Assignment 5

Oral questions

1. Exercises 20.2 and 20.6
2. Exercises 20.4 and 20.8
3. Exercise 20.10
4. Exercise 20.12
5. Exercise 20.14
6. Exercise 20.16a
7. Exercise 20.18

Question to be answered in writing

1. Assume the function $f : \mathbb{R} \to \mathbb{R}$ has limit L at 0. Show that, for any fixed positive number $a > 0$, the function $g : \mathbb{R} \to \mathbb{R}$ given by $g(x) = f(ax)$ has the same limit at 0.

Bonus question

1. (3 points) Find $\lim_{x\to 0} \frac{\sin(x)}{x}$ and prove your claim using geometry. (You are *not allowed* to use L'Hospital's rule or derivatives in any other way.)
2. (3 points) Assume that the sequences $a_1, a_2, a_3 \dots$ and $b_1, b_2, b_3, \dots$ converge to the same limit L. Let $c_1, c_2, c_3, \dots$ be a sequence obtained by "merging" the sequences $a_1, a_2, a_3 \dots$ and $b_1, b_2, b_3, \dots$ in any possible way. (For example, we may have $c_1 = a_1$, $c_2 = a_2$, $c_3 = b_1$, $c_4 = a_3$, $c_5 = a_4$, $c_6 = b_2$, $c_7 = b_3$, and so on.) Prove that the sequence (c_n) converges to the same limit as (a_n) and (b_n).