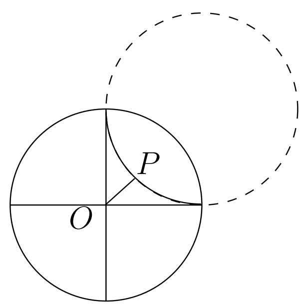
Assignment 12

Oral questions

1. Schweikart's constant is the distance d for which the angle of parallelism is $\Pi(d) = 45^o$. Prove that for the length function of the Poincaré disk model, Schweikart's constant equals $\log(1+\sqrt{2})$. You may use the following formula in your proof. If a point P is at a Euclidean distance r from the center O then its hyperbolic distance from O is

$$d(O,P) = \ln\left(\frac{1+r}{1-r}\right).$$



2. All hyperbolic rotations fixing the point i in the Poincaré upper half plane model are fractional linear transformations $z \mapsto \frac{az+b}{cz+d}$ sending i into i. Using this fact, and assuming that we have scaled our coefficients to satisfy $ad - bc = 1$, show that

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

for some angle θ.

Question to be answered in writing

1. Using $e^{-x} = \tan(\Pi(x)/2)$, prove the following formulas:

$$\sin(\Pi(x)) = \text{sech}(x), \quad \cos(\Pi(x)) = \tanh(x), \quad \tan(\Pi(x)) = \text{csch}(x).$$