

## Test II. (sample)

You have 80 minutes to answer the questions. The usage of books or notes, or communicating with other students is not allowed. Give the simplest possible answer and show all your work. Ask me if you have questions.

- (40 points) There are three clubs at a university. 10 people belong to the first club, 20 to the second, 30 to the third. 5 persons belong to the first and the second club, 10 to the second and the third, and 4 to the first and the third. 2 persons are members of all three clubs. How many people belong to the first club only?
- (40 points) Express  $3x^3 - 2x$  as a polynomial of falling factorials.
- (40 points) Find the coefficient of  $x^6$  in  $\left(\frac{1}{1-x^2}\right)^3$ .
- (40 points) The sequence  $a_0, a_1, \dots$  is given by  $a_0 = a_1 = 1$ , and  $a_{n+2} = 3a_{n+1} - 2a_n$ . Express the generating function  $F(x) = \sum_{n=0}^{\infty} a_n x^n$  as a quotient of two polynomials.
- (40 points) Write  $\frac{x}{1+7x+12x^2}$  as a sum of rational functions whose denominators are linear.
- (40 points) Does the edge-graph of a cube have an Eulerian walk? What if you add the diagonals of each face? What can you say about the starting and endpoint of the existing walks?
- (40 points) Which of the following could be the list of degrees in a (simple) graph? Which could also be a list of degrees in a tree?  
(a) [4, 3, 2, 1]   (b) [3, 3, 2, 1]   (c) [3, 2, 2, 1]   (d) [3, 1, 1, 1].
- (10 points) Which of the following can be used to find bridges in a connected graph: breadth-first search, or depth first search? Which can be used to find the distance between two vertices?
- (10 points) How many trees are there on 10 distinct vertices?
- (10 points) The number of faces of a connected planar graph is 12, the number of its vertices is 20. What is the number of its edges?
- (10 points) Is the Petersen graph planar?

Good luck.

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