Optimal synthesis of six-bar function generators

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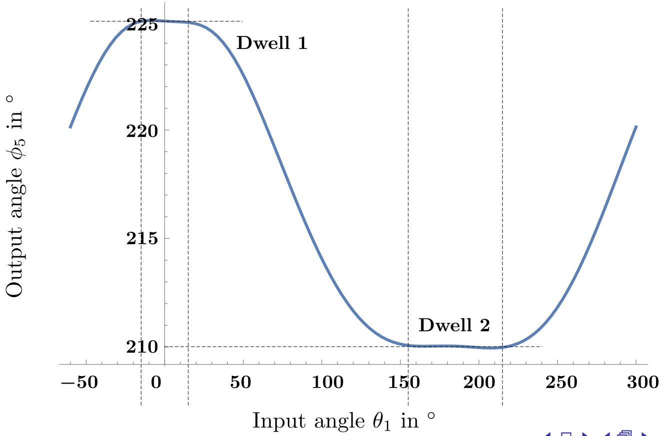
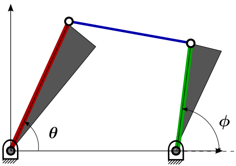
Objective

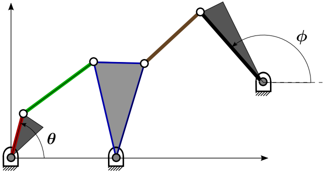
Generating complicated output motions, using simple linkages.



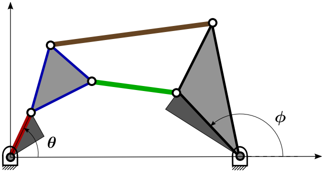
Candidate mechanisms (up to six-bar)



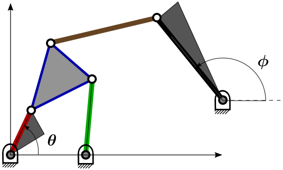
(a) Four-bar: 5 design variables



(b) Watt-II: 9 design variables



(c) Stephenson-II: 12 design variables



(d) Stephenson-III: 11 design variables

Difficulties in using six-bars for function generation

- Specialised computational tools and efforts are required (Stephenson-III: $4^{10}2^8 \approx 268$ million solutions, Watt-II: $4^8 2^4 \approx 1$ million solutions) (Plecnik *et al.*, 2015) computed in 311 hours, using 256 CPU cores
- Infeasible solutions are filtered out later, via simulation-based checks, as Grashof-like mobility conditions do not exist
- Objectives can only be met exactly at a small number of points, in a finite interval

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Present work: a new kinematic formulation of the problem

New contributions:

- Definition of *dual-order* structural error
- Development of mobility conditions
- Elimination of branch-errors at the formulation stage

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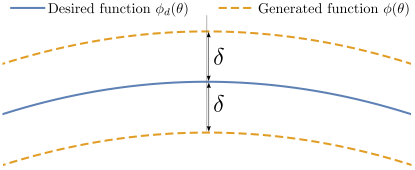
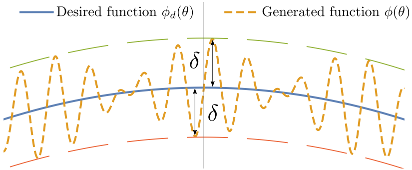
Main results: dual-order structural error





- Zeroth order:

$$\mathcal{E}_0(\theta) = \phi(\theta) - \phi_d(\theta)$$

- First order:

$$\mathcal{E}_1(\theta) = \frac{d\mathcal{E}_0(\theta)}{d\theta} = \frac{d\phi(\theta)}{d\theta} - \frac{d\phi_d(\theta)}{d\theta}$$

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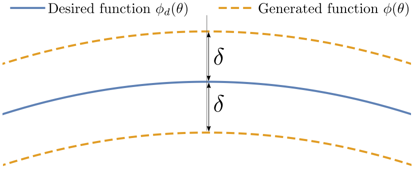

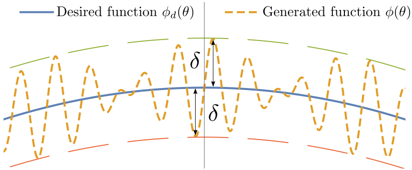

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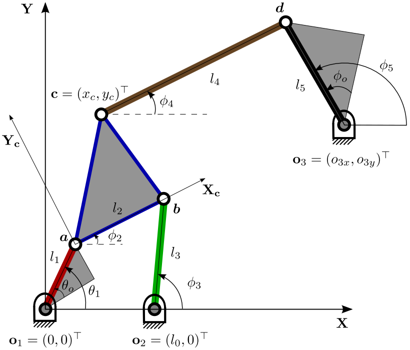
Stephenson-III mechanism



- Input variable: θ_1 (i.e., $\theta_1 - \theta_0$)
- Output variable: ϕ_5 (i.e., $(\phi_5 - \phi_0)$)
- Number of branches: 4
- Number of design variables: 11

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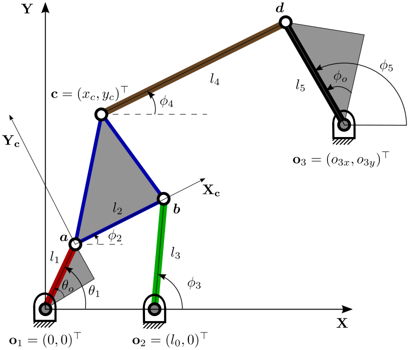

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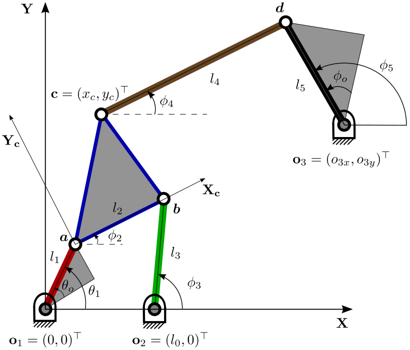

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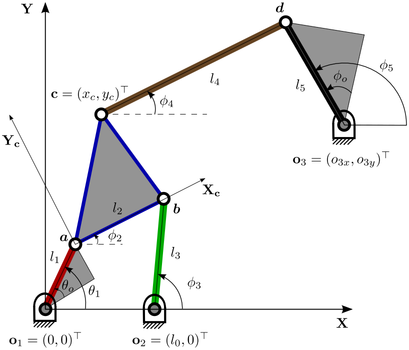

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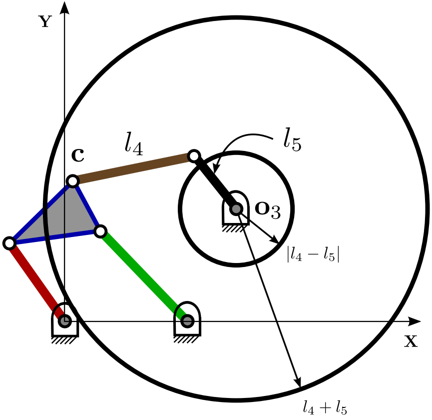
Mobility conditions: feasibility/Assembly criteria



- Feasibility criteria obtained from the RR chain, l_4-l_5:

$$|l_4 - l_5| \leq \overline{\boldsymbol{CO_3}} \leq l_4 + l_5$$

- Mathematical conditions obtained in terms of the design variables *alone*

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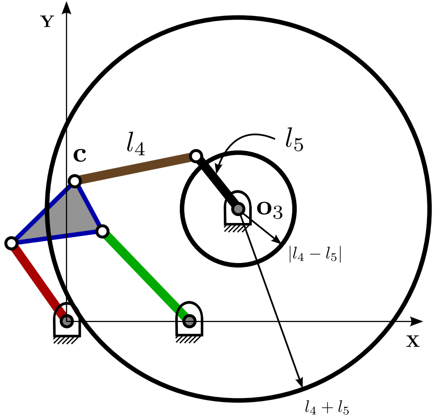

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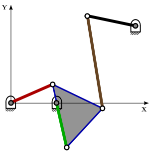
Mobility conditions: criteria for singularity-free motion



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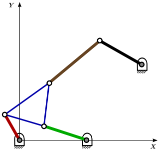
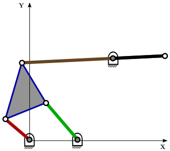
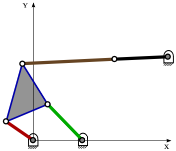


(b) $\phi_3 = \phi_2 - \pi$



(c) $\phi_5 = \phi_4$



(d) $\phi_5 = \phi_4 - \pi$

- Singularity conditions obtained from the rank degeneracy of the Jacobian of the constraint equations, $\boldsymbol{\eta}$ w.r.t. the passive variables $\boldsymbol{\phi}$, $\mathbf{J}_{\boldsymbol{\eta\phi}} = \frac{\partial \boldsymbol{\eta}(\theta_1, \boldsymbol{\phi})}{\partial \boldsymbol{\phi}}$
- Singularity condition:

$$\det(\mathbf{J}_{\boldsymbol{\eta\phi}}) = 0$$

$$\Rightarrow \sin(\phi_2 - \phi_3)\sin(\phi_4 - \phi_5) = 0$$

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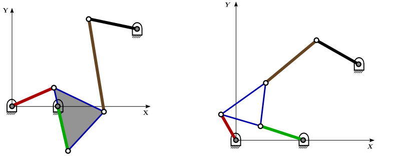

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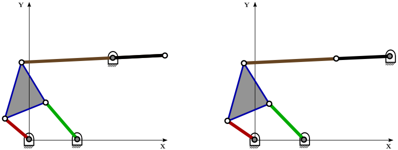

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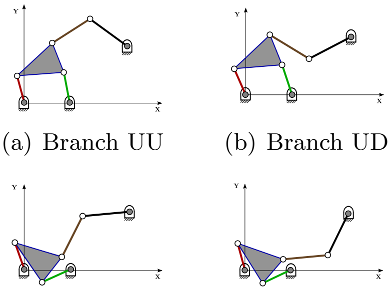
Mobility conditions: Identification of branches



(a) Branch UU (b) Branch UD

(c) Branch DU (d) Branch DD

- Identification of branches through singularity function:
  $s_1 = \sin(\phi_2 - \phi_3)$
  $s_2 = \sin(\phi_4 - \phi_5)$
- Branch identities:
  a) UU: $s_1 < 0$ and $s_2 < 0$
  b) UD: $s_1 < 0$ and $s_2 > 0$
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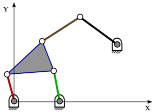
Mobility conditions: Identification of branches



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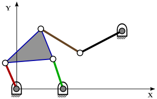

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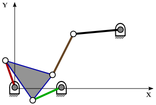

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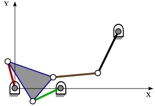

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Formulation of the optimisation problem

$$\text{Minimise} \quad F_1 \triangleq \frac{1}{N}\sum_{j=1}^{N} \mathcal{E}_0^2(\theta_{1j}),$$

$$F_2 \triangleq \frac{1}{N}\sum_{j=1}^{N} \mathcal{E}_1^2(\theta_{1j}), \quad \text{where, } \theta_{1j} \in [\theta_{1i}, \theta_{1f}];$$

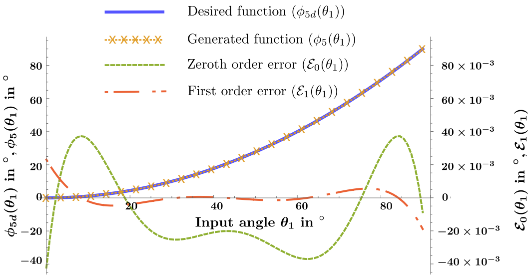
$$\text{subject to} \quad G_{\mathcal{S}_p}(\boldsymbol{x}) \triangleq \mathcal{S}_p > 0,$$

$$G_{\boldsymbol{\mathcal{F}}_q}(\boldsymbol{x}) \triangleq \begin{cases} \mathcal{F}_{qa} > 0, \\ \mathcal{F}_{qb} > 0, \end{cases}$$

$$\text{where, } p = 1, \ldots, 4, \quad q = 1, \ldots, 6,$$

$$x_l \in [a_l, b_l], \quad l = 1, \ldots, 9.$$

Stephenson-III: parabola function



- $\phi_{5d} = \theta_1/90$ $\forall\theta_1 \in [0°, 90°]$
- RMS($\mathcal{E}_0$): 0.026°
- RMS($\mathcal{E}_1$): 0.005
- Sample size, $N = 400$

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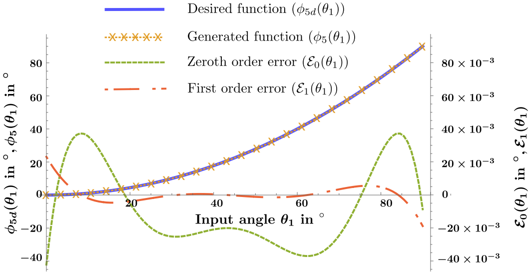

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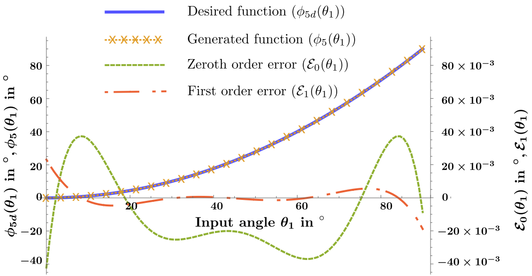

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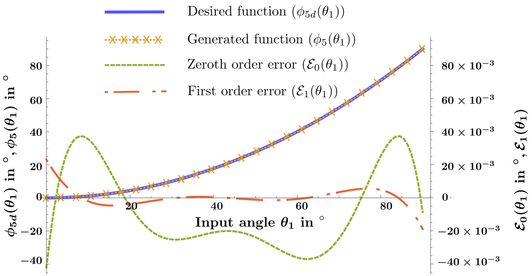

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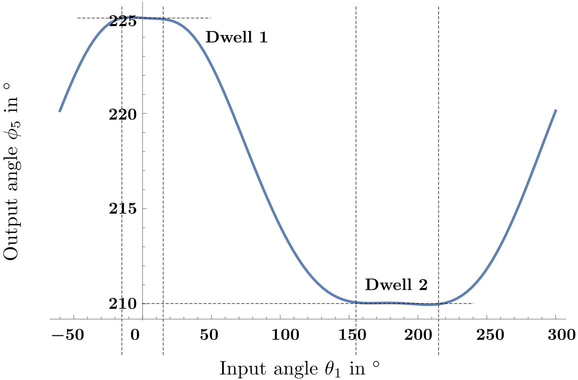
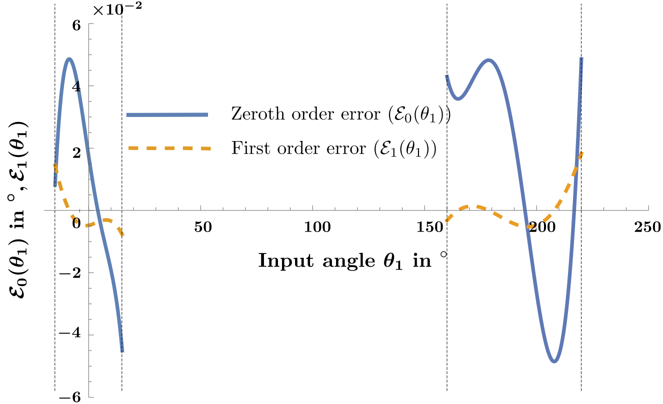
Stephenson-III: double dwell function



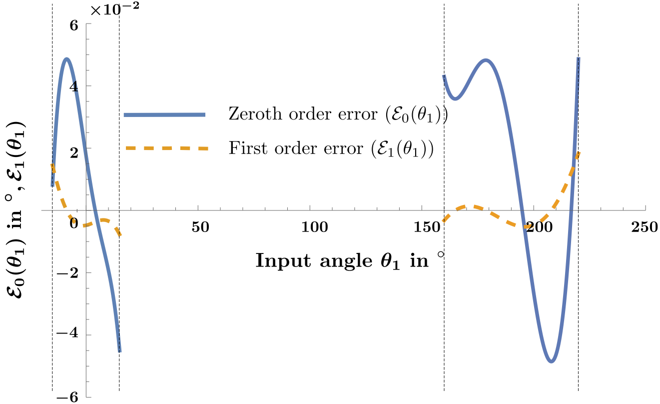
$$\phi_5 = \begin{cases} 225^\circ, & \theta_1 \in [-15^\circ, 15^\circ] \\ 210^\circ, & \theta_1 \in [160^\circ, 220^\circ] \end{cases}$$

Stephenson-III: double dwell function (contd.)



- RMS($\mathcal{E}_0$): 0.039°
- RMS($\mathcal{E}_1$): 0.005

Stephenson-III: double dwell function (contd.)



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Summary of results

Table : Results and comparison with (Plecnik *et al.*, 2014)[3] for parabolic function

Present work		**From [3]**	
$\max \lvert \mathcal{E}_0(\theta_1) \rvert$	0.042°	$\max \lvert \mathcal{E}_0(\theta_1) \rvert$	0.024°
$\mathrm{RMS}\,(\mathcal{E}_0(\theta_1))$	0.025°		
$\max \lvert \mathcal{E}_1(\theta_1) \rvert$	0.023		
$\mathrm{RMS}\,(\mathcal{E}_1(\theta_1))$	0.005		

Summary of results

Table : Results and comparison with (Shiakolas *et al.*, 2005)[13] and (Jagannath *et al.*, 2009)[14] for double dwell function generation

Dwell Period	**Error**	**Present**	**[13]**	**[a] [14]**
$\theta_1 \in [-15°, 15°]$, $\phi_{51} = 225°$	$\max \lvert\mathcal{E}_0(\theta_1)\rvert$(in °)	0.048	0.556	0.044
	$\mathrm{RMS}\,(\mathcal{E}_0(\theta_1))$(in °)	0.030	0.274	-
	$\max\,(\lvert\mathcal{E}_1(\theta_1)\rvert)$	0.014	0.053	0.014
	$\mathrm{RMS}\,(\mathcal{E}_1(\theta_1))$	0.005	0.040	0.005
$\theta_1 \in [160°, 220°]$, $\phi_{52} = 210°$	$\max \lvert\mathcal{E}_0(\theta_1)\rvert$(in °)	0.049	0.254	0.085
	$\mathrm{RMS}\,(\mathcal{E}_0(\theta_1))$(in °)	0.039	0.102	-
	$\max\,(\lvert\mathcal{E}_1(\theta_1)\rvert)$	0.018	0.031	0.006
	$\mathrm{RMS}\,(\mathcal{E}_1(\theta_1))$	0.005	0.012	0.002

[a] In [14] the locations of the dwells were not specified.

Discussions: advantages

- Mobility criteria based on the design variables alone, and deterministic in nature
- Computational time ≈ 12 minutes, scanning all the four branches, on a Intel core i7-4770 CPU running at 3.40 GHz with 8 GB RAM
- Dual-error formulation leads to accurate function generation, with smaller fluctuations in the desired speed
- No specialised computational tools required – general-purpose GA-based optimiser, NSGA-II, has been used in this work, for example

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Discussions: disadvantages/limitations

- Function generation may not need the crank to rotate through a full circle
- Function generation may require mobility of a particular branch pair only

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Thank you for your attention!

Questions/comments?

References

- [3] M. M. Plecnik and J. M. McCarthy, Numerical synthesis of six-bar linkages for mechanical computation, *Journal of Mechanisms and Robotics*, vol. 6, no. 3, p. 031012, 2014.
- [13] P. Shiakolas, D. Koladiya, and J. Kebrle, On the optimum synthe- sis of six-bar linkages using differential evolution and the geometric centroid of precision positions technique, *Mechanism and Machine Theory*, vol. 40, no. 3, pp. 319335, 2005.
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