

ITCS 5356 Intro Machine Learning

Python Stack

Linear Algebra and Optimization in NumPy

Computation Graphs in PyTorch

Razvan C. Bunescu

Computer Science @ CCI

razvan.bunescu@charlotte.edu

Python Programming Stack for Deep Learning

- Python = object-oriented, interpreted, scripting language.
 - imperative programming, with functional programming features.
- NumPy = package for powerful N-dimensional arrays:
 - sophisticated (broadcasting) functions.
 - useful linear algebra, Fourier transform, and random number capabilities.
- SciPy = package for numerical integration and optimization.
- Matplotlib = comprehensive 2D and 3D plotting library.

Python Programming Stack for Deep Learning

- PyTorch = a wrapper of NumPy that enables the use of GPUs and automatic differentiation:
 - **Tensors** similar to NumPy's ndarray, but can also be used on GPU.
- Jupyter Notebook = a web app for creating documents that contain live code, equations, visualizations and markdown text.
- Anaconda = an open-source distribution of Python and Python packages:
 - Package versions are managed through Conda.
 - Install all packages above using Anaconda / Conda install.

Anaconda Install

- **Anaconda:** Installation instructions for various platforms can be found at: <https://docs.anaconda.com/anaconda/install/>
 - For Mac and Linux users, the system PATH must be updated after installation so that ‘conda’ can be used from the command line.
 - Mac OS X:
 - For bash users: `export PATH=~/.anaconda3/bin:$PATH`
 - For csh/tcsh users: `setenv PATH ~/.anaconda3/bin:$PATH`
 - For Linux:
 - For bash users: `export PATH=~/.anaconda3/bin:$PATH`
 - For csh/tcsh users: `setenv PATH ~/.anaconda3/bin:$PATH`
 - It is recommend the above statement be put in the `~/.bashrc` or `~/.cshrc` file, so that it is executed every time a new terminal window is open.
 - To check that conda was installed, running “*conda list*” in the terminal should list all packages that come with Anaconda.

Installing Packages with Conda / Anaconda

- A number of tools and libraries that we will use can be configured from Anaconda:
 - Python 3, NumPy, SciPy, Matplotlib, Jupyter Notebook, Ipython, Pandas, Scikit-learn.
 - PyTorch can be installed from Anaconda, with ‘conda’ from the command line:
 - The actual command line depends on the platform as follows:
 - Using the GUI on pytorch.org, choose the appropriate OS, conda, Python 3.6, CUDA or CPU version.

import numpy as np

- `np.array()`
 - indexing, slices.
- `ndarray.shape`, `.size`, `.ndim`, `.dtype`, `.T`
- `np.zeros()`, `np.ones()`, `np.arange()`, `np.eye()`
 - *dtype* parameter.
 - tuple (shape) parameter.
- `np.reshape()`, `np.ravel()`
- `np.amax()`, `np.maximum()`, `np.sum()`, `np.mean()`, `np.std()`
 - *axis* parameter, also `np.ndarray`
- `np.stack()`, `np.[hv]stack()`, `np.column_stack()`, `np.split()`
- `np.exp()`, `np.log()`,
- <https://docs.scipy.org/doc/numpy/user/quickstart.html>

NumPy: Broadcasting

- Broadcasting describes how numpy treats arrays with different shapes during arithmetic operations.
- The smaller array is “broadcast” across the larger array so that they have compatible shapes, subject to broadcasting rules:
 - NumPy compares their shapes element-wise.
 - It starts with the trailing dimensions, and works its way forward.
 - Two dimensions are compatible when:
 - they are equal, or one of them is 1.
- <https://docs.scipy.org/doc/numpy/user/basics.broadcasting.html>

Other Numpy Functions

- `np.dot()`, `np.vdot()`
 - also `np.ndarray`.
- `np.outer()`, `np.inner()`
- **`import numpy.random as random:`**
 - `randn()`, `randint()`, `uniform()`
- **`import numpy.linalg as la:`**
 - `la.norm()`, `la.det()`, `la.matrix_rank()`, `np.trace()`
 - `la.eig()`, `la.svd()`
 - `la.qr()`, `la.cholesky()`
- <https://docs.scipy.org/doc/numpy/reference/routines.linalg.html>

Implementation: Vectorization of LR

- **Version 3:** Compute gradient, vectorized.

$$\nabla L(\mathbf{w}) = \frac{1}{N} \sum_{n=1}^N (\hat{y}_n - y_n) \mathbf{x}_n$$

```
grad = X.T @ sigmoid(X.dot(w) - y) / N
```

```
def sigmoid(x):  
    return 1 / (1 + np.exp(-x))
```

import scipy

- `scipy.sparse.coo_matrix()`
`groundTruth = coo_matrix((np.ones(N, dtype = np.uint8),
 (labels, np.arange(N))))).toarray()`
- `scipy.optimize:`
 - `scipy.optimize.fmin_l_bfgs_b()`
`theta, _, _ = fmin_l_bfgs_b(softmaxCost, theta,
 args = (numClasses, inputSize, decay, images, labels),
 maxiter = 100, disp = 1)`
 - `scipy.optimize.fmin_cg()`
 - `scipy.minimize`

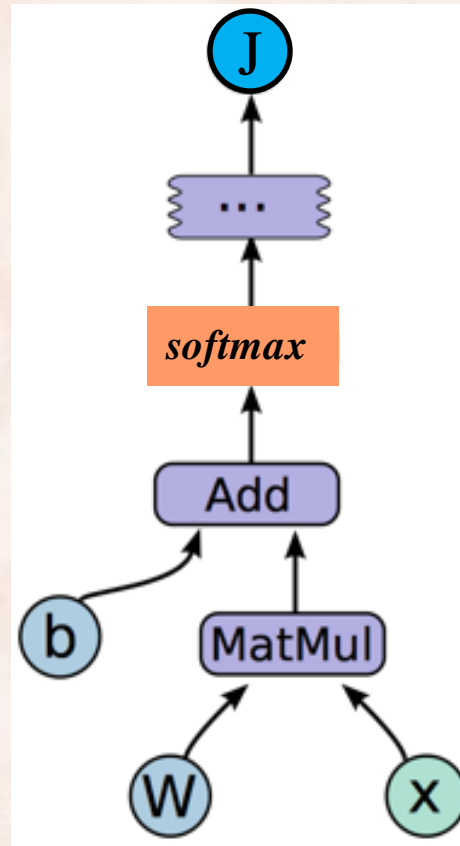
<https://docs.scipy.org/doc/scipy-0.10.1/reference/tutorial/optimize.html>

Towards PyTorch: Graphs of Computations

- A function J can be expressed by the **composition** of **computational elements** from a given set:
 - Mathematical operators:
 - multiplication and additions
 - Elementwise application of mathematical functions:
 - $\max(0, x)$, $\exp(x)$, ...
- The function is defined by a **graph of computations**:
 - A directed acyclic graph, with one node per computational element.
 - Depth of architecture = depth of the graph = longest path from an input node to an output node.

Logistic Regression as a Computation Graph

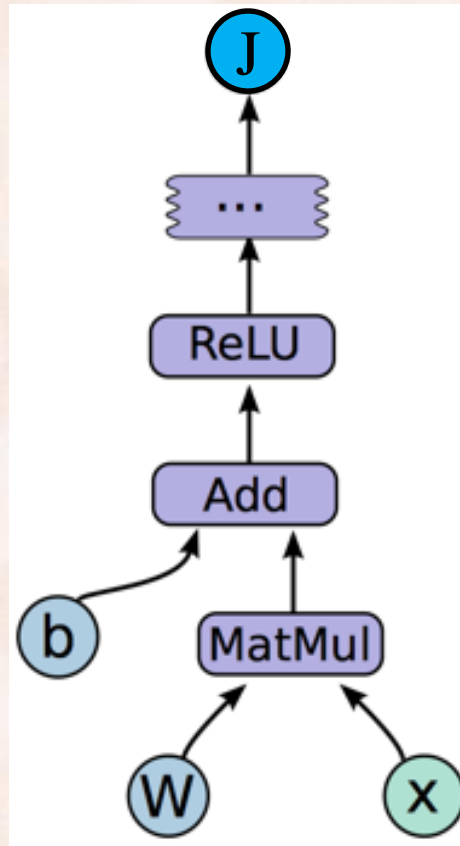
Inference =
*Forward
Propagation*



Learning =
*Backward
Propagation*

Neural Network as a Computation Graph

Inference =
*Forward
Propagation*



Learning =
*Backward
Propagation*

What is PyTorch

- A wrapper of **NumPy** that enables the use of GPUs.
 - **Tensors** similar to NumPy's *ndarray*, but can also be used on GPU.
- A flexible deep learning platform:
 - Deep Neural Networks built on a tape-based **autograd** system:
 - Building neural networks using and replaying a tape recorder.
 - **Reverse-mode auto-differentiation** allows changing the network at runtime:
 - The computation graph is created on the fly.
 - Backpropagation is done on the dynamically built graph.

<http://pytorch.org/about/>

Automatic Differentiation

https://pytorch.org/tutorials/beginner/blitz/autograd_tutorial.html

- Deep learning packages offer automatic differentiation.
- PyTorch has the *autograd* package:
 - *torch.Tensor* the main class; *torch.Function* class also important.
 - When *requires_grad = True*, it tracks all operations on this tensor (e.g. the parameters).
 - An acyclic graph is built **dynamically** that encodes the history of computations, i.e. compositions of functions.
 - TensorFlow compiles **static** computation graphs.
 - To compute the gradient, call *backward()* in a scalar valued Tensor (e.g. the *loss*).

Tensors

- PyTorch **tensors** support the same operations as NumPy.
 - Arithmetic.
 - Slicing and Indexing.
 - Broadcasting.
 - Reshaping.
 - Sum, Max, Argmax, ...
- PyTorch tensors can be converted to NumPy tensors.
- NumPy tensors can be converted to PyTorch tensors.

http://pytorch.org/tutorials/beginner/blitz/tensor_tutorial.html

Autograd

- The **autograd** package provides automatic differentiation for all operations on Tensors.
 - It is a *define-by-run* framework, which means that the gradient is defined by how your code is run:
 - Every single **backprop** iteration can be different.
- **autograd.Tensor** is the central class of the package.
 - Once you finish your computation you can call **.backward()** and have all the gradients computed automatically.

http://pytorch.org/tutorials/beginner/blitz/autograd_tutorial.html

Tensor and Function

- A **Tensor** v has three important attributes:
 - v .**data** holds the raw tensor value.
 - v .**grad** is another Tensor which accumulates the gradient w.r.t. v :
 - The gradient of what?
 - The gradient of any variable u that uses v on which we call u .**backward()**.
 - <http://pytorch.org/docs/master/autograd.html>
 - v .**grad_fn** stores the **Function** that has created the Tensor v :
 - <http://pytorch.org/docs/master/autograd.html>

Multivariate Chain Rule for Differentiation

- Multivariate Chain Rule:

$$f = f(g_1(x), g_2(x), \dots, g_n(x))$$

$$\frac{\nabla f}{\nabla x} = \sum_{i=1}^n \frac{\nabla f}{\nabla g_i} \frac{\nabla g_i}{\nabla x}$$

- Example 2:

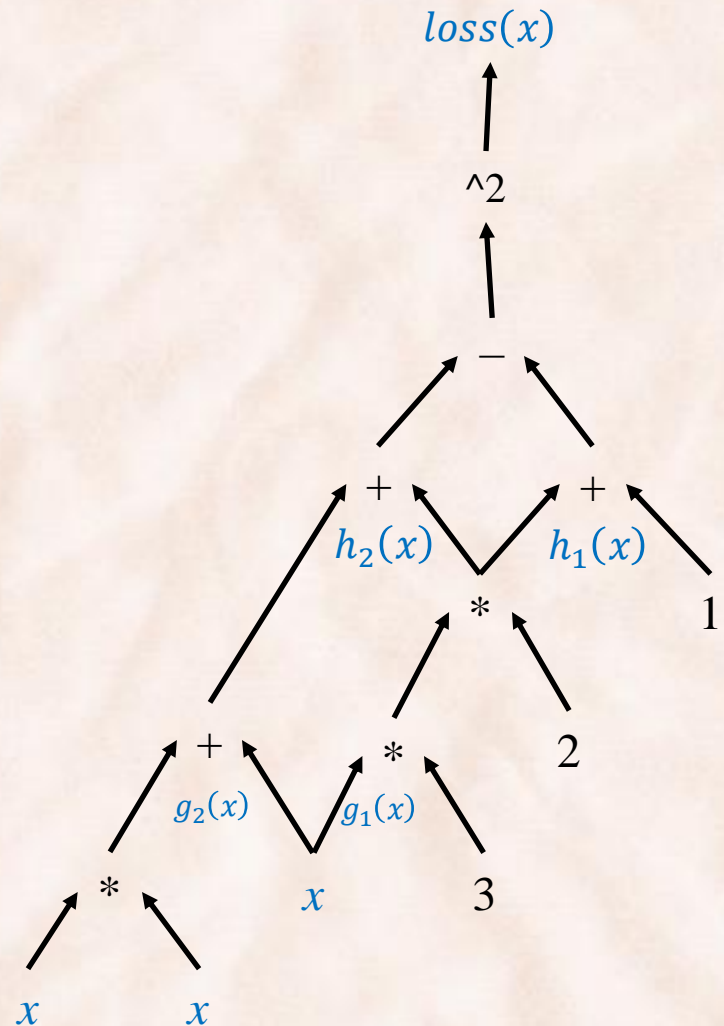
$$loss(x) = (h_1(x) - h_2(x))^2$$

$$h_1(x) = 2g_1(x) + 1$$

$$h_2(x) = 2g_1(x) + g_2(x)$$

$$g_1(x) = 3x$$

$$g_2(x) = x^2 + x$$



$$\text{loss}(x) = (h_1(x) - h_2(x))^2 \quad \frac{\partial \ell}{\partial x} = \frac{\partial \ell}{\partial h_1} \cdot \frac{\partial h_1}{\partial x} + \frac{\partial \ell}{\partial h_2} \cdot \frac{\partial h_2}{\partial x}$$

$$h_1(x) = 2g_1(x) + 1 \rightarrow 7$$

$$h_2(x) = 2g_1(x) + g_2(x) \rightarrow 8$$

$$g_1(x) = 3x \rightarrow 3$$

$$g_2(x) = x^2 + 1 \rightarrow 2$$

$$= 2(h_1 - h_2) \cdot \frac{\partial h_1}{\partial x} + 2(h_2 - h_1) \frac{\partial h_2}{\partial x}$$

$$= 2(h_1 - h_2) \left(\frac{\partial h_1}{\partial x} - \frac{\partial h_2}{\partial x} \right)$$

$$= 2(h_1 - h_2) \left(\frac{\partial h_1}{\partial g_1} \cdot \frac{\partial g_1}{\partial x} - \frac{\partial h_2}{\partial g_1} \frac{\partial g_1}{\partial x} - \frac{\partial h_2}{\partial g_2} \frac{\partial g_2}{\partial x} \right)$$

$$= 2(h_1 - h_2) \left(2 \cdot 3 - 2 \cdot 3 - 1 \cdot (2x + 1) \right) = 2(h_1 - h_2)(-2x - 1)$$

$$= 2(h_2 - h_1)(2x + 1)$$

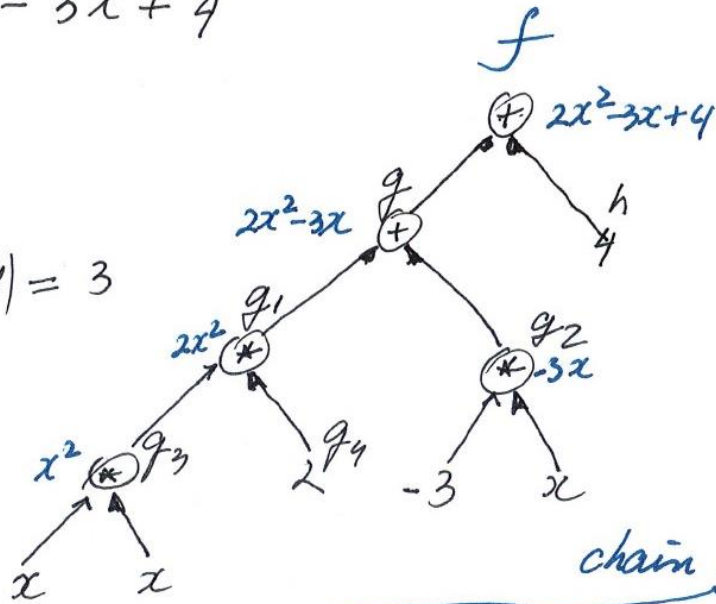
$$x=1 \rightarrow \frac{\partial \ell}{\partial x} = 2(8 - 7)(3) = 6$$

$$f(x) = 2x^2 - 3x + 4$$

$$f'(x) = \frac{\partial f}{\partial x} = 4x - 3$$

forward pass

$$f(1) = 3$$



$x = 1$
backward pass $f'(1) = 1$

$$\text{add}(x, y) = x + y$$

chain rule of differentiation

$$f = \text{add}(g, h) \quad \frac{\partial f}{\partial x} = \frac{\partial \text{add}}{\partial g} \cdot \frac{\partial g}{\partial x} + \frac{\partial \text{add}}{\partial h} \cdot \frac{\partial h}{\partial x}$$

$$= 1 \cdot \frac{\partial g}{\partial x} + 1 \cdot 0 = \frac{\partial g}{\partial x} = 4x - 3$$

$$g = \text{add}(g_1, g_2) \Rightarrow \frac{\partial g}{\partial x} = \frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial x} = 4x - 3$$

$$\frac{\partial \text{mul}}{\partial g_3} \frac{\partial g_3}{\partial x} + \frac{\partial \text{mul}}{\partial g_4} \frac{\partial g_4}{\partial x} = 2 \cdot 2x + g_3 \cdot 0 = 4x$$

PyTorch

- Install using Anaconda:
 - `conda install pytorch torchvision -c pytorch`
 - <http://pytorch.org>
- Install from sources:
 - <https://github.com/pytorch/pytorch#from-source>
- Tutorials:
 - <http://pytorch.org/tutorials/>
 - http://pytorch.org/tutorials/beginner/pytorch_with_examples.html