Hilbert's axioms

Group I: Axioms of connection

I-1 Through any two distinct points A, B, there is always a line m.

I-2 Through any two distinct points A, B, there is not more than one line m.

I-3 On every line there exists at least two distinct points. There exists at least three points that are not on the same line.

I-4 Through any three points not on the same line, there is one and only one plane.

Group II: Axioms of order

$A - B - C$ stands for "B is between A and C".

II-1 If $A - B - C$, then A, B, and C are three distinct points on the same line, and $C - B - A$.

II-2 For any two distinct points A and C, there is at least one point B on the line $\overleftrightarrow{AC}$ such that $A - C - B$.

II-3 If A, B, and C are three points on the same line, then exactly one is between the other two.

II-4 Let A, B, and C be three points that are not on the same line, and let m be a line in the plane containing A, B, and C that does not contain any of the three points. Then, if m contains a point of the segment $\overline{AB}$, it will also contain a point of the segment $\overline{AC}$ or a point of segment $\overline{BC}$.

Group III: Axioms of congruence

III-1 If A and B are distinct points on the line a and if A' is any point on the same or another line a', then it is always possible to find a point B' on a given side of the line a' through A' such that the segment $\overline{AB}$ is congruent to the segment $\overline{A'B'}$.

III-2 If a segment $\overline{A'B'}$ and a segment $\overline{A''B''}$ are congruent to the same segment $\overline{AB}$ then the segment $\overline{A'B'}$ is also congruent to the segment $\overline{A''B''}$ or, briefly, if two segments are congruent to a third segment, then they are congruent to each other.

III-3 On the line a, let $\overline{AB}$ and $\overline{BC}$ be two segments that, except for B, have no point in common. Furthermore, on the same or another line a', let $\overline{A'B'}$ and $\overline{B'C'}$ be two segments that, except for B', have no point in common. In that case if $\overline{AB} \cong \overline{A'B'}$ and $\overline{BC} \cong \overline{B'C'}$, then $\overline{AC} \cong \overline{A'C'}$. (Additivity of segments.)

III-4 If $\angle ABC$ is an angle and if $\overrightarrow{B'C'}$ is a ray, then there is exactly one ray $\overrightarrow{B'A'}$ on each side of $\overleftrightarrow{B'C'}$ such that $\angle A'B'C' \cong \angle ABC$. (Angle construction.)

III-5 If for two triangles $\triangle ABC$ and $\triangle A'B'C'$ the congruences $\overline{AB} \cong \overline{A'B'}$, $\overline{AC} \cong \overline{A'C'}$, and $\angle BAC \cong \angle B'A'C'$ are valid then the congruence $\angle ABC \cong \angle A'B'C'$ is also satisfied.

Group IV: Axiom of parallels

IV-1 Let a be any line and A a point not on it. Then there is at most one line in the plane, determined by a and A, that passes through A and does not intersect a.

Group V: Axioms of continuity

V-1 *Axiom of Archimedes:* If $\overline{AB}$ and $\overline{CD}$ are any segments, then there is a number n such that n copies of $\overline{CD}$ constructed contiguously from A along the ray $\overrightarrow{AB}$ will pass beyond the point B.

V-2 *Postulate of Line Completeness:* An extension of a set of points on a line with its order and congruence relations that would preserve the relations existing among the original elements as well as the fundamental properties of line order and congruence that follow from Axioms I through III and V is impossible.