Study Guide for the Final

This study guide is subject to updates until our last class on April 28, 2020
Last update: Tuesday, April 28, 2020

Definitions and axioms to remember

1. Axioms: Euclid's postulates, Playfair's postulate, hyperbolic parallel postulate.
2. Definitions: angle of parallelism (Definition 6.2.1), defect of a triangle, fractional linear transformations (also conjugate ones!), hyperbolic trigonometric functions, inversion, horoparallel and hyperparallel lines, hypercycles, horocycles.
3. Poincaré disk model and upper half plane model: points, lines, angle, and distance in these models.
4. Spherical geometry: great circles, excess, antipodal points, polar triangle.
5. Projective geometry: Model of the projective plane, points, lines, duality, ideal points, ideal line (notes of April 21). Central perspectivity (Definition 7.6.1), Projective Transformation (Definition 7.6.2)

Statements you should remember with their proof

1. From our textbook: Inverse of a line not passing through the pole of inversion is a circle (Theorem 5.5.6 or handout), only the circle of inversion and the circles orthogonal to it are preserved by inversion (Theorem 5.5.10), existence of the angle of parallelism (Theorem 6.2.1), defect is additive (Theorem 6.4.3), (AAA) congruence in hyperbolic geometry (Theorem 6.4.5, focus on Figure 6.4.3, see also your notes).
2. From lecture and handouts: Ceva's theorem and applications, inversion and all of its properties, exact formulas, with proof. Complex cross ratio is real for points on a circle or line. Existence and uniqueness of a limiting parallel ray, distance function in the Poincaré disk model is additive. Lobachevsky's theorem. Hyperbolic and spherical Pythagorean Theorem and formulas for right triangles, hyperbolic and spherical laws of sines and cosines, except for second hyperbolic law of cosines. Spherical law of cosines for angles (using polarity). Description of hypercycles and horocycles in the Poincaré upper half plane model and in the Poincaré disk models. Fractional linear transformation connecting the two Poincaré models (outline of the main ideas only), Lines are hyperparallel if they are not horoparallel, there is a projective transformation taking 3 given collinear points into 3 other given collinear points, perspectivity preserves the cross-ratio, description of the nine-point circle (see also section 4.8).
3. From homework: angle bisector theorem, value of $\cos(72^o)$, inversion preserves the cross-ratio, inversion preserves angles when inputs are lines, analytic formula for inversion, composing concentric inversions. Fractional linear transformations preserve angles and cross-ratio, description of fractional linear transformations preserving the upper half plane, proof of the hyperbolic Pythagorean theorem, results about polar triangles in spherical geometry.

If a proof was covered in several ways you may choose your favorite one. You may also invent your own proof.

Statements you should know (without proof)

1. From handouts: angle of parallelism depends only on the distance between the point and the line, defect and excess are proportional to the area. Hyperbolic circles are also Euclidean circles in the Poincaré disk model. Second hyperbolic law of cosines. I might ask you to apply the spherical or hyperbolic law of sines or cosines.

What to expect

The exam will be *closed book*. The above guide is meant to help with the mandatory part. For the optional part prepare as if it was another midterm. The mandatory part will be as long as the midterm, the optional part will have only about 5 questions.