HW Assignment 0: Theory

1 Theory (50 + 10 points)

For each problem below, show your work. Clear and complete explanations and proofs of your results are as important as getting the right answer and are required for getting all the points. Problems marked with (∗) are mandatory only for ITCS 8156.

1. [**Derivatives**, 20 points]
   Compute the derivative $f'(x) = \dfrac{\delta f}{\delta x}$ or the gradient vector $\left[\dfrac{\delta f}{\delta x}, \dfrac{\delta f}{\delta y}\right]$ for the following functions $f(x)$ or $f(x, y)$. Make sure you show all your work (derivation steps):

$$f(x) = 4x + 3 \tag{1}$$
$$f(x) = 2x^2 + 3x + 4 \tag{2}$$
$$f(x) = 5x^n + 2x \tag{3}$$
$$f(x) = e^{3x} + 2x \tag{4}$$
$$f(x) = \ln(x + 2) \tag{5}$$
$$f(x) = \ln(3x^2 + 1) \tag{6}$$
$$f(x) = 1/(x^3 + 1) \tag{7}$$
$$f(x, y) = x^3 + 2xy + y^2 \tag{8}$$
$$f(x, y) = 2(4x + y + 3)^2 \tag{9}$$
$$f(x, y) = \ln(1 + e^{3x+y}) \tag{10}$$

2. [**Probability Theory**, 20 points] Suppose that we have three coloured boxes r (red), b (blue), and g (green). Box r contains 3 apples, 4 oranges, and 3 limes, box b contains 1 apple, 1 orange, and 0 limes, and box g contains 3 apples, 3 oranges, and 4 limes. If a box is chosen at random with probabilities $p(r) = 0.2$, $p(b) = 0.2$, $p(g) = 0.6$, and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting an apple? If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box?

3. [**Probability Theory**, 5 points, (∗)] Assume the probability of contracting COVID-19 is 0.09, i.e. $p(C = 1) = 0.09$. Assume the probability that the COVID antibody test of a person is positive given the person has COVID-19 is 0.90, i.e. $p(T = 1|C = 1) = 0.9$ and the probability that the COVID antibody test of a person is negative given the person does not have COVID-19 is 0.99, i.e. $p(T = 0|C = 0) = 0.99$. What is the probability of a person having COVID-19 given that their antibody test is positive, i.e. $p(C = 1|T = 1)$?

4. [**Linear Algebra**, 10 points] Solve the following system of equations:

$$2x_1 - 3x_2 - x_3 = -1 \tag{11}$$
$$x_1 + 2x_2 + 3x_3 = -4 \tag{12}$$
$$4x_1 + x_2 - 2x_3 = 12 \tag{13}$$

5. [**Linear Algebra**, 5 points (∗)] Solve the following system of equations:

$$2x_1 - 3x_2 - x_3 = -1 \tag{14}$$
$$x_1 + 2x_2 + 3x_3 = -4 \tag{15}$$
$$4x_1 + x_2 + 5x_3 = -9 \tag{16}$$

2 Submission

Submit your responses on Canvas as one file named `theory.pdf`. It is recommended to use an editor such as Latex or Word that allows editing and proper formatting of equations. Alternatively, if you choose to write your solutions on paper, submit an electronic scan / photo of it on Canvas. Make sure that your writing is legible and easy to read.