Assignment 12

Oral question

1. Consider the fractional linear transformation $z \mapsto \frac{az+b}{cz+d}$ where $a, b, c, d \in \mathbb{R}$ and $ad-bc \neq 0$. Introduce $z = z_1+z_2 i$ and calculate explicitly the imaginary part of $\frac{az+b}{cz+d}$. Prove that the imaginary part of the image is positive for all $z_2 > 0$ if and only if $ad - bc > 0$.

   Now show that a conjugate fractional linear map $z \mapsto \frac{a\bar{z}+b}{c\bar{z}+d}$ takes the upper half plane into itself if and only if $ad - bc < 0$.

Question to be answered in writing

1. Using $e^{-x} = \tan(\Pi(x)/2)$, prove the following formulas:

$$\sin(\Pi(x)) = \operatorname{sech}(x), \quad \cos(\Pi(x)) = \tanh(x), \quad \tan(\Pi(x)) = \operatorname{csch}(x).$$

2. Find the Poincaré distance between the points $P = 3 + i$ and $Q = (6 + \sqrt{2})/2 + \sqrt{2}/2 \cdot i$ (in the Poincaré upper half plane model).