Assignment 13

Oral questions

1. Find the antiderivative of $\cos^3(x)$.
2. Find the antiderivative of $\frac{1}{(1+x^2)^2}$.
3. Find the antiderivative of $x^2 \cdot \sin(x)$.
4. Find the antiderivative of $\frac{1}{x^2+4x+7}$.
5. Exercise 31.2.
6. The Fibonacci numbers $F_0, F_1, \ldots$ are given by $F_0 = 1$, $F_1 = 1$, and the recursion formula $F_n + F_{n+1} = F_{n+2}$. Find a closed formula for the function whose Taylor series is $\sum_{n=0}^{\infty} F_n \cdot x^n$.

Question to be answered in writing

1. Find the Taylor series expansion of $\arcsin(x)$. (Hint: find the Taylor series of its derivative first, and then integrate term-by-term.)

Bonus question

1. A set S of real numbers has length zero if for all $\varepsilon > 0$ there is a (finite) family of intervals such that S is contained in $\bigcup_{k=0}^{n} I_k$ and the total length of the intervals I_k is less than ε. Consider a bounded function f on $[a, b]$ and let S be the set of numbers where f is not continuous. Prove that f is integrable on $[a, b]$ if the length of S is zero.