Assignment 8

Oral questions

1. Which of the following must always exist even in hyperbolic geometry: the incircle or the circumcircle? (Think of the crossbar theorem and of the possibility of hyperparallel lines being perpendicular bisectors.)
2. Use the additivity of defect to show that all triangles can not have the same positive defect. Is there an upper bound on the defect of a triangle? Compare this to the upper bound on the defect of a quadrilateral.
3. (Euclidean geometry.) Let O be the center of the circle of inversion, P' the inverse of P and Q' the inverse of Q. Assume that O, P, and Q form a triangle. Show that $OPQ_{\triangle}$ is similar to $OQ'P'_{\triangle}$. Use this result to show that inversion preserves the cross-ratio: if A, B, P, and Q are four points distinct from the center O of the circle of inversion and A', B', P', and Q' are their inverses then $(AB, PQ) = (A'B', P'Q')$.

Questions to be answered in writing

1. Let ℓ be the perpendicular bisector of the side AB in the triangle $\triangle ABC$. Let A_1 be the midpoint of the side BC. Let m be the line through A_1 that is perpendicular to ℓ. Show that m contains the midpoint of AC. (Hint: let B_1 be the intersection of the line m with AC. Reflect B_1 about the line ℓ, get B_1', and reflect B_1' about the point A_1 to get B_1''. Show that the length of AB_1 is the same as the length of CB_1'' and then show that $\triangle B_1CB_1''$ is isoceles.) This is a question about *neutral geometry*. You should *not* assume Euclid's fifth postulate in your proof.