1. Two hundred coins are identical except that their weights are an integer number of ounces, $1, 2, 3, \ldots, 200$. An integer set of weights $W = \{w_1, w_2, \ldots, w_k\}$ is required to be used with a two-pan balance so that if you were given just one coin c from the set, you could determine its weight using just c and W. The number of times the balance need be used for any coin is immaterial. What is the size of the smallest set of weights?

2. What is the cardinality of the smallest set of positive integers P such that for each integer n, $1 \leq n \leq 200$, it is possible to find two disjoint subsets A, B of P such that $n = S(A) - S(B)$, where $S(X)$ denotes the sum of the members of the set X.

3. The columns in the table provide the representation of the numbers from 1 to 12 for the systems of enumeration discussed above. Of course, the entries in column A are decimal representations.

A	B	C	D	E	F
1	1	1	1	1	0
2	2	10	10	$1\bar{1}$	1
3	120	100	11	10	10
4	121	101	20	11	2
5	122	1000	21	$1\bar{1}\bar{1}$	100
6	110	1001	100	$1\bar{1}0$	11
7	111	1010	101	$1\bar{1}1$	1000
8	112	10000	110	$10\bar{1}$	3
9	100	10001	111	100	20
10	101	10010	120	101	101
11	102	10100	121	$11\bar{1}$	10000
12	220	10101	200	110	12

Binary numbering

4.

	16	8	4	2	1		32	16	8	4	2	1
0						32						
1						33						
2						34						
3						35						
4						36						
5						37						
6						38						
7						39						
8						40						
9						41						
10						42						
11						43						
12						44						
13						45						
14						46						
15						47						
16						48						
17						49						
18						50						
19	1	0	0	1	1	51						
20						52						
21						53						
22						54						
23						55						
24						56						
25						57						
26						58						
27						59						
28						60						
29						61						
30						62						
31						63						

5. Notice that the only digits needed to write the real numbers in base 6 are $0, 1, 2, 3, 4$ and 5.
   (a) Explain why, using the repeated subtraction method, it can never happen that a power of 6 is subtracted more than 5 times.
   (b) Explain why, using the repeated division method, it can never happen that the remainder is more than 5.
   (c) Explain why, using the repeated multiplication method, it can never happen that the integer part is more than 5.
6. Choose a four-digit base 6 number $\underline{abcd}_6$. Of course the digits a, b, c and d are all in the range $0, 1, 2, \ldots, 5$, and $a \neq 0$.
   (a) Interpret $abcd_6$ to get its decimal equivalent.
   (b) Next use repeated subtraction to find the base 6 representation of the decimal you obtained in part (a).
   (c) Finally, use repeated division on the number obtained in part (a) to get the base 6 representation in a different way.
7. For each of the integers in the first column, use repeated division or repeated subtraction to find the base 2, base 4 and base 8 representations.

n	base 2	base 4	base 8
104	1101000	1220	150
105			
106			
107			
108			
109			
110			
111			
112			

8. Devise a method to find the base 8 and base 4 representations of a number based on its binary (i.e., base 2) representation without converting first to decimal.
9. Devise a method to find the binary representation given its quartic (i.e., base 4) representation without converting first to decimal.
10. Devise a method to find the binary representation given its octal (i.e., base 8) representation without converting first to decimal.

11. Find nonzero digits a, b, c, and d such that $343 \cdot a + 49 \cdot b + 7 \cdot c + d = 2007$. Hint: Is the left side a sum of multiples of powers of 7?

12. In this problem, we explore representation of integers in a negative base. For convenience, we use $b = -6$. Use repeated division to find the base -6 representation of the number 2004. Division by -6 requires some extra care. The crucial observation is that the remainders must always be in the range 0 to 5, just as in the base 6 case. Next see if repeated subtraction works as it did for positive bases.

13. Choose a four-digit base 6 number $.\underline{abcd}_6$. Of course the digits a, b, c and d are all in the range $0, 1, 2, \ldots, 5$, and $a \neq 0$.

    (a) Interpret $.abcd_6$ to get its decimal equivalent.

    (b) Next use repeated subtraction to find the base 6 representation of the decimal you obtained in part (a).

    (c) Finally, use repeated multiplication on the number obtained in part (a) to get the base 6 representation in a different way.

14. Find nonnegative integers a, b, c, and d all less than 6 such that

$$\frac{a}{6} + \frac{b}{36} + \frac{c}{216} + \frac{d}{1296} = \frac{437}{1296}.$$

15. For each of the fractions in the first column, use repeated multiplication or repeated subtraction to find the base 2, base 4 and base 8 representations.

n	base 2	base 4	base 8
$\frac{1}{3}$	$.\overline{01}_2$	$0.\overline{1}_4$	$0.\overline{25}_8$
$\frac{1}{4}$			
$\frac{1}{5}$			
$\frac{2}{7}$			
$\frac{3}{8}$			
$\frac{6}{17}$			