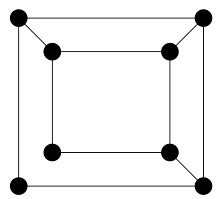
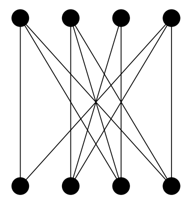
Sample Test I.

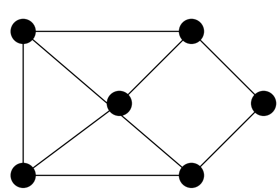
The real test will have less questions and you will have about 75 minutes to answer them. The usage of books or notes, or communicating with other students will not be allowed. You will have to give the simplest possible answer and show all your work. Besides looking at the sample questions below, also review all homework exercises, as questions similar to them might occur on the test.

1. Are the two graphs shown in the picture isomorphic? Justify your claim.





2. State and prove the degree-sum formula.
3. Explain how the degree-sum formula may be used to prove that a “mountain climber puzzle” always has a solution.
4. What is the number of edges in the n-dimensional hypercube?
5. What is the number of edges in the complete bipartite graph $K_{m,n}$?
6. Give a necessary and sufficient condition for a graph to be bipartite and outline the proof of your claim.
7. State and outline the proof of Euler’s formula for connected planar graphs.
8. A connected planar graph has 6 vertices and 3 regions. What is the number of its edges?
9. Prove that in a planar graph the number e of edges does not exceed $3v - 6$, where v is the number of vertices.
10. Use the result in the previous question to show that K_5 is not planar.
11. Use the circle-chord method to show that $K_{3,3}$ is not planar.
12. State Kuratowski’s theorem and be able to apply it to examples such as in Exercise 1.4/3.
13. At least how many times do you need to lift your hand to draw the graph shown below? Justify your answer!



14. Give a necessary and sufficient condition for the existence of an Eulerian cycle in a graph. Outline the proof.

15. Give a necessary and sufficient condition for the existence of an Eulerian trail but no Eulerian cycle in a graph. Explain how this statement may be derived from the previous one.

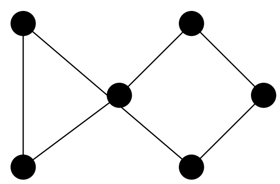
16. For which values of n does the complete graph K_n have an Eulerian cycle?

17. Grinberg's theorem states that a planar graph that has a Hamilton circuit, satisfies the formula:

$$\sum_i (i-2)(r_i - r_i') = 0.$$

Here r_i resp. r_i' is the number of regions that have i sides and lay inside resp. outside the Hamilton circuit.

Use Grinberg's theorem to show that the graph shown below has no Hamilton circuit.



18. Now use the three simple rules about building a Hamilton circuit to show that the graph in the previous question has no Hamilton circuit.

19. Prove that every tournament has a Hamilton path.

20. Define a Gray code on binary strings and write a Gray code of all 8 binary strings of length 3. Explain how this is equivalent to finding a Hamilton circuit in a graph.

Good luck. Gábor Hetyei