1. The number of inches in the perimeter of an equilateral triangle equals the number of square inches in the area of its circumscribed circle. What is the radius, in inches, of the circle?

2. What is the sum of the reciprocals of the roots of the equation $\frac{2003}{2004}x + 1 + \frac{1}{x} = 0$?

3. Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

4. Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

5. Let n be a 5-digit number, and let q and r be the quotient and the remainder, respectively, when n is divided by 100. For how many values of n is $q + r$ divisible by 11?

6. In simplest form, what is

$$\frac{2 - 4 + 6 - 8 + 10 - 12 + 14}{3 - 6 + 9 - 12 + 15 - 18 + 21}?$$

7. Al gets the disease algebritis and must take one green pill and one pink pill each day for two weeks. A green pill costs \$1 more than a pink pill, and Al's pills cost a total of \$546 for the two weeks. How much does one green pill cost?

8. The sum of 5 consecutive even integers is 4 less than the sum of the first 8 consecutive odd counting numbers. What is the smallest of the even integers?

9. The symbolism $\lfloor x \rfloor$ denotes the largest integer not exceeding x. For example, $\lfloor 3 \rfloor = 3$, and $\lfloor 9/2 \rfloor = 4$. Compute

$$\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \cdots + \lfloor \sqrt{16} \rfloor.$$

10. Find the value of x that satisfies the equation $25^{-2} = \frac{5^{48/x}}{5^{26/x} \cdot 25^{17/x}}$.

11. Nebraska, the home of the AMC, changed its license plate scheme. Each old license plate consisted of a letter followed by four digits. Each new license plate consists of the three letters followed by three digits. By how many times is the number of possible license plates increased?

12. A line with slope 3 intersects a line with slope 5 at point $(10, 15)$. What is the distance between the x-intercepts of these two lines?

13. Al, Betty, and Clare split \$1000 among them to be invested in different ways. Each begins with a different amount. At the end of one year, they have a total of \$1500 dollars. Betty and Clare have both doubled their money, whereas Al has managed to lose \$100 dollars. What was Al's original portion?

14. Let $\clubsuit(x)$ denote the sum of the digits of the positive integer x. For example, $\clubsuit(8) = 8$ and $\clubsuit(123) = 1+2+3 = 6$. For how many two-digit values of x is $\clubsuit(\clubsuit(x)) = 3$?

15. Given that $3^8 \cdot 5^2 = a^b$, where both a and b are positive integers, find the smallest possible value for $a + b$.

16. There are 100 players in a single tennis tournament. The tournament is single elimination, meaning that a player who loses a match is eliminated. In the first round, the strongest 28 players are given a bye, and the remaining 72 players are paired off to play. After each round, the remaining players play in the next round. The match continues until only one player remains unbeaten. The total number of matches played is divisible by which of the four primes 2, 5, 7 or 11?

17. A restaurant offers three desserts, and exactly twice as many appetizers as main courses. A dinner consists of an appetizer, a main course, and a dessert. What is the least number of main courses that a restaurant should offer so that a customer could have a different dinner each night in the year 2003?

18. What is the largest integer that is a divisor of

$$(n+1)(n+3)(n+5)(n+7)(n+9)$$

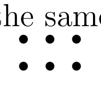
for all positive even integers n?

19. The first four terms in an arithmetic sequence are $x+y$, $x-y$, xy, and $\frac{x}{y}$, in that order. What is the fifth term?

20. How many distinct four-digit numbers are divisible by 3 and have 23 as their last two digits?

21. What is the value of x if $|x-1| = |x-2|$?

22. A set of three points is randomly chosen from the grid shown. Each three point set has the same probability of being chosen. What is the probability that the points lie on the same straight line?



23. Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and grand-daughters have no daughters?

24. A grocer stacks oranges in a pyramid-like stack whose rectangular base is 5 oranges by 8 oranges. Each orange above the first level rests in a pocket formed by four oranges below. The stack is completed by a single row of oranges. How many oranges are in the stack?

25. A game is played with tokens according to the following rule. In each round, the player with the most tokens gives one token to each of the other players and also places one token in the discard pile. The game ends when some player runs out of tokens. Players A, B, and C start with 15, 14, and 13 tokens, respectively. How many rounds will there be in the game?

26. Coin A is flipped three times and coin B is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?

27. A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

28. Henry's Hamburger Heaven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties, and any collection of condiments. How many different kinds of hamburgers can be ordered?

29. At a party, each man danced with exactly three women and each woman danced with exactly two men. Twelve men attended the party. How many women attended the party?

30. The average value of all the pennies, nickels, dimes, and quarters in Paula's purse is 20 cents. If she had one more quarter, the average value would be 21 cents. How many dimes does she have in her purse?

31. Given that $-4 \leq x \leq -2$ and $2 \leq y \leq 4$, what is the largest possible value of $\frac{x+y}{x}$?

32. Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

33. A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term in the geometric progression?

34. Let f be a function with the following properties:

    $(i)\quad f(1) = 1$, and $(ii)\quad f(2n) = n \times f(n)$, for any positive integer n. What is the value of $f(2^{100})$? Express your answer as a power of 2.

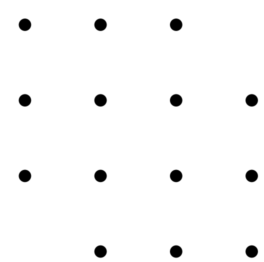
35. At each basketball practice last week, Jenny made twice as many free throws as she made at the previous practice. At her fifth practice she made 48 free throws. How many free throws did she make at the first practice?

36. A standard six-sided die is rolled, and P is the product of the five numbers that are visible. What is the largest number that is certain to divide P?

37. In the expression $c \cdot a^b - d$, the values of a, b, c, and d are 0, 1, 2, and 3, although not necessarily in that order. What is the maximum possible value of the result?

0.1 Bonus Problems

38. Two opposite corner dots from a 4×4 array have been removed, as shown in the diagram below. How many different squares are there all of whose vertices are among the 14 dots? What if you replace the two corner dots?



39. Consider all $a \times b \times c$ boxes where a, b and c are integers such that $1 \leq a, b, c \leq 5$. An $a_1 \times b_1 \times c_1$ box fits inside an $a_2 \times b_2 \times c_2$ box if and only if $a_1 \leq a_2,\ b_1 \leq b_2,\ c_1 \leq c_2$. Determine the largest number of the boxes under consideration such that none of them fits inside another subject to

    (a) the condition $1 \leq a \leq b \leq c \leq 5$.

    (b) no additional condition.

    (c) Consider the planar version of the problem with rectangles.

40. On a circle, there are 100 blue points and 1 red point. Rahul counts the number of convex polygons that can be drawn by joining only blue vertices. Anish counts the number of convex polygons which include the red point among its vertices. What is the positive difference between Rahul's number and Anish's number?