

1. Chicken McNuggets can be purchased in quantities of 6, 9, and 20 pieces. You can buy exactly 15 pieces by purchasing a 6 and a 9, but you can't buy exactly 10 McNuggets. What is the largest number of McNuggets that can NOT be purchased?
2. How many base-10 three-digit numbers are also three digit numbers in both base-9 and base-11? (AMC 10)
3. Determine the base  $(-2)$  representation of 34.
4. (MA $\Theta$ ) Find the largest integer  $d$  for which there are no nonnegative integer solutions  $(a, b, c)$  which satisfy the equation  $5a + 7b + 11c = d$ .
5. On a true-false test of 100 items, every question that is a multiple of 4 is true, and all others are false. If a student marks every item that is a multiple of 3 false and all others true, how many of the 100 items will be correctly answered? (MathCounts)
6. How many whole numbers  $n$ , such that  $100 \leq n \leq 1000$ , have the same number of odd factors as even factors? (MathCounts)
7. Find the base  $(-4)$  representation of  $33\frac{1}{3}$ .
8. (ARML) For a positive integer  $n$ , let  $C(n)$  be the number of pairs of consecutive 1s in the binary representation of  $n$ . For example,  $C(183) = C(101101112) = 3$ . Compute  $C(1) + C(2) + C(3) + \dots + C(256)$ .
9. The integers 1 through 2010 are written on a white board. The integer 1 is erased. Every integer that is either 7 or 11 greater than an erased integer will be erased. At the end of the process what is the largest integer remaining on the board? (MathCounts)
10. Determine the last two digits of  $7^{7^7}$ .
11. (IMO) Find the smallest natural number  $n$  which has the following properties:
  - (a) Its decimal representation has 6 as the last digit.
  - (b) If the last digit 6 is erased and placed in front of the remaining digits, the resulting number is four times as large as the original number  $n$ .
12. To weigh an object by using a balance scale, Brady places the object on one side of the scale and places enough weights on each side to make the two sides of the scale balanced. Brady's set of weights contains the minimum number necessary to measure the whole-number weight of any object from 1 to 40

pounds, inclusive. What is the greatest weight, in pounds, of a weight in Brady's set? (MathCounts)

13. A game of solitaire is played as follows. After each play, according to the outcome, the player receives either  $a$  or  $b$  points, where  $a$  and  $b$  are positive integers such that  $a > b$ . The player's score accumulates from play to play. She notices that there are 35 unattainable scores and that one of these is 58. Find  $a$  and  $b$  as an ordered pair  $(a, b)$ . (Putnam)
14. Let  $S$  be a subset of  $\{1, 2, 3, \dots, 50\}$  such that no pair of distinct elements in  $S$  has a sum divisible by 7. What is the maximum number of elements in  $S$ ?
15. Find the sum of all the integers  $N > 1$  with the properties that each prime factor of  $N$  is either 2, 3, or 5, and  $N$  is not divisible by any perfect cube greater than 1.
16. Find the least positive integer  $n$  for which  $\frac{n-13}{5n+6}$  is a non-zero reducible fraction. (AHSME)
17. Let  $n$  be the smallest positive integer that is a multiple of 75 and has exactly 75 positive integer divisors, including 1 and itself. Find  $n/75$ . (AIME)
18. A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. For example, after traveling one mile the odometer changed from 000039 to 000050. If the odometer now reads 002005, how many miles has the car actually traveled? (AMC 12)
19. Begin with the 200-digit number 9876543210987...43210, which repeats the digits 0 – 9 in reverse order. From the left, choose every third digit to form a new number. Repeat the same process with the new number. Continue the process repeatedly until the result is a two-digit number. What is the resulting two-digit number? (MathCounts)
20. How many integers from 1 to 1992 inclusive have a base three representation that does not contain the digit 2? (Mandelbrot)
21. Find the remainder when  $3^3 \cdot 33^{33} \cdot 333^{333} \cdot 3333^{3333}$  is divided by 100. (Purple Comet)
22. (1992 AHSME 17) The two-digit integers from 19 to 92 are written consecutively to form the large integer

$$N = 192021 \dots 909192.$$

Suppose that  $3^k$  is the highest power of 3 that is a factor of  $N$ . What is  $k$ ?

23. Find the number of integers  $n, 1 \leq n \leq 25$  such that  $n^2 + 3n + 2$  is divisible by 6.
24. Prove that  $2^n + 6 \cdot 9^n$  is always divisible by 7 for any positive integer  $n$ .
25. The positive integers  $N$  and  $N^2$  both end in the same sequence of four digits  $abcd$  when written in base 10, where digit  $a$  is not zero. Find the three-digit number  $abc$ .
26. When  $30!$  is computed, it ends in 7 zeros. Find the digit that immediately precedes these zeros.
27. Let  $S$  be the set of integers between 1 and  $2^{40}$  whose binary expansions have exactly two 1's. If a number is chosen at random from  $S$ , the probability that it is divisible by 9 is  $p/q$ , where  $p$  and  $q$  are relatively prime positive integers. Find  $p + q$ .
28. Find the remainder when  $2^{2019}$  is divided by 29.
29. One of Euler's conjectures was disproved in the 1960s by three American mathematicians when they showed there was a positive integer such that  $133^5 + 110^5 + 84^5 + 27^5 = n^5$ . Find the value of  $n$ .
30. Let  $a_1 = a_2 = a_3 = 1$  and for all  $n > 3$ , let  $a_n = a_{n-3} + a_{n-2} + a_{n-1}$ . What is the units digit of  $a_{2019}$ .
31. Find the smallest positive integer  $n$  such that 11 divides  $2^n + 1$  and 13 divides  $3^n - 1$ .
32. What is the largest even integer that cannot be written as the sum of two odd composite numbers?