$$y^{(i)} = U^T x^{(i)} \qquad Y = [y^{(1)} | \cdots | y^{(m)}] \qquad X = [x^{(1)} | \cdots | x^{(m)}]$$

$$Y = U^T X$$

$\hookrightarrow$ mean normalized

$$\sum_{i=1}^{m} x^{(i)} = \vec{0}$$

$$\boxed{\frac{1}{m} Y Y^T} = \frac{1}{m} U^T X X^T U = U^T \underbrace{\left(\frac{1}{m} X X^T\right)}_{\Sigma} U$$

$$\Sigma = \frac{1}{m} X X^T$$

$$\Sigma_Y = U^T \Sigma U = \Lambda = \text{diag}(\lambda_1, \dots, \lambda_\Delta)$$

$\Delta \times \Delta$

$U \rightarrow$ eigenvectors of Σ

$$\Sigma U = \Lambda U \quad (\Sigma u_j = \lambda_j u_j)$$

$$= \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_\Delta \end{bmatrix}$$

$$U^T \Sigma U = U^T \Lambda U$$

$$= \Lambda = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_\Delta \end{bmatrix}$$

$$\Sigma_Y[i,j] \rightarrow \text{cov}(y_i, y_j)$$