Welcome

Design and Analysis of Algorithms

CS404/504

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Course Description

Course Description:

This course provides an introduction to the modern study of computer algorithms. Through this course students should be able to:

1) Analyze algorithm performance using complexity measurement.
2) Master major algorithms design techniques such as divide and conquer, greedy and dynamic programming.
3) Apply above approaches to solve a variety of practical problems such as sorting and selection, graph problems, and other optimization problems.
4) Understand the theory of NP-completeness.

What is an algorithm?

Definition: An **algorithm** is a computational procedure that takes values as *input* and produces values as *output*, in order to solve a well defined *computational problem*.

- The statement of the *problem* specifies a desired relationship between the *input* and the *output*.
- The algorithm specifies how to achieve that *input/output* relationship.
- A particular value of the *input* corresponds to an instance of the *problem*.

How to describe an algorithm?

We use Pseudo code:

1) **assignment** statement:
variable := value

2) **for loop**:
for variable := value1 **to** value2 **do** {
    <statement 1>;
    <statement 2>;
    ...
}

3) **while loop**:
while <condition> **do** {
    <statement 1>;
    <statement 2>;
    ..
}

4) **repeat loop**:
repeat {
    <statement 1>;
    <statement 2>;
    ..
} **until** < condition >

How to describe an algorithm?

We use Pseudo code:

5) **if-then** statement:

```
if < condition > {
   <statement 1>;
   <statement 2>;
   ..
}
```

6) **if-then-else** statement:

```
if < condition > {
   <statement 1>;
   <statement 2>;
   ..
} else {
   <statement 1'>;
   <statement 2'>;
   ..
}
```

Examples

1) **Find the maximum**: given an array of n numbers a[1..n], what is the maximum?

```
algorithm max(a, n)
{
    max := a[1];
    for i:= 2 to n do
        if (a[i] > max)
            max := a[i];

    return max;
}
```

Examples, cont'd

2) **Calculate the sum**: given an array of n numbers a[1..n], what is their sum?

```
algorithm sum(a, n)
{
    result := a[1];
    for i := 2 to n do
        result := a[i] + result;

    return result;
}
```

Examples, cont'd

3) **Sorting**:

Input: a sequence of n numbers a_1, a_2 ... a_n.

Output: a reordering of the input sequence such that in the resulting sequence each number is larger than all the numbers before, and smaller than the numbers after.

Output: a permutation π s.t. $a_{\pi(1)} \leq a_{\pi(2)} \leq ... \leq a_{\pi(n)}$

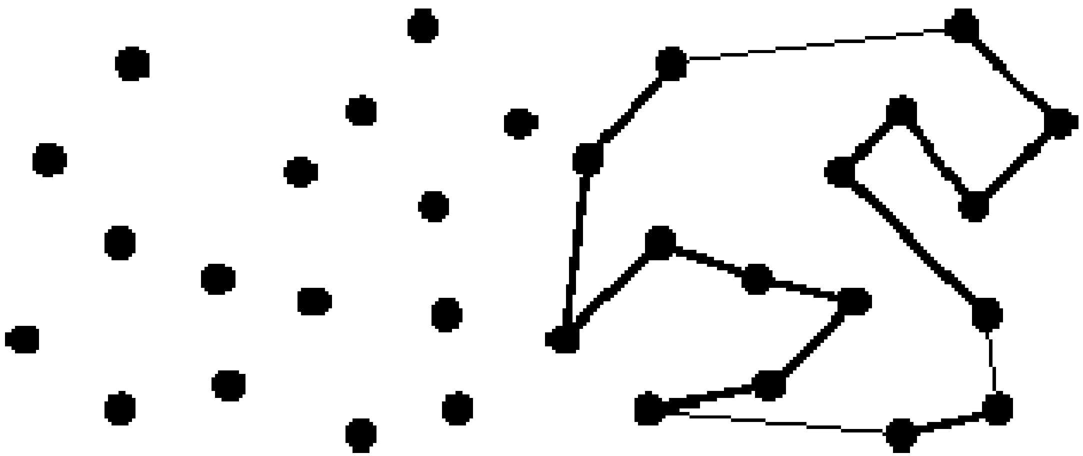
What kind of algorithms are we looking for?

1. **Correct**.
   How to prove an algorithm is correct/incorrect?

2. **Efficient**. Including time and space efficiency.

Correctness is not obvious!

An example: Travelling Salesman Problem (TSP)



Nearest Neighbor Tour

Algorithm: NNT($\mathcal{P} = \{P_1, ..., P_n\}$)

Pick and visit an initial vertex P_i

Set current vertex P to P_i

while there are still unvisited vertices in $\mathcal{P}$ *do*

    Let V be the closest vertex to P that is unvisited

    Visit V

    Set current vertex P to V

Return to P_i from P

Is that correct?

What if the input is as follows?

A correct solution

- We could try all possible orderings of the points, then select the ordering which minimizes the total length.

- **How many possibilities do we need to check?** $N!$

  How big is $N!$?

  When $N = 10$, $N! = 3628800$, $N = 20$,
  $N! = 2.4329 * 10^{18}$, $N = 100, N! = 9.32 * 10^{157}$.