Sample Final exam questions (mandatory part)

The actual final exam will have at most 9 mandatory questions and 5 optional questions. The optional questions will be similar to the ones on the (sample) tests, and need to be answered only if you do not want me to re-use your average test score. The list of questions below is supposed to help you prepare for the mandatory part of the final.

1. Write up the multiplication table of $\mathbb{Z}_7$.
2. Provide a formula for the number of bases in an n-dimensional vector space over $GF[q]$ and *prove* its validity.
3. What is the number of 3-dimensional subspaces of a 7-dimensional space over the field $\mathbb{Z}_5 = GF[5]$?
4. Identify each subset of a set S with its characteristic vector over $\mathbb{Z}_2$. What set-operation corresponds to taking the sum of two characteristic vectors? What vector operation corresponds to taking the intersection of two sets?
5. List the elements of the cycle subspace and of the cutset subspace for the graph shown in Fig. 5.6 (page 63) of our textbook.
6. Assume that $C_1 = [A_1, B_1]$ and $C_2 = [A_2, B_2]$ are cuts. Prove that their symmetric difference is a disjoint union of cuts. (Draw a picture.) Use this observation to describe the elements of the cutset subspace.
7. Describe the elements of the cycle subspace of a graph.
8. Prove that a graph is a disjoint union of cycles if and only if the degree of every vertex is even. How does this observation help to prove that the cycle subspace is closed under vector addition?
9. In the proof of the description of the cycle space and of the cutspace we never showed closure under scalar multiplication. Why? How can you describe subspaces of a vector space over the field $\mathbb{Z}_2$?
10. Explain how a spanning tree may be used to find a basis of the cycle and cutset spaces. Illustrate the method by listing the fundamental cycles and cuts for the graph shown below:

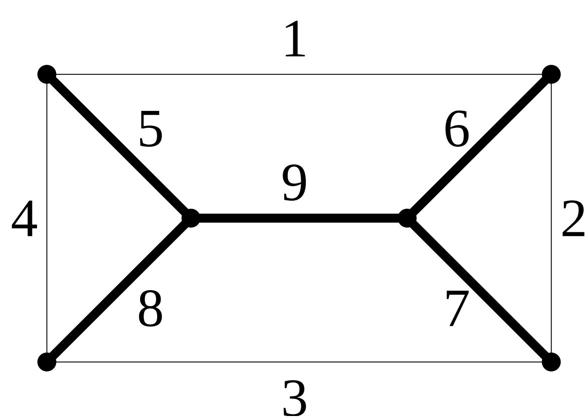

Figure 1: Graph with selected spanning tree

11. Based on the answer to the previous question, what can you say about the dimension of the cycle space and the cutset space? Are these spaces complements of each other?

12. Prove that every cycle has an even number of edges in common with every cutset. What orthogonality relation follows from this observation?

13. Prove that there is a number $R(p, q)$ such that any painting of the edges of $K_{R(p,q)}$ with two colors contains either a monochromatic K_p of the first color, or a monocohrmatic K_q of the second color. Use your proof to estimate the Ramsey number $R(4, 3)$.

14. Find the actual value of $R(4, 3)$

15. Using the formula for $R(T, K_n)$ (where T is any tree), find $R(P_3, K_4)$ and $R(K_{1,s}, K_3)$.

16. Assume you color the edges of an infinite complete graph with finitely many colors. What can you say about the size of the largest monochromatic complete subgraph?

17. According to Turán's theorem, how can you describe a graph on n vertices that contains no K_{r+1} and has the maximum number of edges? What is the largest number of edges you can have in a graph with 11 vertices, containing no K_3?

Good Luck. Gábor Hetyei