Multi-Robot Collision Avoidance under Uncertainty with Probabilistic Safety Barrier Certificates

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Motivation

Increasing popularity of autonomous systems in real-world applications demands for explicit and formally provable design to ensure safety under uncertainties such as *measurement error* and *motion disturbances*.

Contribution: we proposed the **Probabilistic Safety Barrier Certificates (PrSBC)** using control barrier functions ***under uncertainty*** to define the *admissible* action space that produces ***probabilistic safe*** robot controller with ***formally provable guarantee***. Given any nominal control policy, the method entails minimally modifying the policy to determine *alternative safe controllers* that live in the safety-certified action space by PrSBC constraints.

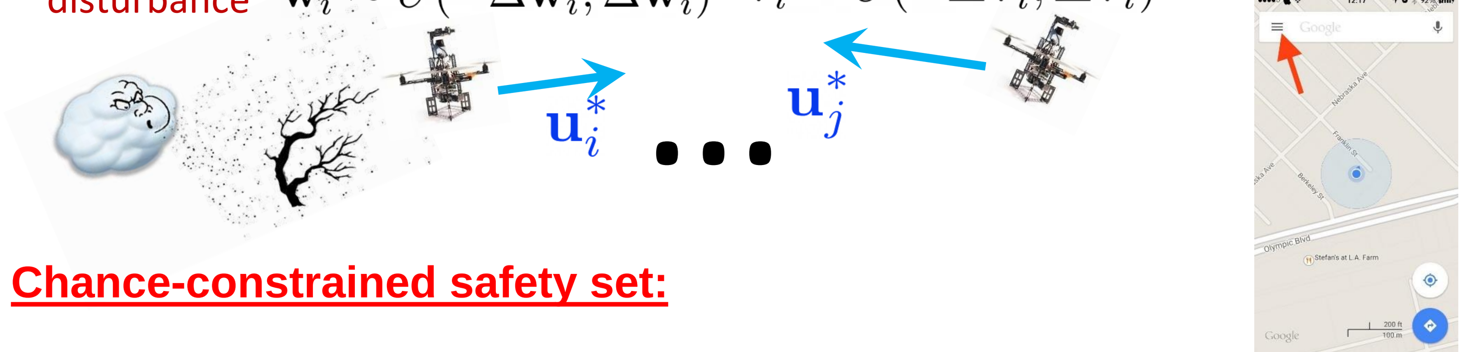
- No assumptions about the uncertainty model are required other than finite support, which also enables *worst-case guarantee*. Unbounded uncertainty model can also be adapted to PrSBC framework with safety guarantee
- *Formally provable* probabilistic safety of PrSBC throughout the continuous time scale
- Experimental results with simulated aerial robots in AirSim using *centralized* and *decentralized* PrSBC computation
- Future work in integrating dynamical system learning with PrSBC safety constraints to guarantee safe learning to control

Problem Formulation

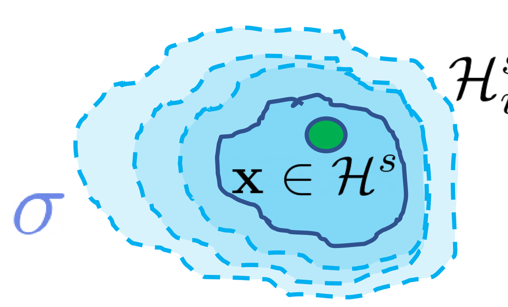
System dynamics:

$\dot{\mathbf{x}}_i = f_i(\mathbf{x}_i, \mathbf{u}_i) + \mathbf{w}_i$, $\hat{\mathbf{x}}_i = \mathbf{x}_i + \mathbf{v}_i$

Motion disturbance $\mathbf{w}_i \sim U(-\Delta\mathbf{w}_i, \Delta\mathbf{w}_i)$; Noisy localization $\mathbf{v}_i \sim U(-\Delta\mathbf{v}_i, \Delta\mathbf{v}_i)$



Chance-constrained safety set:



$$\mathcal{H}^s_{i,j}: \quad h^s_{i,j}(\mathbf{x}) = ||x_i - x_j||^2 - R_s^2 \geq 0$$

$$\Pr(\mathbf{x}_i, \mathbf{x}_j \in \mathcal{H}^s_{i,j}) \geq \sigma, \quad \forall i > j$$

Optimization-based safe control:

$$\mathbf{u}_i^*(\mathbf{x}) \sim \pi_\theta^* \longrightarrow \min_{\mathbf{u}} \Sigma_{i=1}^N ||\mathbf{u}_i - \mathbf{u}_i^*||^2$$

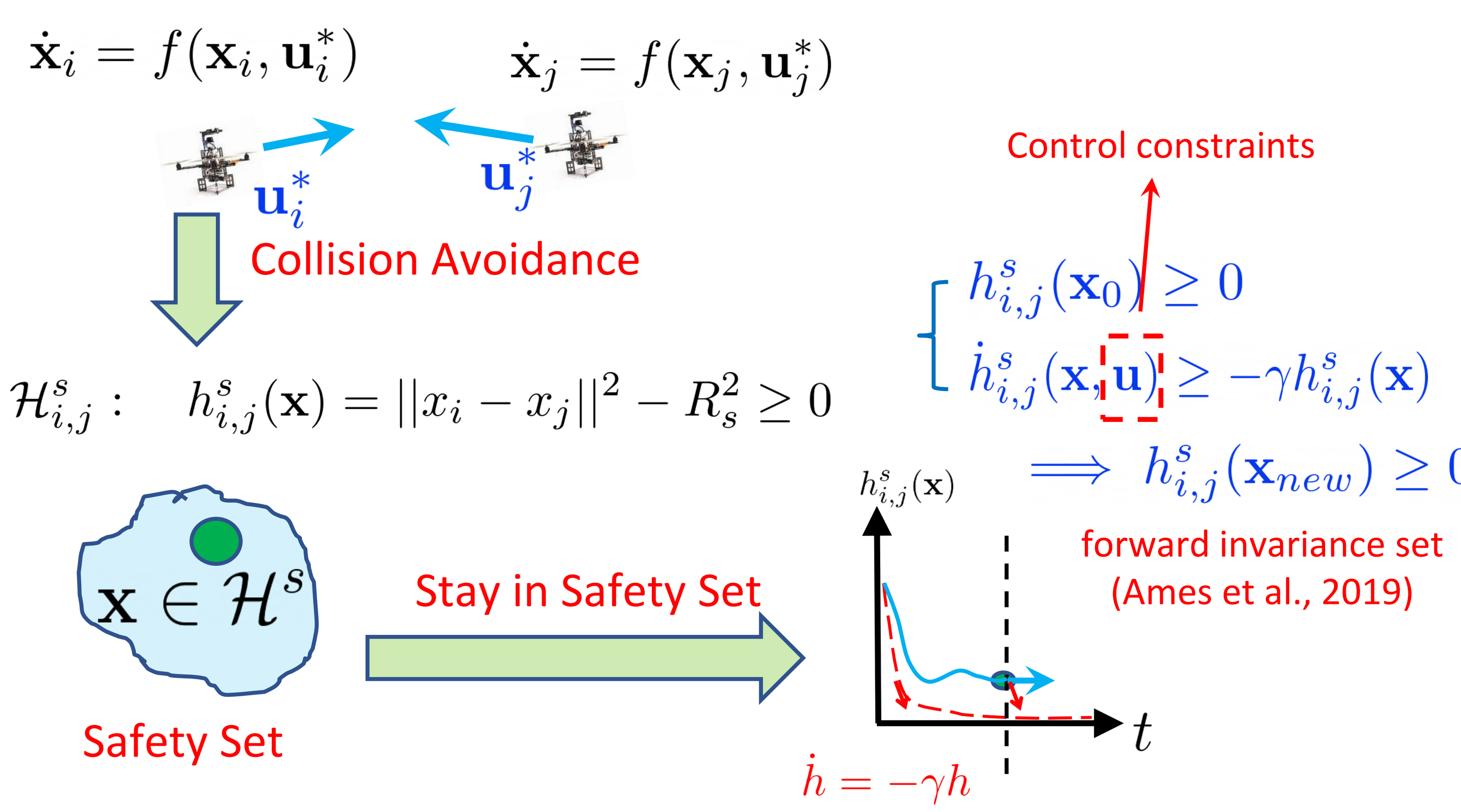
$$s.t. \ \Pr(\mathbf{x}_i, \mathbf{x}_j \in \mathcal{H}^s_{i,j}) \geq \sigma, \quad \forall i > j$$

Nominal robot control policy — Chance-constrained safety constraints

Given a nominal control policy, how to minimally modify it for provable safety under measurement and motion uncertainty?

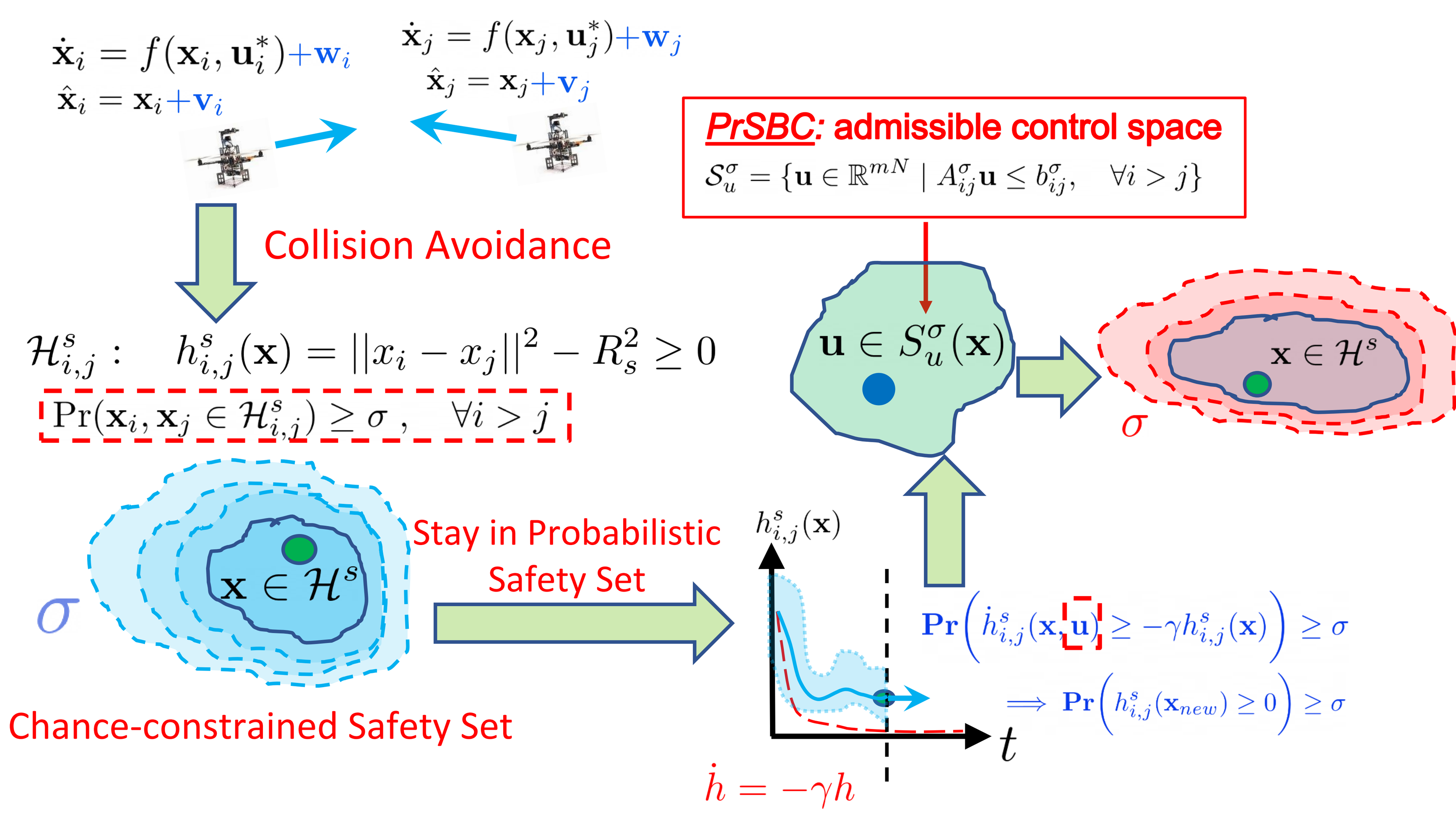
Approach

Set forward invariance using control barrier function (deterministic):



Robots stay in the safety set *H* as long as 1) they are initially safe, and 2) control constraints hold true at all times (Safety Barrier Certificates) [Wang et al., 2017]

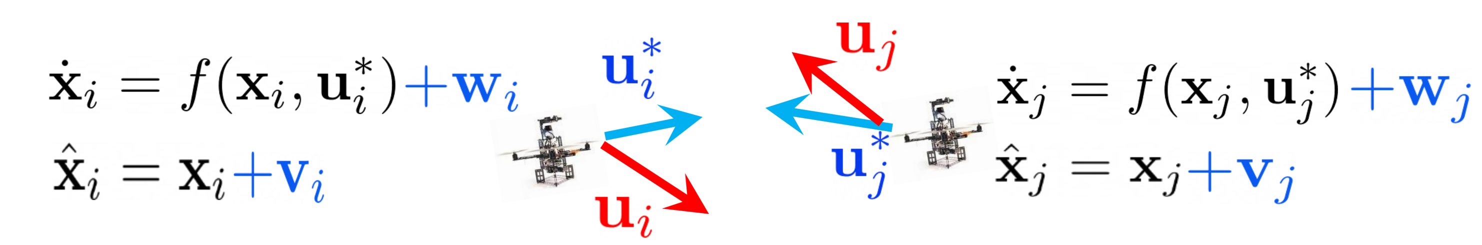
Define PrSBC constraints for forward invariant probabilistic safety:



Robots stay in the *chance-constrained safety set* as long as 1) they are initially safe, and 2) *PrSBC control constraints* hold true at all times

Selecting actions from PrSBC control space will ensure probabilistic safety!

Optimization-based Safe Controller



$$\mathbf{u}_i^*(\mathbf{x}) \sim \pi_\theta^* \longrightarrow \min_{\mathbf{u}} \Sigma_{i=1}^N ||\mathbf{u}_i - \mathbf{u}_i^*||^2 \longrightarrow \mathbf{u}_i(\mathbf{x}) \sim \pi_{safe}$$

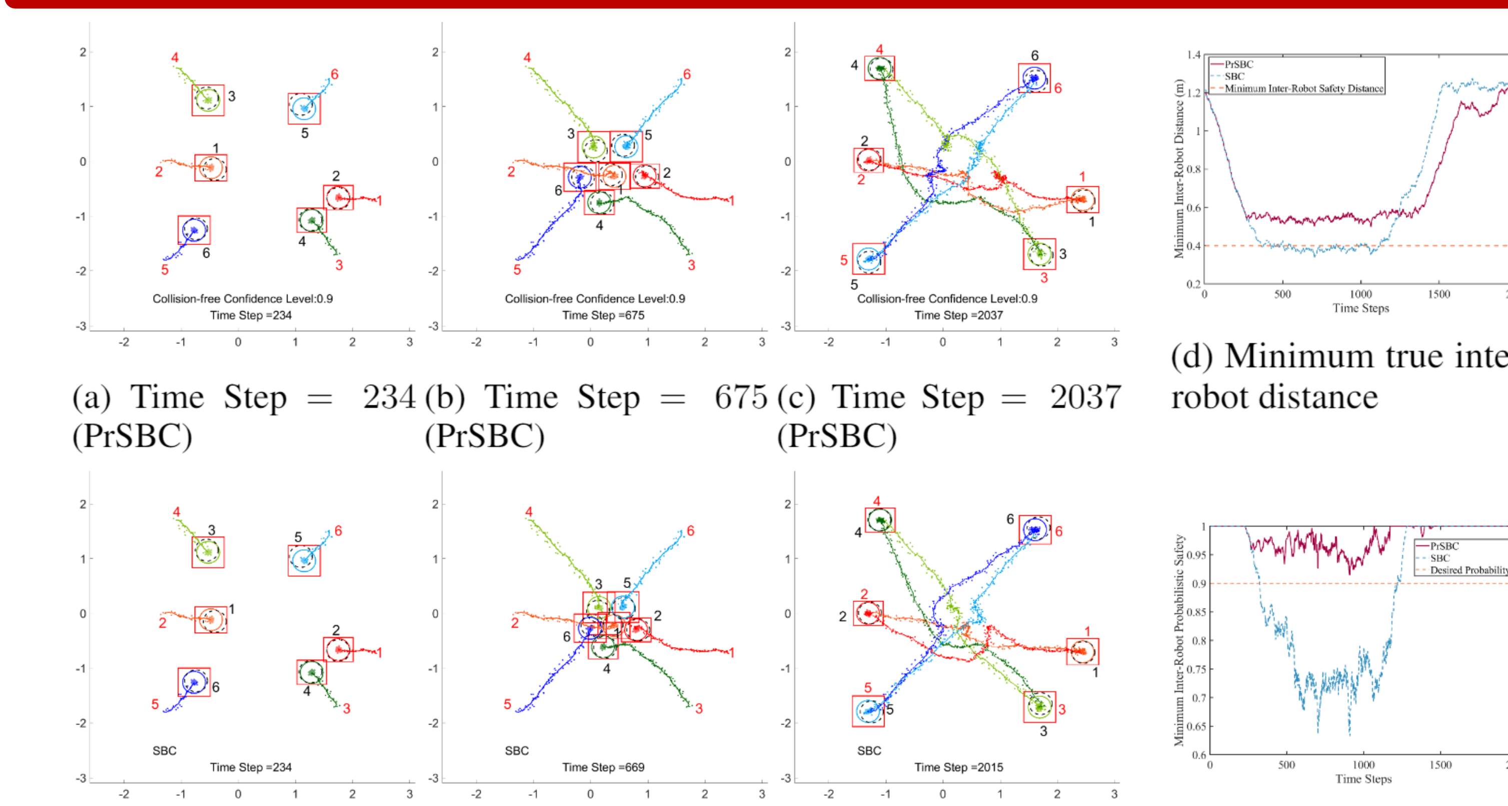
$$s.t. \quad \mathbf{u} \in S_u^\sigma(\mathbf{x})$$

Nominal robot control policy — PrSBC Control Constraints

$$\Pr(\mathbf{x}_i, \mathbf{x}_j \in \mathcal{H}^s_{i,j}) \geq \sigma, \quad \forall i > j$$

Guaranteed probabilistic safety

Results

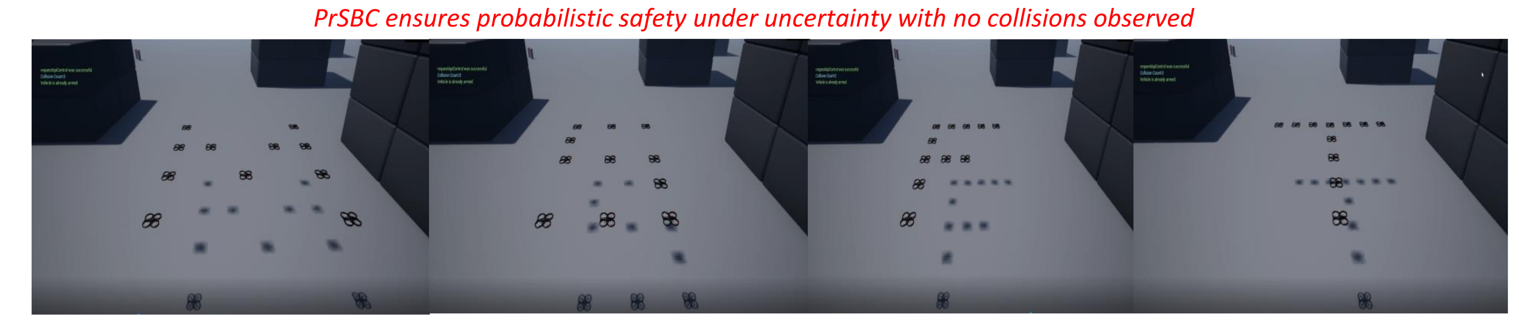


(a) Time Step = 234 (PrSBC) (b) Time Step = 675 (PrSBC) (c) Time Step = 2037 (PrSBC) (d) Minimum true inter-robot distance

(e) Time Step = 234 (SBC) (f) Time Step = 669 (SBC) (g) Time Step = 2015 (SBC) (h) Minimum inter-robot probabilistic safety

6 robots move to swap positions while ensuring probabilistic safety (PrSBC v.s. SBC)

PrSBC ensures probabilistic safety under uncertainty with no collisions observed



Experiment results on Microsoft AirSim (safely move to form M-S-F-T formation)

References

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