Assignment 13

Oral questions

1. Using $e^{-x} = \tan(\Pi(x)/2)$, prove the following formulas:

$$\sin(\Pi(x)) = \operatorname{sech}(x), \quad \cos(\Pi(x)) = \tanh(x), \quad \tan(\Pi(x)) = \operatorname{csch}(x).$$

2. Explain why a dilation, centered at the origin, represents a congruence in the Poincaré half plane model. Show that each such dilation may be written as a composition of two inversions, where both circles are centered at the origin. Keeping in mind that these inversions correspond to reflections, help visualize the congruence represented by a dilation by comparing it to the composition of two reflections about two parallel lines in the Euclidean plane.

Questions to be answered in writing

1. Find the Poincaré distance between the points $P = 3 + i$ and $Q = (6 + \sqrt{2})/2 + \sqrt{2}/2 \cdot i$ (in the Poincaré upper half plane model).

2. Find the angles of the triangle whose sides are 3, 4, and 5. (Use the hyperbolic law of cosines.)