# Assignment 2

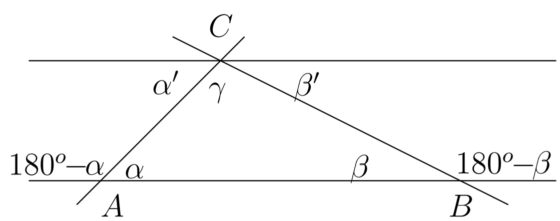
## Oral questions

1. List all primitive Pythagorean triples $(a, b, c)$ that satisfy $b = 20$. (Here $b$ is one of the legs.) How many primitive Pythagorean triples satisfy $b = 30$?

2. Given a line $\ell$ and a point $P$ on it, define the following relation for on the points of $\ell \setminus \{P\}$: $A \sim B$ if $P$ is not between $A$ and $B$, that is, $A * P * B$ is false. Prove that this relation is an equivalence relation.

3. Prove Lemma 3.2 on page 33. (Hint: use Postulate 8.) Use your proof to devise a method to construct $Q$ using compass and ruler.

4. Complete the following proof of the theorem stating that the sum of the angles of a triangle $ABC$ is $180^o$. We draw parallel line to $AB$ through $C$ and use the notation introduced in the picture.




Applying Euclid's fifth postulate to the line $AB$ and the angles $180^o - \alpha$ and $\alpha'$ yields $180^o - \alpha + \alpha' \geq 180^o$. As a consequence we must have $\alpha' \geq \alpha$. Similarly, applying Euclid's fifth postulate to the line $BC$ and the angles $180^o - \beta$ and $\beta'$ yields $180^o - \beta + \beta' \geq 180^o$, and so $\beta' \geq \beta$. Hence we obtain

$$\alpha + \beta + \gamma \leq \alpha' + \beta' + \gamma \leq 180^o.$$

Use Euclid's fifth postulate directly in two more situations to show that $\alpha + \beta + \gamma$ is also greater than equal to $180^o$.

## Question to be answered in writing

1. Identify the points in the plane with their Cartesian coordinates. Define the distance of the point $P := (x_1, y_1)$ and $Q := (x_2, y_2)$ by the formula $d(P, Q) := ||x_1 - x_2| + |y_1 - y_2||$. Prove that this function satisfies the axioms given in Definition 3.1 on page 29. Draw a picture of the "unit circle centered at the origin", i.e., the set $\{(x, y) : ||x - 0| + |y - 0|| = 1\}$.