HW Assignment 7, Theory

ITCS 6156 students solve one of problems 1 and 2, and one of problems 3 and 4. Clearly mark which problem you want to be graded. ITCS 8156 students need to solve all problems.

1 PCA (30 points)

Let $y^{(k)} = U^T x^{(k)}$ be the rotation of $x^{(k)}$ through PCA, where U is the matrix of eigenvectors of the covariance matrix of the zero-mean dataset $X = [x^{(1)}, ..., x^{(m)}]$. Show that:

$$\frac{1}{m}\sum_{k=1}^{m}(y_j^{(k)})^2 = \lambda_j \tag{1}$$

2 PCA (30 points)

Prove that PCA is invariant to the scaling of the data, i.e. it will return the same eigenvectors regardless of the scaling of the input. More formally, if you multiply each feature vector $x^{(i)}$ by some positive number $c > 0$ (thus scaling every feature in every training example by the same number), PCA's output eigenvectors will not change.

3 Whitening (20 points)

Let X be a dataset of m D-dimensional samples and Y be the PCA whitening of X. Let R be an arbitrary orthogonal matrix, i.e. $RR^T = R^T R = I$. Prove that the sample covariance matrix of the rotated data RY is equal to the identity matrix I.

4 Sample Covariance Matrix (20 points)

Let X be a dataset of m D-dimensional samples and Σ its sample covariance matrix. Prove that all eigenvalues of Σ are positive real numbers.

5 Submission

Submit your responses on Canvas as one file named `theory.pdf`. It is recommended to use an editor such as Latex or Word or Jupyter-Notebook that allows editing and proper formatting of equations. Alternatively, if you choose to write your solutions on paper, submit an electronic scan / photo of it on Canvas. Make sure that your writing is legible and the scan has good quality.