- Sum-of-squares error function:

$$\underbrace{J(\mathbf{w})}_{[w_0, w_1]^T} = \frac{1}{2N} \sum_{n=1}^{N} \big(\underbrace{h_{\mathbf{w}}(x_n)}_{w_0 + w_1 x_n} - t_n \big)^2 \quad (\text{convex in } \mathbf{w})$$

- Training $\mathbf{w} \Longleftrightarrow$ solving $\hat{\mathbf{w}} = \underset{\mathbf{w}}{\operatorname{argmin}}\, J(\mathbf{w}) \Rightarrow$

$$\nabla_{\mathbf{w}} J = 0 \Rightarrow \left[\frac{\partial J}{\partial w_0}, \frac{\partial J}{\partial w_1} \right] = [0, 0]$$

① $$\frac{\partial J}{\partial w_0} = 0 \Rightarrow \frac{1}{2N} \sum_{n=1}^{N} 2 \big(h_{\mathbf{w}}(x_n) - t_n\big) \cdot \frac{\partial \big(h_{\mathbf{w}}(x_n) - t_n\big)}{\partial w_0} = 0$$

$$\Rightarrow \frac{1}{\cancel{2}N} \sum_{n=1}^{N} \cancel{2} \left(w_0 + w_1 x_n - t_n \right) \cdot 1 = 0 \quad / * N$$

$$\Rightarrow \sum_{n=1}^{N} (w_0 + w_1 x_n) = \sum_{n=1}^{N} t_n$$

$$\Rightarrow \boxed{w_0 N + w_1 \sum_{n=1}^{N} x_n = \sum_{n=1}^{N} t_n}$$

② $$\frac{\partial J}{\partial w_1} = 0 \Rightarrow \frac{1}{\cancel{2}N} \sum_{n=1}^{N} \cancel{2} \left(w_0 + w_1 x_n - t_n \right) \cdot x_n = 0 \quad / * N$$

$$\Rightarrow \sum_{n=1}^{N} (w_0 + w_1 x_n) x_n = \sum_{n=1}^{N} t_n x_n$$

$$\Rightarrow \boxed{w_0 \sum_{n=1}^{N} x_n + w_1 \sum_{n=1}^{N} x_n^2 = \sum_{n=1}^{N} t_n x_n}$$

$\Rightarrow$ training $\Longleftrightarrow$ solving a system of 2 linear equations in 2 variables.

$$\underbrace{\begin{bmatrix} N & \sum_{n=1}^{N} x_n \\ \sum_{n=1}^{N} x_n & \sum_{n=1}^{N} x_n^2 \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} w_0 \\ w_1 \end{bmatrix}}_{w} = \underbrace{\begin{bmatrix} \sum_{n=1}^{N} t_n \\ \sum_{n=1}^{N} t_n x_n \end{bmatrix}}_{T}$$

A and T are known, solve for w in $Aw = T$

$$\Rightarrow \boxed{w = A^{-1}T}$$