1. In this game, two children take turns breaking up a 6 square by 8 square rectangular chocolate bar. They break the bar only at the divisions between the squares. If the bar breaks into several pieces, they keep breaking one piece at a time until only the squares remain. The first player who cannot make a break is the loser. Who will win?

2. There are three piles of stones, one with 10 stones, one with 15 stones and one with 20 stones. At each stage, a player can choose a pile and divide it into two smaller piles. The loser is the first player who cannot do this. Who wins and why?

3. Two players take turns putting rooks on a chess board so they cannot capture each other. The winner is the last to put down a rook. Who will win and how? Rooks move along rows and along columns.

4. Change the rooks in problem 4 to knights. Two players alternate placing knights on a chess board so that no knight can capture another. The first player who is unable to move loses. What if the board was 7×7?

5. You are participating in a raffle where there are 11 baskets and 10 players who take turns, one after another, distributing 5 tickets among the baskets. Assuming perfect play by all parties, what is the expected number of wins for the last player? Round your answer to the nearest tenth.

6. Ten 1's and ten 2's are written on a board. In one turn, a player may erase any two digits. If they are identical, they are replaced by the single digit 2 and if they are different, they are replaced by the single digit 1. The first player wins if a 1 is left at the end and the second wins if a 2 is left. Who wins and why?

7. Recall the subtraction game in which two players start with two positive integers a and b written on a board. The first player subtracts one of the numbers on the board from a larger one, and write down the new difference. At each stage, the next player finds a positive difference between two numbers that is not already written on the board and writes it on the board. The first player who cannot find a new positive difference loses. For each of the pairs listed below, write down all the numbers that will eventually appear on the board, and use this information to state whether the game will be won by the first player or the second.

   (a) 35 and 42

   (b) 36 and 42

(c) 39 and 42

(d) 40 and 42

8. Begin with a stack of chips. On your turn, you may take either 2 or 3 chips from the stack. We consider three versions of this game.

   a. last player wins.

   b. last player loses.

   c. the last player wins if he takes the last counter and loses otherwise.

   In each case you are the first player. Decide how many chips you should start with so that you can win the game.

9. Now change the moves from 2 or 3 to 3 or 4 chips? What if you are only allowed to take 5 or 11 chips? [See A. Bogolmony of `http://cut-the-knot.com`]

10. In the game of squares and circles, begin with a collection of squares and circles. For example, you might start with three circles and a square. At each step, cross out any two shapes. If the shapes you just crossed out are the same, draw one square. If they are different, draw one circle. Eventually, there is only one shape left. You win if the final shape is a circle. What can you say about what happens in this game? What about with different starting situations?

11. In the previous problem, what if we have squares, circles, and triangles, and the rule is that you may only cross out two different shapes and then draw the third shape. Can you win (ie, end with just one circle) if you start with three circles ($c = 3$) and a square ($s = 1$)?

    What about other starting situations, like four of each shape for example? What about if you can reverse the rule when you wish, crossing out one shape and drawing one each of the other two shapes? What if the rules are changed so that when you cross out two different shapes you then draw two copies of the third shape? The goal is to get all the shapes the same. What starting situations enable you to eventually win?

12. The Mad Veterinarian [http://bumblebeagle.org/madvet/index.html, which incidentally also has some great solution discussions] has three machines. One converts a cat into two dogs and a mouse (or vice-versa): $1C \leftrightarrow 2D1M$. A second machine does $1D \leftrightarrow 1C1M$, and a third machine does $1M \leftrightarrow 1C3D$. The general puzzle is to start with just one animal and replicate it: whats the fewest cats (more than one) that you can turn one cat into (with no mice

or dogs left around)? Or, even more generally, starting with one cat can you describe all the combinations of animals you can end up with?

13. Heres a two-player game for a change: player 1 writes a sequence of ten positive integers. Then player 2 writes a $+$ or a $-$ sign in each of the nine spaces between the integers. In the end, if the final numeric result is odd, player 1 wins, and if even, player 2 wins.
    (a) Who should win this game, and how?
    (b) What if player 1 is given a bag with a certain collection of numbers, each of which can be used only once? For example, if they have a bag containing the numbers 1 through 12? 1 through 11? 1 through 10?
    (c) What if player 2 can use exactly one multiplication sign $\cdot$, and eight plus or minus signs, in the nine spaces?
    (d) What if player 2 gets exactly two multiplication signs (and 7 + or signs)?

14. Another one-player game: Start with a stack of n boxes. At each move, as long as any stacks have more than one box, split one stack into two parts, say x boxes into y and z, and score yz points. How should you split them in order to maximize your score? What is the maximum score for each n?

15. Coin-flipping: Begin with some number of coins, say four for example, and set them on the table in a line, with a given starting sequence like $HHTH$ for example. At each move, you may flip any two adjacent coins. You win if the final arrangement of the coins is all heads.

16. Coin-splitting: Begin with an infinite strip of squares, and a penny on one spot. At each move, you may either split the penny (remove it and put a penny on each adjacent spot) or merge two pennies (remove two pennies with exactly one space between them and put one on the space between; in other words, undoing the splitting operation). You may have any number of pennies on a given spot (but each move only splits one penny or merges two pennies into one). Starting with one penny, can you split and merge to end up with just one penny on the board in a different spot? What different spots are possible?