

Birkhoff's axioms

Primitives: point, line, distance, angle.

- I The points A, B, \dots of any line are in 1 : 1 correspondence with the real numbers such that $|x_b - x_a| = d(A, B)$ for all points A and B .
- II One and only one line ℓ contains any two distinct points P and Q .
- III The half lines (or rays) ℓ, m, n, \dots through any point O can be put into 1 : 1 correspondence with the real numbers $a \pmod{2\pi}$ so that if A and B are points (other than O) of ℓ and m , respectively, the difference $a_\ell - a_m \pmod{2\pi}$, of numbers associated with the line ℓ and m is $m\angle AOB$ (i.e., the measure of $\angle AOB$).
- IV If in two triangles $\triangle ABC$ and $\triangle A'B'C'$ and for some constant $k > 0$, $d(A, B) = kd(A', B')$, $d(A, C) = kd(A', C')$ and $m\angle B'A'C' = m\angle BAC$ then also $d(B, C) = kd(B', C')$, $m\angle C'B'A' = m\angle CBA$ and $m\angle A'C'B' = m\angle ACB$.