

Rules you are allowed to use in formal proofs

Logical equivalences

1. $\neg(\neg P) \Leftrightarrow P$ (*Double Negation*)
2. $P \vee Q \Leftrightarrow Q \vee P$ (*Commutative Law*)
3. $P \wedge Q \Leftrightarrow Q \wedge P$ (*Commutative Law*)
4. $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$ (*Associative Law*)
5. $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$ (*Associative Law*)
6. $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ (*Distributive Law*)
7. $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ (*Distributive Law*)
8. $P \rightarrow Q \Leftrightarrow \neg P \vee Q$
9. $P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$ (*Contrapositive*)
10. $P \leftrightarrow Q \Leftrightarrow Q \leftrightarrow P$
11. $P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$
12. $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ (*De Morgan's Law*)
13. $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$ (*De Morgan's Law*)
14. $\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$
15. $\neg(P \leftrightarrow Q) \Leftrightarrow (P \wedge \neg Q) \vee (Q \wedge \neg P)$

Logical implications

1. $(P \rightarrow Q) \wedge P \Rightarrow Q$ (*Modus Ponens*)
2. $(P \rightarrow Q) \wedge \neg Q \Rightarrow \neg P$ (*Modus Tollens*)
3. $P \wedge Q \Rightarrow P$ (*Simplification*)
4. $P \Rightarrow P \vee Q$ (*Addition*)
5. $(P \vee Q) \wedge \neg P \Rightarrow Q$ (*Modus Tollendo Ponens*)
6. $P \leftrightarrow Q \Rightarrow P \rightarrow Q$ (*Biconditional-Conditional*)
7. $(P \rightarrow Q) \wedge (Q \rightarrow P) \Rightarrow P \leftrightarrow Q$ (*Conditional-Biconditional*)
8. $(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow P \rightarrow R$ (*Hypothetical Syllogism*)
9. $(P \rightarrow Q) \wedge (R \rightarrow S) \Rightarrow (P \vee R) \rightarrow (Q \vee S)$ (*Constructive Dilemma*)

Given P and Q you may also write $P \wedge Q$ (*Adjunction*).