

PROBLEM DEPARTMENT

ASHLEY AHLIN* AND HAROLD REITER†

This department welcomes problems believed to be new and at a level appropriate for the readers of this journal. Old problems displaying novel and elegant methods of solution are also invited. Proposals should be accompanied by solutions if available and by any information that will assist the editor. An asterisk () preceding a problem number indicates that the proposer did not submit a solution.*

All correspondence should be addressed to Harold Reiter, Department of Mathematics, University of North Carolina Charlotte, 9201 University City Boulevard, Charlotte, NC 28223-0001 or sent by email to hbreiter@email.uncc.edu. Electronic submissions using L^AT_EX are encouraged. Other electronic submissions are also encouraged. Please submit each proposal and solution preferably typed or clearly written on a separate sheet (one side only) properly identified with name, affiliation, and address. Solutions to problems in this issue should be mailed to arrive by February 1, 2006. Solutions identified as by students are given preference.

Problems for Solution.

1109. *Proposed by Cecil Rousseau, University of Memphis, Memphis, TN*

- (a) Determine the value of $1 - \frac{2^5}{2!} + \frac{3^5}{3!} - \frac{4^5}{4!} + \dots$.
- (b) Generalize to

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^m}{n!}.$$

The sum of the generalized series can be expressed in terms of m as a finite sum of known numbers.

1110. *Proposed by David Wells, Penn State New Kensington, Upper Burrell, PA*

Let $p(x) = ax^2 + bx + c$ where a, b , and c are real numbers. Let $q(x) = p(p(x))$. Find necessary and sufficient conditions on a, b , and c such that the number of zeros of q is none, one, two, three, and four.

1111. *Proposed by Paul S. Bruckman, Sointula, BC, Canada*

- (a) Let p be any odd prime and $S(m) = \sum_{k=1}^p k^m$, $m = 1, 2, \dots$. Prove that $p|S(m)$ for any $m = 1, 2, \dots, p - 2, p$, and that $S(p - 1) \equiv -1 \pmod{p}$.
- (b) Let $\sigma(m)$ denote the elementary symmetric function of order m , on the numbers $1, 2, \dots, p$. Prove that $p|\sigma(m)$, $m = 1, 2, \dots, p - 2, p$, and $\sigma(p - 1) \equiv -1 \pmod{p}$.

1112. *Proposed by Harry Sedinger, St. Bonaventure University, St. Bonaventure, NY*

Without using a calculator or computer, determine whether or not the positive integer $9753^{2468} + 3579^{8642} + 9357^{2468} + 9573^{8642}$ is a perfect square.

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1122. *Proposed by Arthur Holshouser, Charlotte, NC*

- A. (S, \cdot) is a binary operator that satisfies $\forall x, y \in S, (xy)x = y$. This is what Donald Knuth calls a grope.
- (a) Is it provable that $\forall a, b \in S, \exists a$ unique $x \in S$ such that $xa = b$?
 - (b) Is it provable that $\forall a, b \in S, \exists a$ unique $x \in S$ such that $ax = b$?
- B. Now suppose that (\bar{S}, \cdot) satisfies $\forall x, y \in \bar{S}, (xy)y = x$. Answer (a) and (b) for (\bar{S}, \cdot) also.