1. **(E)** Factor 2001 into primes to get $2001 = 3 \cdot 23 \cdot 29$. The largest possible sum of three distinct factors whose product is 2001 is the one which combines the two largest prime factors, namely $I = 23 \cdot 29 = 667, M = 3,$ and $O = 1$, so the largest possible sum is $1 + 3 + 667 = 671$.

2. **(A)** $2000(2000^{2000}) = (2000^1)(2000^{2000}) = 2000^{(1+2000)} = 2000^{2001}$. All the other options are greater than 2000^{2001}.

3. **(B)** Since Jenny ate 20% of the jellybeans remaining each day, 80% of the jellybeans are left at the end of each day. If x is the number of jellybeans in the jar originally, then $(0.8)^2x = 32$. Thus $x = 50$.

4. **(C)** The sequence of units digits is

$$1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, 7, 7, 4, 1, 5, 6, \ldots.$$

The digit 6 is the last of the ten digits to appear.

5. **(C)** Since $x < 2$, it follows that $|x - 2| = 2 - x$. If $2 - x = p$, then $x = 2 - p$. Thus $x - p = 2 - 2p$.

6. **(C)** There are five prime numbers between 4 and 18: $5, 7, 11, 13,$ and 17. Hence the product of any two of these is odd and the sum is even. Because $xy-(x+y) = (x-1)(y-1)-1$ increases as either x or y increases (since both x and y are bigger than 1), the answer must be an odd number that is no smaller than $23 = 5 \cdot 7 - (5 + 7)$ and no larger than $191 = 13 \cdot 17 - (13 + 17)$. The only possibility among the options is 119, and indeed $119 = 11 \cdot 13 - (11 + 13)$.

7. **(E)** If $\log_b 729 = n$, then $b^n = 729 = 3^6$, so n must be an integer factor of 6; that is, $n = 1, 2, 3,$ or 6. Since $729 = 729^1 = 27^2 = 9^3 = 3^6$, the corresponding values of b are $3^6, 3^3, 3^2,$ and 3.

8. **(C)** Calculating the number of squares in the first few figures uncovers a pattern. Figure 0 has $2(0) + 1 = 2(0^2) + 1$ squares, figure 1 has $2(1) + 3 = 2(1^2)+3$ squares, figure 2 has $2(1+3)+5 = 2(2^2)+5$ squares, and figure 3 has $2(1 + 3 + 5) + 7 = 2(3^2) + 7$ squares. In general, the number of unit squares in figure n is

$$2(1 + 3 + 5 + \cdots + (2n - 1)) + 2n + 1 = 2(n^2) + 2n + 1.$$

Therefore, the figure 100 has $2(100^2) + 2 \cdot 100 + 1 = 20201$.

OR

Each figure can be considered to be a large square with identical small pieces deleted from each of the four corners. Figure 1 has $3^2 - 4(1)$ unit squares, figure 2 has $5^2 - 4(1+2)$ unit squares, and figure 3 has $7^2 - 4\cdot(1+2+3)$ unit squares. In general, figure n has

$$(2n + 1)^2 - 4(1 + 2 + \ldots + n) = (2n + 1)^2 - 2n(n + 1) \text{ unit squares.}$$

Thus figure 100 has $201^2 - 200(101) = 20201$ unit squares.

OR

The number of unit squares in figure n is the sum of the first n positive odd integers plus the sum of the first $n+1$ positive odd integers. Since the sum of the first k positive odd integers is k^2, figure n has $n^2 + (n + 1)^2$ unit squares. So figure 100 has $100^2 + 101^2 = 20201$ unit squares.

9. **(C)** Note that the integer average condition means that the sum of the scores of the first n students is a multiple of n. The scores of the first two students must be both even or both odd, and the sum of the scores of the first three students must be divisible by 3. The remainders when $71, 76, 80, 82$, and 91 are divided by 3 are $2, 1, 2, 1$, and 1, respectively. Thus the only sum of three scores divisible by 3 is $76 + 82 + 91 = 249$, so the first two scores entered are 76 and 82 (in some order), and the third score is 91. Since 249 is 1 larger than a multiple of 4, the fourth score must be 3 larger than a multiple of 4, and the only possibility is 71, leaving 80 as the score of the fifth student.

10. **(E)** Reflecting the point $(1, 2, 3)$ in the xy-plane produces $(1, 2, -3)$. A half-turn about the x-axis yields $(1, -2, 3)$. Finally, the translation gives $(1, 3, 3)$.

11. **(E)** Combine the three terms over a common denominator and replace ab in the numerator with $a-b$ to get

$$\begin{aligned}\frac{a}{b}+\frac{b}{a}-ab &= \frac{a^2+b^2-(ab)^2}{ab}\\ &= \frac{a^2+b^2-(a-b)^2}{ab}\\ &= \frac{a^2+b^2-(a^2-2ab+b^2)}{ab}\\ &= \frac{2ab}{ab}=2.\end{aligned}$$

OR

Note that $a=a/b-1$ and $b=1-b/a$. It follows that $\frac{a}{b}+\frac{b}{a}-ab=\frac{a}{b}+\frac{b}{a}-\left(\frac{a}{b}-1\right)\left(1-\frac{b}{a}\right)=\frac{a}{b}+\frac{b}{a}-\left(\frac{a}{b}+\frac{b}{a}-2\right)=2.$

12. **(E)** Note that

$$AMC+AM+MC+CA=(A+1)(M+1)(C+1)-(A+M+C)-1=pqr-13,$$

where p,q, and r are positive integers whose sum is 15. A case-by-case analysis shows that pqr is largest when $p=5, q=5$, and $r=5$. Thus the answer is $5\cdot 5\cdot 5-13=112$.

13. **(C)** Suppose that the whole family drank x cups of milk and y cups of coffee. Let n denote the number of people in the family. The information given implies that $x/4+y/6=(x+y)/n$. This leads to

$$3x(n-4)=2y(6-n).$$

Since x and y are positive, the only positive integer n for which both sides have the same sign is $n=5$.

OR

If Angela drank c cups of coffee and m cups of milk, then $0<c<1$ and $m+c=1$. The number of people in the family is $6c+4m=4+2c$, which is an integer if and only if $c=\frac{1}{2}$. Thus, there are 5 people in the family.

14. **(E)** If x were less than or equal to 2, then 2 would be both the median and the mode of the list. Thus $x > 2$. Consider the two cases $2 < x < 4,$ and $x \geq 4$.

Case 1: If $2 < x < 4$, then 2 is the mode, x is the median, and $\frac{25+x}{7}$ is the mean, which must equal $2 - (x-2)$, $\frac{x+2}{2}$, or $x + (x-2)$, depending on the size of the mean relative to 2 and x. These give $x = \frac{3}{8}, x = \frac{36}{5}$, and $x = 3$, of which $x = 3$ is the only value between 2 and 4.

Case 2: If $x \geq 4$, then 4 is the median, 2 is the mode, and $\frac{25+x}{7}$ is the mean, which must be $0, 3,$ or 6. Thus $x = -25, -4,$ or 17, of which 17 is the only one of these values greater than or equal to 4.

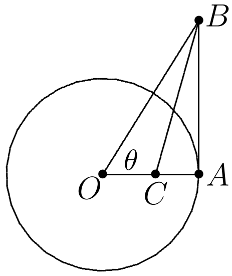
Thus the x-values sum to $3 + 17 = 20$.

15. **(B)** Let $x = 9z$. Then $f(3z) = f(9z/3) = f(3z) = (9z)^2 + 9z + 1 = 7$. Simplifying and solving the equation for z yields $81z^2 + 9z - 6 = 0$, so $3(3z + 1)(9z - 2) = 0$. Thus $z = -1/3$ or $z = 2/9$. The sum of these values is $-1/9$.

Note. The answer can also be obtained by using the sum-of-roots formula on $81z^2 + 9z - 6 = 0$. The sum of the roots is $-9/81 = -1/9$.

16. **(D)** Suppose each square is identified by an ordered pair (m, n), where m is the row and n is the column in which it lies. In the original system, each square (m, n) has the number $17(m-1) + n$ assigned; in the renumbered system, it has the number $13(n-1) + m$ assigned to it. Equating the two expressions yields $4m - 3n = 1$, whose acceptable solutions are $(1, 1), (4, 5), (7, 9), (10, 13)$, and $(13, 17)$. These squares are numbered $1, 56, 111, 166,$ and 221, respectively, and the sum is 555.

17. **(D)** The fact that $OA = 1$ implies that $BA = \tan\theta$ and $BO = \sec\theta$. Since $\overline{BC}$ bisects $\angle ABO$, it follows that $\frac{OB}{BA} = \frac{OC}{CA}$, which implies $\frac{OB}{OB+BA} = \frac{OC}{OC+CA} = OC$. Substituting yields



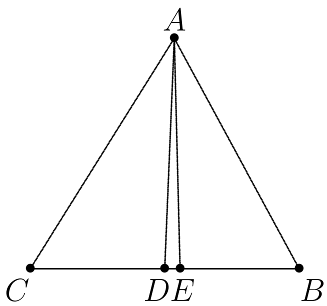
$$OC = \frac{\sec\theta}{\sec\theta + \tan\theta} = \frac{1}{1+\sin\theta}.$$

OR

Let $\alpha = \angle CBO = \angle ABC$. Using the *Law of Sines* on triangle BCO yields $\dfrac{\sin\theta}{BC} = \dfrac{\sin\alpha}{OC}$, so $OC = \dfrac{BC\sin\alpha}{\sin\theta}$. In right triangle ABC, $\sin\alpha = \dfrac{1-OC}{BC}$. Hence $OC = \dfrac{1-OC}{\sin\theta}$. Solving this for OC yields $OC = \dfrac{1}{1+\sin\theta}$.

18. **(A)** Note that, if a Tuesday is d days after a Tuesday, then d is a multiple of 7. Next, we need to consider whether any of the years $N-1, N, N+1$ is a leap year. If N is not a leap year, the 200^{th} day of year $N+1$ is $365-300+200 = 265$ days after a Tuesday, and thus is a Monday, since 265 is 6 larger than a multiple of 7. Thus, year N is a leap year and the 200^{th} day of year $N+1$ is another Tuesday (as given), being 266 days after a Tuesday. It follows that year $N-1$ is not a leap year. Therefore, the 100^{th} day of year $N-1$ precedes the given Tuesday in year N by $365-100+300 = 565$ days, and therefore is a Thursday, since $565 = 7\cdot 80 + 5$ is 5 larger than a multiple of 7.

19. **(C)** By Heron's Formula the area of triangle ABC is $\sqrt{(21)(8)(7)(6)}$, which is 84, so the altitude from vertex A is $2(84)/14 = 12$. The midpoint D divides $\overline{BC}$ into two segments of length 7, and the bisector of angle BAC divides $\overline{BC}$ into segments of length $14(13/28) = 6.5$ and $14(15/28) = 7.5$ (since the angle bisector divides the opposite side into lengths proportional to the remaining two sides). Thus the triangle ADE has base $DE = 7 - 6.5 = 0.5$ and altitude 12, so its area is 3.



20. **(B)** Note that $(x+1/y)+(y+1/z)+(z+1/x)=4+1+7/3=22/3$ and that

$$\begin{aligned} 28/3 = 4\cdot 1\cdot 7/3 &= (x+1/y)(y+1/z)(z+1/x) \\ &= xyz+x+y+z+1/x+1/y+1/z+1/(xyz) \\ &= xyz+22/3+1/(xyz). \end{aligned}$$

It follows that $xyz+1/(xyz)=2$ and $(xyz-1)^2=0$. Hence $xyz=1$.

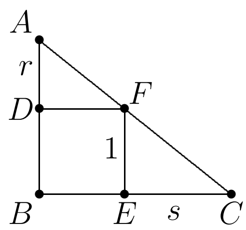
OR

By substitution,

$$4 = x+\frac{1}{y} = x+\frac{1}{1-1/z} = x+\frac{1}{1-3x/(7x-3)} = x+\frac{7x-3}{4x-3}.$$

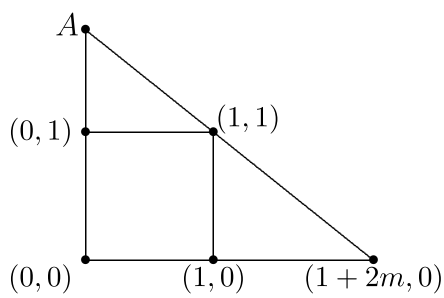
Thus $4(4x-3) = x(4x-3)+7x-3$, which simplifies to $(2x-3)^2 = 0$. Accordingly, $x = 3/2, z = 7/3-2/3 = 5/3$, and $y = 1-3/5 = 2/5$, so $xyz = (3/2)(2/5)(5/3) = 1$.

21. **(D)** Without loss of generality, let the side of the square have length 1 unit and let the area of triangle ADF be m. Let $AD = r$ and $EC = s$. Because triangles ADF and FEC are similar, $s/1 = 1/r$. Since $\frac{1}{2}r = m$, the area of triangle FEC is $\frac{1}{2}s = \frac{1}{2r} = \frac{1}{4m}$.

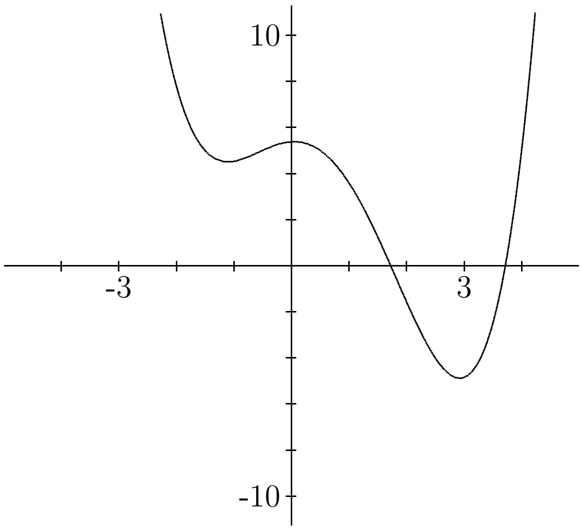


OR

Let $B=(0,0)$, $E=(1,0)$, $F=(1,1)$, and $D=(0,1)$ be the vertices of the square. Let $C=(1+2m,0)$, and notice that the area of $BEFD$ is 1 and the area of triangle FEC is m. The slope of the line through C and F is $-\dfrac{1}{2m}$; thus, it intersects the y-axis at $A=\left(0,1+\dfrac{1}{2m}\right)$. The area of triangle ADF is therefore $\dfrac{1}{4m}$.



22. **(C)** First note that the quartic polynomial can have no more real zeros than the two shown. (If it did, the quartic $P(x) - 5$ would have more than four zeros.) The sum of the coefficients of P is $P(1)$, which is greater than 3. The product of all the zeros of P is the constant term of the polynomial, which is the y-intercept, which is greater than 5. The sum of the real zeros of P (the sum of the x-intercepts) is greater than 4.5, and $P(-1)$ is greater than 4. However, since the product of the real zeros of P is greater than 4.5 and the product of all the zeros is less than 6, it follows that the product of the non-real zeros of P is less than 2, making it the smallest of the numbers.



23. **(B)** In order for the sum of the logarithms of six numbers to be an integer k, the product of the numbers must be 10^k. The only prime factors of 10 are 2 and 5, so the six integers must be chosen from the list $1, 2, 4, 5, 8, 10, 16, 20, 25, 32, 40$. For each of these, subtract the number of times that 5 occurs as a factor from the number of times 2 occurs as a factor. This yields the list $0, 1, 2, -1, 3, 0, 4, 1, -2, 5, 2$. Because 10^k has just as many factors of 2 as it has of 5, the six chosen integers must correspond to six integers in the latter list that sum to 0. Two of the numbers must be -1 and -2, because there are only two zeros in the list, and no number greater than 2 can appear in the sum, which must therefore be$(-2) + (-1) + 0 + 0 + 1 + 2 = 0$. It follows that Professor Gamble chose $25, 5, 1, 10$, one number from $\{2, 20\}$, and one number from $\{4, 40\}$. There are four possible tickets Professor Gamble could have bought and only one is a winner, so the probability that Professor Gamble wins the lottery is $1/4$.

OR

As before, the six integers must be chosen from the set $S = \{1, 2, 4, 5, 8, 10, 16, 20, 25, 32, 40\}$. The product of the smallest six numbers in S is $3{,}200 > 10^3$, so the product of the numbers on the ticket must be 10^k for some $k \geq 4$. On the other hand, there are only six factors of 5 available among the numbers in S, so the product p can only be $10^4, 10^5$, or 10^6.

Case 1, $p = 10^6$. There is only one way to produce 10^6, since all six factors of 5 must be used and their product is already 10^6, leaving 1 as the other number: $1, 5, 10, 20, 25, 40$.

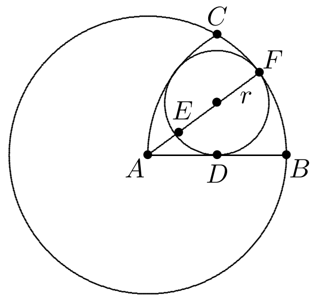
Case 2, $p = 10^5$. To produce a product of 10^5 we must use six numbers that include five factors of 5 and five factors of 2 among them. We cannot use both 20 and 40, because these numbers combine to give five factors of 2 among them and the other four numbers would have to be odd (whereas there are only three odd numbers in S). If we omit 40, we must include the other multiples of 5 $(5, 10, 20, 25)$ plus two numbers whose product is 4 (necessarily 1 and 4). If we omit 20, we must include $5, 10, 25,$ and 40, plus two numbers with a product of 2 (necessarily 1 and 2).

Case 3, $p = 10^4$. To produce a product of 10^4 we must use six numbers that include four factors of 5 and four factors of 2 among them. So that there are only four factors of 2, we must include $1, 5, 25, 2,$ and 10. These include two factors of 2 and four factors of 5, so the sixth number must contain two factors of 2 and no 5's, so must be 4.

Thus there are four lottery tickets whose numbers have base-ten logarithms with an integer sum: $\{1, 5, 10, 20, 25, 40\}, \{1, 2, 5, 10, 25, 40\}, \{1, 2, 4, 5, 10, 25\}$, and $\{1, 4, 5, 10, 20, 25\}$. Professor Gamble has a $1/4$ probability of being a winner.

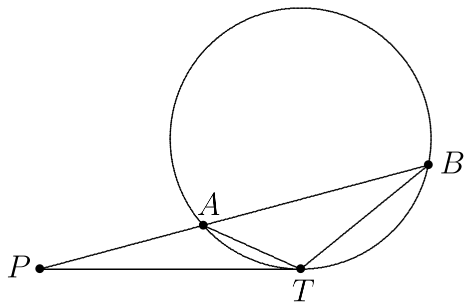
24. **(D)** Construct the circle with center A and radius AB. Let F be the point of tangency of the two circles. Draw $\overline{AF}$, and let E be the point of intersection of $\overline{AF}$ and the given circle. By the *Power of a Point Theorem*, $AD^2 = AF \cdot AE$ (see Note below). Let r be the radius of the smaller circle. Since $\overline{AF}$ and $\overline{AB}$ are radii of the larger circle, $AF = AB$ and $AE = AF - EF = AB - 2r$. Because $AD = AB/2$, substitution into the first equation yields

$$(AB/2)^2 = AB \cdot (AB - 2r),$$



or, equivalently, $\dfrac{r}{AB} = \dfrac{3}{8}$. Points A, B, and C are equidistant from each other, so $\overset{\frown}{BC} = 60°$ and thus the circumference of the larger circle is $6 \cdot$ (length of $\overset{\frown}{BC}$) $= 6 \cdot 12$. Let c be the circumference of the smaller circle. Since the circumferences of the two circles are in the same ratio as their radii, $\dfrac{c}{72} = \dfrac{r}{AB} = \dfrac{3}{8}$. Therefore $c = \frac{3}{8}(72) = 27$.

Note. From any exterior point P, a secant PAB and a tangent PT are drawn. Consider triangles PAT and PTB. They have a common angle P. Since angles ATP and PBT intercept the same arc $\overset{\frown}{AT}$, they are congruent. Therefore triangles PAT and PTB are similar, and it follows that $PA/PT = PT/PB$ and $PA \cdot PB = PT^2$. The number PT^2 is called *the power of the point* P with respect to the circle. Intersecting secants, tangents, and chords, paired in any manner create various cases of this theorem, which is sometimes called *Crossed Chords*.



25. **(E)** The octahedron has 8 congruent equilateral triangular faces that form 4 pairs of parallel faces. Choose one color for the bottom face. There are 7 choices for the color of the top face. Three of the remaining faces have an edge in common with the bottom face. There are $\binom{6}{3} = 20$ ways of choosing the colors for these faces and two ways to arrange these on the three faces (clockwise and counterclockwise). Finally, there are $3! = 6$ ways to fix the last three colors. Thus the total number of distinguishable octahedrons that can be constructed is $7 \cdot 20 \cdot 2 \cdot 6 = 1680$.

OR

Place a cube inside the octahedron so that each of its vertices touches a face of the octahedron. Then assigning colors to the faces of the octahedron is equivalent to assigning colors to the vertices of the cube. Pick one vertex and assign it a color. Then the remaining colors can be assigned in $7!$ ways. Since three vertices are joined by edges to the first vertex, they are interchangeable by a rotation of the cube, hence the answer is $7!/3 = 1680$.