## Sample Final Exam Questions

This list of sample final exam questions is subject to updates until we review for the test Last update: December 15, 2020

The final exam will be comprehensive, covering the material taught throughout the entire semester. This collection of sample final exam questions covers only what we learned after Test 3. Please also refer to the previous study guides when preparing for the final exam.

1. In a survey of a 120 students, 55 like apples, 55 like oranges and 70 like bananas. 30 students like apples and oranges, 30 students like bananas and oranges, 30 students like apples and bananas. 10 students like all three fruits. How many students are there who like only oranges? How many students don't like any of these fruits?
2. Find the probability of rolling 1 or 2 on a fair die.
3. You roll a pair of fair dice. Find the probability that the sum of the two numbers rolled is 7 .
4. Find the probability of $\bar{X}$, given $P(X)=0.2$. Here $\bar{X}$ denotes the complement of $X$.
5. Below is a partially complete probability model. Enter the probability for the final outcome.

| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |
| 0.2 | 0.2 | 0.1 |  |

6. Find $n(A \cap \bar{B})$, given $n(A)=10, n(B)=20$ and $n(A \cap B)=7$.
7. Find $n(A \cup B)$, given $n(A)=10, n(B)=20$ and $n(A \cap B)=5$.
8. State the probability of a certain event and the probability of an impossible event.
9. Find $P(A \cup B)$ if $P(A)=0.2, P(B)=0.3$ and $A$ and $B$ are disjoint events.
10. Find $P(A \cap \bar{B})$ if $P(A)=0.4, P(B)=0.2$ and $A$ and $B$ are independent events.
11. Find $P(A \cup B)$ if $P(A)=0.2, P(B)=0.3$ and $A$ and $B$ are independent events.
12. The odds for Lightning defeating Sleeping Beauty in a horse race are $3: 4$. What is the probability that Lightning will defeat Sleeping Beauty?
13. What are the odds against raining tomorrow if the probability of raining tomorrow is $60 \%$ ?
14. Find $P(A \cup B)$, given $P(A)=0.1, P(B)=0.2$ and $P(A \mid B)=0.3$.
15. Find $P(A)$, given $P(A \cap B)=0.1$ and $P(B \mid A)=0.3$.
16. Find the minimum, the maximum and the median of the sequence $5,1,3,2,4,5$.
17. Find the mean of 29 copies of 1,31 copies of 2 , and 40 copies of 3 .
18. In a course, homework accounts for $20 \%$ of the course grade, each of the two tests accounts for $25 \%$, and the final counts for $30 \%$. Joe scored $80 \%$ on the homework, $60 \%$ on the first test, $45 \%$ on the second, and $95 \%$ on the final. What is Joe's overall performance in the course?
19. Find the mean, the population variance and the standard deviation of the sequence $5,1,3,2,4,5,7,8$. Round your answers to three decimal digits.

## Solutions:

1. The answers may be found by filling out the diagram below.


First we fill in the number 10 at the intersection of the three sets. Next we find the number of students who like exactly two fruits, by subtraction. 30 students like apples and oranges, 10 like all three fruits, so $30-10=20$ students like apples and oranges but not bananas. Similarly the number of students liking only apples and bananas is 20 and the number of students liking only oranges and bananas is also 20. Next we determine the number of students liking only one fruit. 55 students like apples, from this we must subtract the numbers already filled into the circle representing the apples and we get $55-(20+20+10)=5$. Similarly, the number of students liking only oranges is $55-(20+20+10)=5$ and the number of students liking only bananas is $70-(20+20+10)=20$. The sum of all numbers in the circles is 100 , hence $120-100=20$ students don't like any of these fruits.
2. There are 6 outcomes, each equally likely, and 2 of these are favorable. The probability is $2 / 6=1 / 3$.
3. There are 6 ways to get a total of 7 on the two dice: $1+6,2+5,3+4,4+3,5+2$ and $6+1$. There are $6 \times 6$ possible outcomes, each has the same probability. The probability of rolling a total of 7 points is $6 / 36=1 / 6$.
4. $P(\bar{X})=1-P(X)=1-0.2=0.8$.
5. The sum of the probabilities of the possible outcomes must be 1. The missing probability is $1-(0.2+0.2+0.1)=0.5$.
6. $n(A \cap \bar{B})=n(A)-n(A \cap B)=10-7=3$.
7. $n(A \cap \bar{B})=n(A)+n(B)-n(A \cap B)=10+20-5=25$.
8. The probability of a certain event is 1 , the probability of an impossible event is 0 .
9. If $A$ and $B$ are disjoint events then $P(A \cap B)=0$ and we have $P(A \cup B)=P(A)+$ $P(B)-P(A \cap B)=0.2+0.3-0=0.5$.
10. If $A$ and $B$ are independent events then $P(A \cap B)=P(A) \cdot P(B)=0.4 \cdot 0.2=0.08$ and we have $P(A \cap \bar{B})=P(A)-P(A \cap B)=0.4-0.08=0.32$.
11. If $A$ and $B$ are independent events then $P(A \cap B)=P(A) \cdot P(B)=0.2 \cdot 0.3=0.06$ and we have $P(A \cup B)=P(A)+P(B)-P(A \cap B)=0.2+0.3-0.06=0.44$.
12. We know that the odds of $m$ to $n$ correspond to a probability $m /(m+n)$. We have $m=3$ and $n=4$, so the probability is $3 / 7$.
13. We know that the odds of $m$ to $n$ correspond to a probability $m /(m+n)$. The probability of raining tomorrow is $60 / 100$. If the odds for raining tomorrow are $m$ to $n$ then $m=60$ and $m+n=100$. This gives $n=40$, that is, the odds for raining tomorrow is 60 to 40. We obtain the odds against by swapping the two numbers, that is, the odds against raining tomorrow is 40 to 60 .
14. We have $P(A \mid B)=\frac{P(A \cap B)}{P(B)}$ and so $P(A \cap B)=P(A \mid B) \cdot P(B)=0.3 \cdot 0.2=0.06$. Thus $P(A \cup B)=P(A)+P(B)-P(A \cap B)=0.1+0.2-0.06=0.24$.
15. We have $P(B \mid A)=\frac{P(A \cap B)}{P(A)}$ and so

$$
P(A)=\frac{P(A \cap B)}{P(B \mid A)}=\frac{0.1}{0.3}=\frac{1}{3} .
$$

16. First we put the numbers into increasing order: $1,2,3,4,5,5$. The minimum is 1 , the maximum is 5 and the median is $(3+4) / 2=3.5$.
17. The count is $29+31+40=100$, the mean is

$$
\bar{x}=\frac{29 \cdot 1+31 \cdot 2+40 \cdot 3}{100}=\frac{211}{100}=2.11 .
$$

18. 

$$
0.2 \cdot 80 \%+0.25 \cdot 60 \%+0.25 \cdot 45 \%+0.3 \cdot 95 \%=70.75 \%
$$

19. The mean is

$$
\bar{x}=\frac{5+1+3+2+4+5+7+8}{6}=\frac{35}{8}=4.375 .
$$

The deviations from the mean are

$$
\begin{array}{lll}
5-4.375=0.625, & 1-4.375=-3.375, & 3-4.375=-1.375,
\end{array} \quad 2-4.375=-2.375, ~ 子=2.375=3-4.375=2.625, \quad 8-4.375=3.625 .
$$

The sample variance is

$$
\begin{aligned}
s^{2} & =\frac{0.625^{2}+(-3.375)^{2}+(-1.375)^{2}+(-2.375)^{2}+(-0.375)^{2}+0.625^{2}+2.625^{2}+3.625^{2}}{7} \\
& =\frac{39.875}{7} \approx 5.6964 .
\end{aligned}
$$

The standard deviation is

$$
s=\sqrt{\frac{39.875}{7}} \approx 2.387
$$

