Study guide for test 1

The test may have less questions and you will have about 75 minutes to answer them. You will have to give the simplest possible answer and show all your work. The questions below are sample questions. Besides trying to answer these questions, make sure you also review all homework exercises. The test may also have questions similar to those exercises. During the test, the usage of books or notes, or communicating with other students will not be allowed.

1. Suppose you are looking for the intersection of two lines by solving a system of linear equations. You find that the system is inconsistent. What can you say about the two lines?

2. Find the intersection of the two lines given in Exercise 1.1/3, by solving the appropriate system of linear equations.

3. Write the coefficient matrix and the augmented matrix for the linear system in Exercise 1.1/13.

4. List the row operations you may use to reduce an augmented matrix into echelon form.

5. For what value(s) of h is the following matrix the augmented matrix of a consistent linear system? If the system is consistent, is the solution unique?
$$\begin{pmatrix} 1 & 0 & 1 & 3 \\ 2 & 0 & h & 10 \end{pmatrix}$$

6. In Exercise 1.2/1 describe which matrices are only in echelon form and which are in reduced echelon form.

7. Suppose you have an augmented matrix of a linear system in reduced echelon form. Describe in terms of the pivot columns which are the free variables, when is the system consistent, and when is the solution unique.

8. Find the general solution of the linear system, whose augmented matrix is given in Exercise 1.1/13.

9. After setting up and solving the appropriate linear system, write the vector $\begin{bmatrix} -1 \\ 6 \\ -2 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$ or explain why it is not possible.

10. After setting up and solving the appropriate linear system, write the vector $\begin{bmatrix} -1 \\ 8 \\ -2 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$ or explain why it is not possible.

11. Restate the question in Exercise 1.4/13 in the language of the previous two questions.

12. Using Theorem 4 in Section 1.4 (you will have to memorize this), solve Exercises 1.4/21 and 1.4/22.

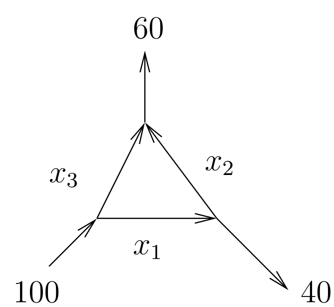
13. Write the solution of the homogeneous system in Exercise 1.5/5 in parametric vector form.

14. Write the solution of the system in Exercise 1.5/15 in parametric vector form and explain how it is related to the solution of the corresponding homogeneous system.

15. Write up the linear system you would use to balance the chemical equation given in Exercise 1.6/9.

16. Suppose you have to balance a chemical equation and you arrive at the following solution. $x_1 = 2/3 \cdot x_3$, $x_2 = 1/6 \cdot x_3$, x_3 is free, $x_4 = 3/10 \cdot x_3$. What coefficients would you choose to have the smallest possible whole numbers?

17. Find the general flow pattern for the network shown in the figure. Assuming that the flows are all nonnegative, what is the lowest possible value for x_1?



18. Write down the matrix of the following transformations, mapping $\mathbb{R}^2$ into $\mathbb{R}^2$: counterclockwise rotation about the origin by 45 degrees, reflection about the x_1 axis, reflection about the line $x_1 = x_2$, vertical expansion by a factor of 3, horizontal contraction by a factor of 1/2, refection through the origin, horizontal shear taking the point $\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]$ into $\left[\begin{smallmatrix}3\\1\end{smallmatrix}\right]$.

19. Write down the matrix of the vertical projection of each point of $\mathbb{R}^3$ onto the plane $x_3 = 0$.

Concluding remarks: For the moment, we have skipped the economic applications in Section 1.6, linear independence (Section 1.7), and we covered selected parts of Sections 1.8 and 1.9, see the last three sample questions.

Good luck. Gábor Hetyei