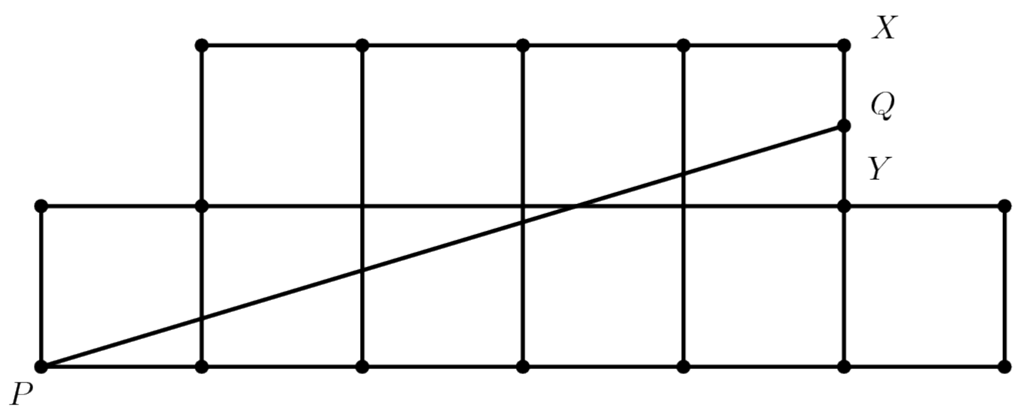
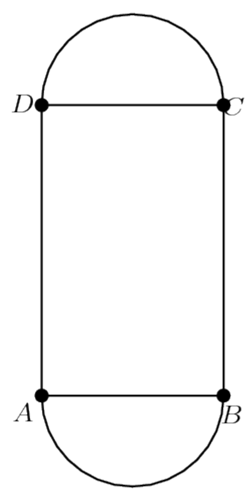
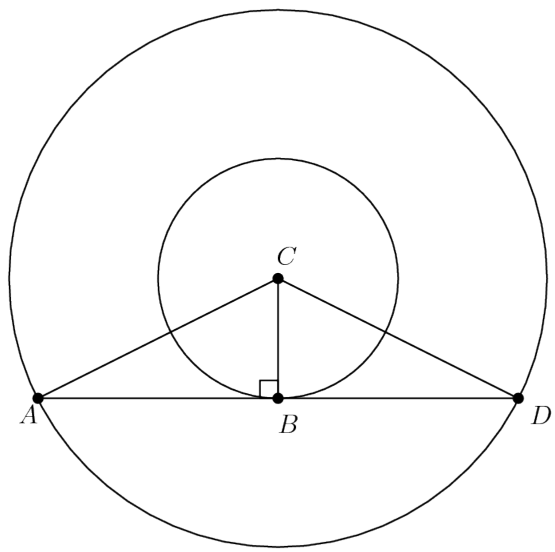
1. The diagram shows an octagon consisting of 10 unit squares. The portion below $\overline{PQ}$ is a unit square and a triangle with base 5. If $\overline{PQ}$ bisects the area of the octagon, what is the ratio $\frac{XQ}{QY}$?



2. A decorative window is made up of a rectangle with semicircles on either end. The ratio of AD to AB is $3:2$, and AB is 30 inches. What is the ratio of the area of the rectangle to the combined areas of the semicircles?



3. The two circles pictured have the same center C. Chord $\overline{AD}$ is tangent to the inner circle at B, AC is 10, and chord $\overline{AD}$ has length 16. What is the area between the two circles?

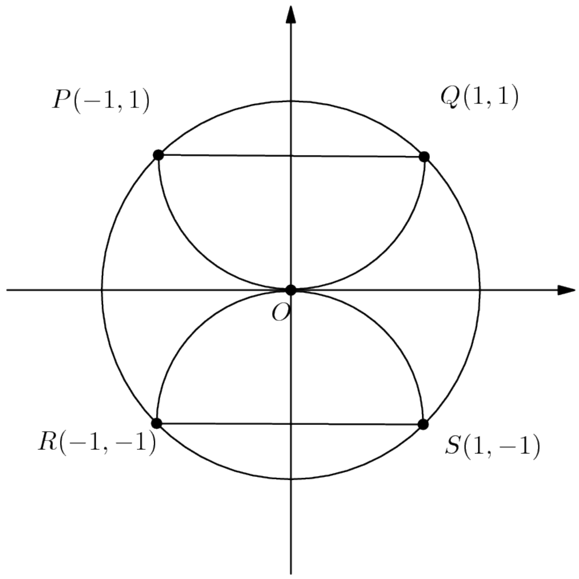


4. In a room, 2/5 of the people are wearing gloves, and 3/4 of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and a glove?

5. Hui is an avid reader. She bought a copy of the best seller Math is Beautiful. On the first day, Hui read 1/5 of the pages plus 12 more, and on the second day she read 1/4 of the remaining pages plus 15 pages. On the third day she read 1/3 of the remaining pages plus 18 pages. She then realized that there were only 62 pages left to read, which she read the next day. How many pages are in this book?

6. The hundreds digit of a three-digit number is 2 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the units digit of the result?

7. Semicircles POQ and ROS pass through the center of circle O. What is the ratio of the combined areas of the two semicircles to the area of circle O?



8. What is the correct ordering of the three numbers, 10^8, 5^{12}, and 2^{24}?

9. Everyday at school, Jo climbs a flight of 6 stairs. Jo can take the stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2. In how many ways can Jo climb the stairs?

10. Crystal has a running course marked out for her daily run. She starts this run by heading due north for one mile. She then runs northeast for one mile, then southeast for one mile. The last portion of her run takes her on a straight line back to where she started. How far, in miles, is this last portion of her run?

11. Tony works 2 hours a day and is paid \$0.50 per hour for each full year of his age. During a six month period Tony worked 50 days and earned \$630. How old was Tony at the end of the six month period?

12. A palindrome, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x + 32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?

13. Marvin had a birthday on Tuesday, May 27 in the leap year 2008. In what year will his birthday next fall on a Saturday?

14. The length of the interval of solutions of the inequality $a \leq 2x + 3 \leq b$ is 10. What is $b - a$?

15. Logan is constructing a scaled model of his town. The city's water tower stands 40 meters high, and the top portion is a sphere that holds 100,000 liters of water. Logan's miniature water tower holds 0.1 liters. How tall, in meters, should Logan make his tower?

16. Angelina drove at an average rate of 80 km/h and then stopped 20 minutes for gas. After the stop, she drove at an average rate of 100 km/h. Altogether she drove 250 km in a total trip time of 3 hours including the stop. Which equation could be used to solve for the time t in hours that she drove before her stop?

17. For a real number x, define $\heartsuit(x)$ to be the average of x and x^2. What is $\heartsuit(1) + \heartsuit(2) + \heartsuit(3)$?

18. A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days of the week could be the first day of this month?

19. A circle is centered at O, $\overline{AB}$ is a diameter and C is a point on the circle with $\angle COB = 50^\circ$. What is the degree measure of $\angle CAB$?

20. A triangle has side lengths 10, 10, and 12. A rectangle has width 4 and area equal to the area of the triangle. What is the perimeter of this rectangle?

21. A ticket to a school play cost x dollars, where x is a whole number. A group of 9th graders buys tickets costing a total of \$48, and a group of 10th graders buys tickets costing a total of \$64. How many values for x are possible?

22. Lucky Larry's teacher asked him to substitute numbers for a, b, c, d, and e in the expression $a-(b-(c-(d+e)))$ and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincidence. The numbers Larry substituted for a, b, c, and d were 1, 2, 3, and 4, respectively. What number did Larry substitute for e?

23. Shelby drives her scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?

24. A shopper plans to purchase an item that has a listed price greater than \$100 and can use any one of the three coupons. Coupon A gives 15% off the listed price, Coupon B gives \$30 off the listed price, and Coupon C gives 25% off the amount by which the listed price exceeds \$100. Let x and y be the smallest and largest prices, respectively, for which Coupon A saves at least as many dollars as Coupon B or Coupon C. What is $y-x$?

25. At the beginning of the school year, 50% of all students in Mr. Wells' math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, x% of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x?

26. What is the sum of all the solutions of $x=|2x-|60-2x||$?

27. The average of the numbers $1, 2, 3, \cdots, 98, 99$, and x is $100x$. What is x?

28. On a 50-question multiple choice math contest, students receive 4 points for a correct answer, 0 points for an answer left blank, and -1 point for an incorrect answer. Jesse's total score on the contest was 99. What is the maximum number of questions that Jesse could have answered correctly?

29. Mr. Earl E. Bird gets up every day at 8:00 AM to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time?

30. Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. What is the number of possible values of k?

31. Two different positive numbers a and b each differ from their reciprocals by 1. What is $a + b$?

32. The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. What is the largest integer that can be an element of this collection?

33. Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, ..., 10\}$. What is the probability that Sergio's number is larger than the sum of the two numbers chosen by Tina?

34. Several sets of prime numbers, such as $\{7, 83, 421, 659\}$ use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?

35. Suppose that a and b are digits, not both nine and not both zero, and the repeating decimal $0.\overline{ab}$ is expressed as a fraction in lowest terms. How many different denominators are possible?

36. Consider the sequence of numbers: $4, 7, 1, 8, 9, 7, 6, \ldots$ For $n > 2$, the n-th term of the sequence is the units digit of the sum of the two previous terms. Let S_n denote the sum of the first n terms of this sequence. What is the smallest value of n for which $S_n > 10,000$?

37. Compute the sum of all the roots of $(2x+3)(x-4)+(2x+3)(x-6) = 0$

38. The product of three consecutive positive integers is 8 times their sum. What is the sum of their squares?

39. How many values of x such that $8xy - 12y + 2x - 3 = 0$ is true for all values of y.

40. The number $25^{64} \cdot 64^{25}$ is the square of a positive integer N. In decimal representation, the sum of the digits of N is

41. The positive integers $A, B, A - B$, and $A + B$ are all prime numbers. What is the sum of these four primes?

42. For how many integers n is $\dfrac{n}{20-n}$ the square of an integer?

43. Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

44. Suppose that $\{a_n\}$ is an arithmetic sequence with

$$a_1 + a_2 + \cdots + a_{100} = 100 \text{ and } a_{101} + a_{102} + \cdots + a_{200} = 200.$$

What is the value of $a_2 - a_1$?

45. Let a, b, and c be real numbers such that $a-7b+8c = 4$ and $8a+4b-c = 7$. Then $a^2 - b^2 + c^2$ is

46. Andy's lawn has twice as much area as Beth's lawn and three times as much area as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower. If they all start to mow their lawns at the same time, who will finish first?

47. The digits $1, 2, 3, 4, 5, 6, 7, 8$, and 9 are to be written in the squares so that every row and every column with three numbers has a total of 13. Two numbers have already been entered. What is the number in the square marked ?