This study guide is subject to updates until our lass class on Tuesday December 2, 2025
Last update: November 26, 2025

The actual final exam will have a mandatory and an optional section. The optional questions will be similar to the ones on the previous (sample) tests, and need to be answered only if you do not want me to re-use your average test score. The questions below are supposed to help you prepare for the mandatory part of the final.

1. Find a minimum cut and a maximum flow for the network shown in Figure 1. **Show all your work!**

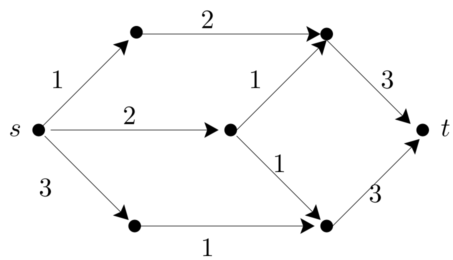


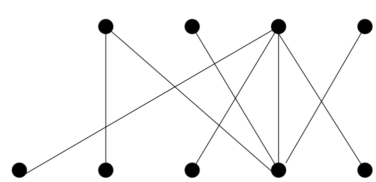
Figure 1: Network with source s and sink t

2. State the Ford-Fulkerson theorem for network flows. Explain how the network flow algorithm may be used to prove it for integer capacities.

3. State Menger's theorem (edge version) and explain how network flows may be used to prove them. (See your notes and "messenger problems" in the book). You only need to state, how would you set up the network associated to the graph.

4. Suppose the vertices in Figure 1 are cities, and the capacities indicate how many wagons can travel along that edge each day. Suppose wagons can leave each city once a day and each edge takes 2 days to travel. Explain how would you find the number of wagons that can be sent from s to t in 12 days. (Only describe how would you set up the problem.)

5. Find a maximum size matching and a minimum size cover in the bipartite graph below by stating and solving a related network flow problem. *Show all your work!*



6. Explain how the question of finding a maximum size matching and a minimum size edge cover in a bipartite graph may be translated into the question of finding a maximum flow and a minimum

cut in a network flow. Prove that the maximum size of a matching is the same as the minimum size of a cover in a bipartite graph.

7. State and prove Hall's Theorem.

8. State Birkhoff's Theorem and write the following doubly stochastic matrix as a convex combination of permutation matrices:

$$\begin{pmatrix} 1/6 & 1/2 & 1/3 \\ 1/2 & 1/6 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

9. Solve a transportation problem, just like in the following exercises in our textbook: 4.5/1,3,5,7. *Show all your work!*

10. Prove that a transportation problem has always an optimal solution S, for which the associated set of edges $E(S)$ contains no circuit.

11. State and prove the formula expressing the cost of a spanning tree solution to the transportation problem in terms of the supply and demand numbers and the price values associated to the vertices. Explain how these prices are calculated, and why they are uniquely defined.

12. State and prove the formula on the equality between the transportation cost and the price difference between the income at the stores and the cost at the warehouses for the price system associated to a spanning tree solution.

13. Explain why you may need edges with a zero flow on them in a spanning tree solution of the transportation problem. How could the use of the northwest corner rule force the presence of such an edge even in the initial solution?

14. Suppose you find an edge that could be added to a spanning tree solution in the algorithm finding an optimal solution. Explain how you would rearrange the flow and what would you do if you end up with several edges in the circuit created having zero flow on them.

15. Define a kernel of a progressively finite game and show that a player who starts at a non-kernel position has a winning strategy.

16. Find the Grundy number of the NIM game with pile vector $(1, 2, 2, 3)$. Which player has a winning stategy?

Good Luck. Gábor Hetyei