Name: **Student ID:**

Sample Test 2

The actual test will have only 5 questions.

1. Prove that the vertex chromatic number of a graph is less than equal to the maximum degree plus one.
2. Five colors are necessary to properly color the vertices of a graph G, but four colors suffice to color any proper subgraph of it. What can you say about $\delta(G)$ (=the minimum degree)? Prove your estimate.
3. The graph G is not complete and does not contain an odd cycle. According to Brooks' theorem, what is the sharpest upper bound for $\chi(G)$, in terms of $\Delta(G)$?
4. Prove that the chromatic polynomial of a tree is $x(x-1)^{v-1}$, where v is the number of vertices.
5. State and prove the recursion formula for the chromatic polynomial of a graph (in terms of deleting and contracting a select edge).
6. Using the formula mentioned in the previous question, find the chromatic polynomial of the graphs shown in Fig. 7.4. on page 96 of our textbook.
7. Prove by induction the formula for the chromatic polynomial of a cycle, stated in exercise 7.3.5. (I will provide the formula if I ask this.)
8. State the Vizing-Gupta theorem on the edge chromatic number of a graph.
9. What is the edge-chromatic number of a d-dimensional hypercube?
10. What is the edge-chromatic number of a tree?
11. Prove that $K_{3,3}$ is not planar (without referring to Kuratowski's theorem).
12. Name the two graphs to which any other non-planar graph may be reduced by deleting vertices and edges and by contracting out vertices of degree two. (Kuratowski's theorem.)
13. State and prove Euler's formula for planar graphs.
14. Find the dual of the planar graph shown in the picture.

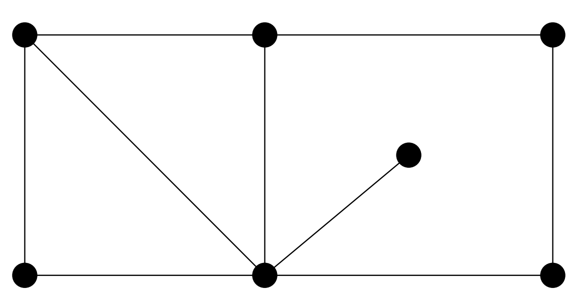
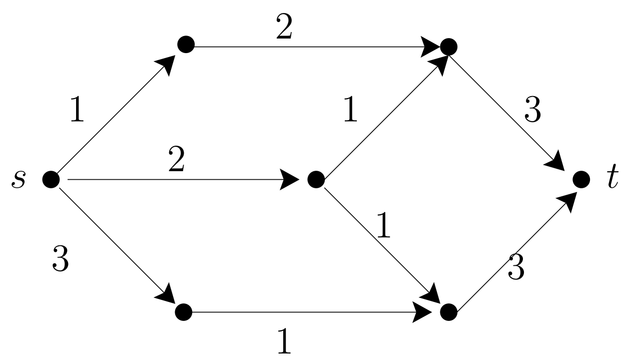


Figure 1: A planar graph

15. Prove that every planar graph can be colored using at most five colors. Is this the sharpest known upper bound?
16. What assumption do you have to make about a planar map for the four-color theorem to be applicable?

17. In a planar graph every face has at least five sides. Prove that this graph satisfies $3f \leq 2v - 4$. (Here f is the number of faces and v is the number of vertices.)

18. Find a minimum cut and a maximum flow for the graph shown below. **Show all your work!**



Good Luck. Gábor Hetyei