Study Guide for the Midterm Exam

Last update: October 3, 2024

Definitions, notions and axioms to remember

1. Axioms: know how to interpret Euclid's postulates (page 38-39), Birkhoff's and Hilbert's axioms, but if I ask about them I will provide a copy of those, Playfair's postulate (page 48), Elliptic Parallel Postulate (page 78), Euclidean parallel postulate (page 128).
2. Definitions and notions: incidence, betweenness, rays, line segments, congruence (see Section 3.2), defect of a triangle, defect of a polygon, and the following triangle centers: centroid, orthocenter, circumcenter. Sensed ratio, parallelism. You should also be able to use inner products (aka dot products).

Statements you should remember with their proof

1. From our textbook: Two distinct lines can not intersect in more than one point (section 2.4), Isoceles Triangle Theorem (Theorem 3.2.7), Perpendicular Bisector Theorem (Theorem 3.2.8), Exterior Angle Theorem (Theorem 3.2.9), triangle congruence conditions (Theorems 3.3.1, 3.3.3 and 3.3.5), Alternate Interior Angle Theorem (Theorem 3.4.1), Euclid's fifth postulate is equivalent to the Euclidean parallel postulate (Theorem 3.4.5), Saccheri-Legendre Theorem (Theorem 3.5.1, also the proofs of the lemmas used, e.g., Lemma 3.5.3), base of a Saccheri quadrilateral is not longer than the summit (Theorem 3.6.6), shortest distance between the base and the summit of a Saccheri quadrilateral is the segment connecting the midpoints (Theorem 3.6.9), if a rectangle exist, then there is a rectangle with two arbitrarily large sides (Theorems 3.6.11 and 3.6.12), if one triangle has angle sum 180^o then the geometry is Euclidean (Theorems 3.6.13 through 3.6.18), sum of all angles in a triangle is 180^o (Theorem 4.2.2), Euclidean exterior angle theorem (Corollary 4.2.3), opposite sides of a parallelogram are congruent (Theorem 4.2.4), parallel transversals theorem (Theorem 4.2.5).
2. From lecture and handouts: defect of triangles is additive, isometries of the plane, SSS congruence (Theorem 3.3.9), Euclid's fifth postulate is equivalent to parallelism being an equivalence relation (Theorem 3.4.7), properties of the Saccheri and Lambert quadrilaterals (statements 3.6.1, 3.6.2, 3.6.3, 3.6.4, 3.6.5, 3.6.7), least distance between the base and summit of a Saccheri quadrilateral is at the common perpendicular (Theorem 3.6.9), theorems on the existence of a rectangle (Theorems 3.6.12 and 3.6.13), if there is one rectangle then all triangles have angle sum 180^o (Theorem 3.6.15), the Euclidean parallel postulate is equivalent to every triangle having angle sum 180^o. Ceva's theorem (see also Theorem 4.7.4), Median Concurrence Theorem (Theorem 4.2.7), area formulas (Theorems 4.3.2 through 4.3.6), parallel transversal theorem (Theorem 4.2.5), "Star Trek lemma" (Theorem 4.5.11).
3. From homework: sum of the interior angles of a triangle from Euclid's fifth postulate, generalization of the Pasch theorem to polygons, , distance formula, midpoint formula, Median Concurrence Theorem (see also Theorem 4.2.7), existence of the Euler line.

If a proof was covered in several ways you may choose your favorite one. You may also invent your own proof.

Statements you should know (without proof)

1. From our textbook: first 6 consequences of negating Euclid's fifth postulate on page 76, defect measures area elementary facts about congruence (Theorems 3.2.1 through 3.2.4) Pasch Axiom (Theorem 3.2.5), Crossbar Theorem (Theorem 3.2.6), The converse of the Isoceles triangle theorem (Theorem 3.3.2), Theorem 3.3.6, Triangle Inequality (Theorem 3.3.7), Hinge Theorem (Theorem 3.3.8), equivalent forms of Euclid's fifth postulate (theorems in section 3.4, only the proof of Theorems 3.4.1 and 3.4.7 are required), generalization of the parallel transversal theorem (Theorem 4.2.5)
2. From lecture: description of the projective plane.
3. From homework: sides between the two right angles are not longer than the opposite sides in a Lambert quadrilateral (Theorem 3.6.8), converse of the "Star Trek lemma" (Theorem 4.5.11).

What to expect

The exam will be *closed book*. You will have 80 minutes. Some questions may ask you to state and prove a theorem from the list I gave, others may be exercises similar to your homework assignments. There may be questions about examples, whether they have certain properties.