f(x) = \ln(3x^2 + 5x + 1)$$

8. **(Product & Exponential Composition)**

$$f(x) = x^3 e^{-2x}$$

9. **(Quotient & Logarithmic Composition)**

$$f(x) = \frac{\ln(x)}{x^3} \quad (x > 0)$$

10. **(Radical & Chain Rule)**

$$f(x) = \sqrt{x^2 + 9}$$

11. **(Polynomial, Product & Exponential)**

$$f(x) = (x^2 + 2x)e^{-x}$$

12. **(Quotient & Exponential Composition)**

$$f(x) = \frac{e^{3x}}{2x + 1} \quad \left(x \neq -\frac{1}{2}\right)$$

Section B: Two-Variable Gradients

Compute the gradient vector $\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$ for each function:

13. **(Foundational Bivariate Polynomial)**

$$f(x, y) = 3x^3y^2 - 5x^2 + 4y^3 - 7$$

14. **(Linear Exponential Composition)**

$$f(x, y) = e^{2x+3y}$$

15. **(Logarithm of Sum of Squares)**

$$f(x, y) = \ln(x^2 + y^2) \quad ((x, y) \neq (0, 0))$$

16. **(Polynomial & Rational Combination)**

$$f(x, y) = x^2y + \frac{x}{y} \quad (y \neq 0)$$

17. **(Polynomial Product & Exponential)**

$$f(x, y) = x^3y^2 + e^{2x+y}$$

18. **(Product, Exponential & Reciprocal)**

$$f(x, y) = x^2ye^{3x} + \frac{1}{y} \quad (y \neq 0)$$

19. **(Composite Logarithmic & Rational)**

$$f(x, y) = \ln(x^2 + y) + \frac{x}{y}$$

Section C: Three-Variable Gradients

Compute the gradient vector $\nabla f(x, y, z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$ for each function:

20. **(Symmetric Trivariate Polynomial)**

$$f(x, y, z) = x^3 + y^3 + z^3 - 3xyz$$

21. **(Monomial Product)**

$$f(x, y, z) = x^2y^3z^4$$

22. **(Linear 3D Exponential)**

$$f(x, y, z) = e^{x+2y+3z}$$

23. **(Mixed Polynomial, Exponential & Logarithm)**

$$f(x, y, z) = x^2yz + e^{xz} + \ln(y) \quad (y > 0)$$

24. **(Product & Logarithmic Argument)**

$$f(x, y, z) = x^2yz^3 + \ln(x + 2y + 3z)$$

25. **(Rational & Exponential Composition)**

$$f(x, y, z) = \frac{xy^2}{z + 1} + e^{x-z} \quad (z \neq -1)$$

Part 2: Solutions and Step-by-Step Derivations

Solutions for Section A (Questions 1–12)

Solution 1

Apply the power rule term-by-term:

$$f'(x) = 4(5x^4) - 3(3x^2) + 7(1) - 0 = 20x^4 - 9x^2 + 7$$

Solution 2

Rewrite $-3/x$ as $-3x^{-1}$ and differentiate:

$$f'(x) = 5e^x + 2\left(\frac{1}{x}\right) - 3(-1x^{-2}) = 5e^x + \frac{2}{x} + \frac{3}{x^2}$$

Solution 3

Apply the product rule $(uv)' = u'v + uv'$ with $u = 3x^2 + 1$ and $v = 2x - 5$:

$$u' = 6x, \quad v' = 2$$

$$f'(x) = 6x(2x - 5) + (3x^2 + 1)(2) = 12x^2 - 30x + 6x^2 + 2 = 18x^2 - 30x + 2$$

Solution 4

Apply the quotient rule $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ with $u = x^2 + 4$ and $v = x - 1$:

$$u' = 2x, \quad v' = 1$$

$$f'(x) = \frac{2x(x-1) - (x^2+4)(1)}{(x-1)^2} = \frac{2x^2 - 2x - x^2 - 4}{(x-1)^2} = \frac{x^2 - 2x - 4}{(x-1)^2}$$

Solution 5

Apply the chain rule $\frac{d}{dx}[u^n] = nu^{n-1}u'$ with $u = 5x^3 - 2$:

$$u' = 15x^2$$

$$f'(x) = 4(5x^3 - 2)^3 \cdot (15x^2) = 60x^2(5x^3 - 2)^3$$

Solution 6

Apply the chain rule $\frac{d}{dx}[e^u] = e^u u'$ with $u = 4x - 1$:

$$u' = 4$$

$$f'(x) = 4e^{4x-1}$$

Solution 7

Apply the chain rule $\frac{d}{dx}[\ln(u)] = \frac{u'}{u}$ with $u = 3x^2 + 5x + 1$:

$$u' = 6x + 5$$
$$f'(x) = \frac{6x + 5}{3x^2 + 5x + 1}$$

Solution 8

Apply the product rule to $u = x^3$ and $v = e^{-2x}$:

$$u' = 3x^2, \quad v' = -2e^{-2x}$$
$$f'(x) = 3x^2e^{-2x} + x^3(-2e^{-2x}) = x^2e^{-2x}(3 - 2x)$$

Solution 9

Apply the quotient rule to $u = \ln(x)$ and $v = x^3$:

$$u' = \frac{1}{x}, \quad v' = 3x^2$$
$$f'(x) = \frac{\left(\frac{1}{x}\right)x^3 - \ln(x)(3x^2)}{(x^3)^2} = \frac{x^2 - 3x^2\ln(x)}{x^6} = \frac{x^2(1 - 3\ln(x))}{x^6} = \frac{1 - 3\ln(x)}{x^4}$$

Solution 10

Rewrite as $f(x) = (x^2 + 9)^{1/2}$ and use the chain rule:

$$f'(x) = \frac{1}{2}(x^2 + 9)^{-1/2} \cdot (2x) = \frac{x}{\sqrt{x^2 + 9}}$$

Solution 11

Apply the product rule to $u = x^2 + 2x$ and $v = e^{-x}$:

$$u' = 2x + 2, \quad v' = -e^{-x}$$
$$f'(x) = (2x + 2)e^{-x} + (x^2 + 2x)(-e^{-x}) = e^{-x}(2x + 2 - x^2 - 2x) = (2 - x^2)e^{-x}$$

Solution 12

Apply the quotient rule to $u = e^{3x}$ and $v = 2x + 1$:

$$u' = 3e^{3x}, \quad v' = 2$$
$$f'(x) = \frac{3e^{3x}(2x + 1) - e^{3x}(2)}{(2x + 1)^2} = \frac{e^{3x}(6x + 3 - 2)}{(2x + 1)^2} = \frac{(6x + 1)e^{3x}}{(2x + 1)^2}$$

Solutions for Section B (Questions 13–19)

Solution 13

Differentiate with respect to x (holding y constant) and y (holding x constant):

$$\frac{\partial f}{\partial x} = 9x^2y^2 - 10x, \quad \frac{\partial f}{\partial y} = 6x^3y + 12y^2$$
$$\nabla f(x,y) = \left(9x^2y^2 - 10x,\ 6x^3y + 12y^2\right)$$

Solution 14

Apply the chain rule for exponential functions:

$$\frac{\partial f}{\partial x} = 2e^{2x+3y}, \quad \frac{\partial f}{\partial y} = 3e^{2x+3y}$$
$$\nabla f(x,y) = \left(2e^{2x+3y},\ 3e^{2x+3y}\right)$$

Solution 15

Using $\frac{\partial}{\partial x}[\ln(u)] = \frac{1}{u}\frac{\partial u}{\partial x}$ with $u = x^2 + y^2$:

$$\frac{\partial f}{\partial x} = \frac{2x}{x^2+y^2}, \quad \frac{\partial f}{\partial y} = \frac{2y}{x^2+y^2}$$
$$\nabla f(x,y) = \left(\frac{2x}{x^2+y^2},\ \frac{2y}{x^2+y^2}\right)$$

Solution 16

Rewrite as $f(x,y) = x^2y + xy^{-1}$:

$$\frac{\partial f}{\partial x} = 2xy + y^{-1} = 2xy + \frac{1}{y}$$
$$\frac{\partial f}{\partial y} = x^2 - xy^{-2} = x^2 - \frac{x}{y^2}$$
$$\nabla f(x,y) = \left(2xy + \frac{1}{y},\ x^2 - \frac{x}{y^2}\right)$$

Solution 17

Differentiate each component:

$$\frac{\partial f}{\partial x} = 3x^2y^2 + 2e^{2x+y}, \quad \frac{\partial f}{\partial y} = 2x^3y + e^{2x+y}$$
$$\nabla f(x,y) = \left(3x^2y^2 + 2e^{2x+y},\ 2x^3y + e^{2x+y}\right)$$

Solution 18

For $\frac{\partial f}{\partial x}$, apply product rule to x^2e^{3x}:

$$\frac{\partial f}{\partial x} = y\left(2xe^{3x} + 3x^2e^{3x}\right) = ye^{3x}(3x^2 + 2x)$$

For $\frac{\partial f}{\partial y}$, differentiate linearly:

$$\frac{\partial f}{\partial y} = x^2e^{3x} - \frac{1}{y^2}$$

$$\nabla f(x, y) = \left(ye^{3x}(3x^2 + 2x),\ x^2e^{3x} - \frac{1}{y^2}\right)$$

Solution 19

Apply partial differentiation rules:

$$\frac{\partial f}{\partial x} = \frac{2x}{x^2 + y} + \frac{1}{y}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x^2 + y} - \frac{x}{y^2}$$

$$\nabla f(x, y) = \left(\frac{2x}{x^2 + y} + \frac{1}{y},\ \frac{1}{x^2 + y} - \frac{x}{y^2}\right)$$

Solutions for Section C (Questions 20–25)

Solution 20

Compute partial derivatives for each variable:

$$\frac{\partial f}{\partial x} = 3x^2 - 3yz, \quad \frac{\partial f}{\partial y} = 3y^2 - 3xz, \quad \frac{\partial f}{\partial z} = 3z^2 - 3xy$$

$$\nabla f(x, y, z) = \left(3x^2 - 3yz,\ 3y^2 - 3xz,\ 3z^2 - 3xy\right)$$

Solution 21

Treat remaining variables as constants for each partial derivative:

$$\frac{\partial f}{\partial x} = 2xy^3z^4, \quad \frac{\partial f}{\partial y} = 3x^2y^2z^4, \quad \frac{\partial f}{\partial z} = 4x^2y^3z^3$$

$$\nabla f(x, y, z) = \left(2xy^3z^4,\ 3x^2y^2z^4,\ 4x^2y^3z^3\right)$$

Solution 22

Apply chain rule to the exponential function e^u where $u = x + 2y + 3z$:

$$\frac{\partial f}{\partial x} = e^{x+2y+3z}, \quad \frac{\partial f}{\partial y} = 2e^{x+2y+3z}, \quad \frac{\partial f}{\partial z} = 3e^{x+2y+3z}$$

$$\nabla f(x, y, z) = \left(e^{x+2y+3z},\ 2e^{x+2y+3z},\ 3e^{x+2y+3z}\right)$$

Solution 23

Compute each partial derivative separately:

$$\frac{\partial f}{\partial x} = 2xyz + ze^{xz}$$

$$\frac{\partial f}{\partial y} = x^2z + \frac{1}{y}$$

$$\frac{\partial f}{\partial z} = x^2y + xe^{xz}$$

$$\nabla f(x, y, z) = \left(2xyz + ze^{xz},\ x^2z + \frac{1}{y},\ x^2y + xe^{xz}\right)$$

Solution 24

Let $u = x + 2y + 3z$:

$$\frac{\partial f}{\partial x} = 2xyz^3 + \frac{1}{x + 2y + 3z}$$

$$\frac{\partial f}{\partial y} = x^2z^3 + \frac{2}{x + 2y + 3z}$$

$$\frac{\partial f}{\partial z} = 3x^2yz^2 + \frac{3}{x + 2y + 3z}$$

$$\nabla f(x, y, z) = \left(2xyz^3 + \frac{1}{x + 2y + 3z},\ x^2z^3 + \frac{2}{x + 2y + 3z},\ 3x^2yz^2 + \frac{3}{x + 2y + 3z}\right)$$

Solution 25

Compute partial derivatives with respect to x, y, and z:

$$\frac{\partial f}{\partial x} = \frac{y^2}{z + 1} + e^{x-z}$$

$$\frac{\partial f}{\partial y} = \frac{2xy}{z + 1}$$

$$\frac{\partial f}{\partial z} = xy^2\left(-(z + 1)^{-2}\right) + e^{x-z}(-1) = -\frac{xy^2}{(z + 1)^2} - e^{x-z}$$

$$\nabla f(x, y, z) = \left(\frac{y^2}{z + 1} + e^{x-z},\ \frac{2xy}{z + 1},\ -\frac{xy^2}{(z + 1)^2} - e^{x-z}\right)$$