Study Guide for Test 1.

The real test will have less questions and you will have about 80 minutes to answer them. The usage of books or notes, or communicating with other students will not be allowed. You will have to give the simplest possible answer and show all your work. Below you find sample questions and indications which theorems and proofs you will have to remember from the book. Review also all past homework questions as questions similar to them might appear on the test.

1. Define prime, composite, even and odd numbers.
2. Define the relation "x divides y".
3. Using only the notion of distance define "C is between A and B". (A, B, and C are points in the plane.)
4. Transform the statement "all rabbits run fast" into an "if-then" statement.
5. Explain why a mathematician believes that "the present king of France is bald". (Hint: think of vacuous truths".)
6. Prove that if a is a multiple of 2 and b is a multiple of 4 then $a + b$ is even. Is the converse also true?
7. Find a counterexample to the statement "if the sum of two integers is even, then both integers are even".
8. Use a truth table to verify that $(x \vee y) \wedge z$ is equivalent to $(x \wedge z) \vee (y \wedge z)$.
9. Give an example of a tautology, and of a contradiction.
10. Express the exclusive or relation using $\wedge$, $\vee$ and negation.
11. Express $A \rightarrow B$ using $\wedge$, $\vee$ and negation.
12. State the multiplication principle.
13. Find the number of lists of length k whose elements are chosen from a pool of n possible elements, if repetitions are permitted. What if they are not?
14. Express $\prod_{k=3}^{100}(2k + 1)$ using factorials.
15. How many padlock combinations of length 5 are there? (There are 10 numbers on a padlock and no number in the code can be equal or adjacent to the previous letter.)
16. Which of 2 and $\{2\}$ is an element of $\{1, 2, 3\}$ and which is a subset?
17. If A has 10 elements, how many elements are there in the powerset of A?

18. Which of the following is true for integers $\forall x \exists y (x = y)$ or $\exists x \forall y (x = y)$? Prove the true statement, give a counterexample to the false one.

19. Negate the sentence $\forall x \exists y \exists z (xy = z)$.

20. If $\forall x \exists y P(x, y)$ is true, can we conclude that $\exists x \exists y P(x, y)$ is also true? Why?

21. How do you define the equality of two sets, and why does this definition imply that there is only one empty set?

22. Let $A = \{x \in \mathbb{Z} : \exists y \in \mathbb{Z}(x = y^2)\}$ and let $B = \{x \in \mathbb{Z} : x \geq 0\}$. Which set is a subset of the other? Prove your statement.

23. Prove that $|A \cup B| = |A| + |B| - |A \cap B|$ and explain how this implies the addition principle.

24. Find the number of multiples of 4 or 14 between 1 and $10,000$.

25. Let $A = \{x \in \mathbb{Z} : 4|x\}$ and $B = \{x \in \mathbb{Z} : 6|x\}$. Describe the symmetric difference $A \triangle B$.

26. Under which circumstances is the Cartesian product $A \times B$ equal to $B \times A$? Write an if-and-only-if statement and prove it.

Good luck. Gábor Hetyei